

# Final Review

Jacob M. Montgomery

Quantitative Political Methodology

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- ▶ Poster session

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- ▶ 1% Extra credit for completing.

# Exam checklist

- ▶ Peer evaluation form
- ▶ Pencils/erasers
- ▶ One sheet of paper
- ▶ Calculator with memory cleared

# Overview of today

- ▶ Big picture overview of the class
- ▶ What test do I use here again?
- ▶ Questions?
- ▶ In-class review problems (answers will be posted to BB)



What is statistics?

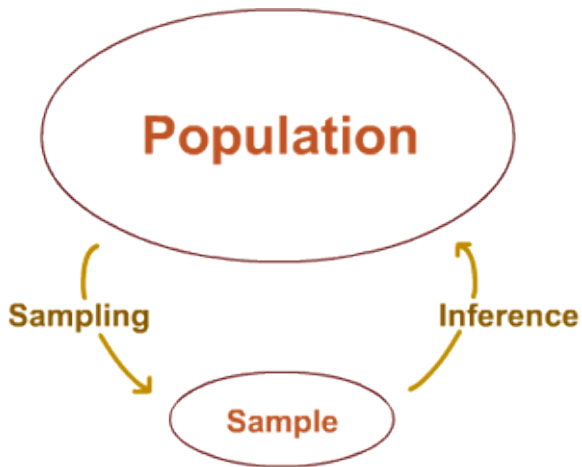
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## What is statistics?

*A body of methods for collecting and analyzing data.*

Let's break that down.

- ▶ **Research design:** Gathering data that will allow us to answer research questions by testing empirical hypotheses.
- ▶ **Description:** Summarizing the *data*.
- ▶ **Inference:** Using data to make (probabilistic) statements about the *real world*. Testing our hypotheses.



# (1) What kind of data are we gathering?

<b>Quantitative</b>		<b>Qualitative (categorical)</b>	
<i>Continuous</i>	<i>Discrete</i>	<i>Continuous</i>	<i>Discrete</i>

Interval

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Interval	ex., Income		

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Nominal	NA	NA	NA	

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(2) Now that we have this data, how can we summarize it?

Sample mean

$$\bar{y} = \frac{y_1 + y_2 + \dots + y_n}{n} = \frac{1}{n} \sum_{i=1}^n y_i$$

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Median

*The observation that falls in the center of an ordered sample.*

Mode

*The most frequently occurring value*

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Range

$$(\max(y) - \min(y)) \equiv |\min(y) - \max(y)|$$

Standard deviation

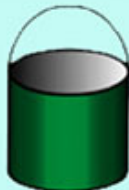
$$S = \sqrt{S^2} = \sqrt{\frac{\sum_{i=1}^n (y_i - \bar{y})^2}{n - 1}}$$

Interquartile range (IQR)

$75^{th}$  percentile -  $25^{th}$  percentile



Probability: Given the information in the pail, what is in your hand?



Statistics: Given the information in your hand, what is in the pail?

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- ▶  $\hat{\beta}, \hat{\alpha}, F, \chi^2, \bar{y}_2 - \bar{y}_1$

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We want to say something useful about the unknown population parameter. One way to do this is to set up a confidence interval.

Point estimation

*A sample statistic that gives a good guess about a population parameter.*

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## Confidence coefficient

*The probability that an interval would contain the parameter with repeated sampling.*

This is calculated as:  $\text{Point Estimate} \pm (Z \text{ or } T) \times \text{Standard error}$

(4) Now that we have these statistics, how can we make inferences? Hypothesis tests

We calculate statistics with known sampling distributions to:

- ▶ Make systematic statements about population parameters for a single population.
- ▶ Make systematic statements about how population parameters are related to explanatory variables

(5) Now that we have these statistics, how can we make inferences? Hypothesis tests

Five steps of hypothesis testing:

1. Make/state assumptions about our data
2. Formulate null and alternative hypotheses
3. Calculate the appropriate test statistic
4. Calculate the p-value
5. Draw a conclusion

P-value

*A P-Value is a measure of surprise.*



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## P-value

*A P-Value is a measure of surprise. We ask, "If the null hypothesis is true, how likely is it that we would observe a test-statistic this extreme **or more?**"*

## (6) Using the appropriate test for your data

Resp. Variable	Explanatory Variables	Measure	Statistical Test(s)	Sample Size
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3	$\frac{\hat{\pi} - \pi_0}{\sqrt{\frac{\pi_0(1-\pi_0)}{n}}}$	Standard normal (Z)

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4	$\frac{(\bar{y}_2 - \bar{y}_1) - 0}{\sqrt{\frac{S_2^2}{n_2} + \frac{S_1^2}{n_1}}}$	Standard normal (Z)

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4	$\frac{(\bar{y}_2 - \bar{y}_1) - 0}{\sqrt{\frac{S_2^2}{n_2} + \frac{S_1^2}{n_1}}}$	Standard normal (Z)
5	$\frac{(\bar{y}_2 - \bar{y}_1) - 0}{\hat{\sigma} \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$	t-dist, $df = n - 2$
$\hat{\sigma} = \sqrt{\frac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{n_1 + n_2 - 2}}$		

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6	$\chi^2 = \sum \frac{(f_0 - f_e)^2}{f_e}$	$\chi^2$ , $df = (rows - 1)(cols - 1)$
7	F-statistic	$F$ , $df_1 = k$ , $df_2 = n - (k + 1)$

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8 (bivariate)	$\hat{\beta} = \frac{\frac{\hat{\sigma}}{\sqrt{(X_i - \bar{X})^2}} \sum_{i=1}^n (X_i - \bar{X})(Y_i - \bar{Y})}{\sum_{i=1}^n (X_i - \bar{X})^2}$ $\hat{\sigma} = \sqrt{\frac{SSE}{n-2}} = \sqrt{\frac{\sum (Y_i - \hat{Y}_i)^2}{n-2}}$	<i>t - dist, df = n - 2</i>

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	$\hat{\alpha} = \frac{\bar{Y} - \hat{\beta}\bar{X}}{\hat{\sigma}_{\hat{\alpha}}}$	$t - dist, df = n - 2$
9 (multivariate)	$\frac{\hat{\beta}}{\hat{\sigma}_{\hat{\beta}}}$	$t - dist, df = n - (k + 1)$
	$\frac{\hat{\alpha}}{\hat{\sigma}_{\hat{\alpha}}}$	$t - dist, df = n - (k + 1)$

## Using regression to establish causality

1. Experiments (Dummies)
2. Statistical control
3. Statistical control (Fixed effects dummies)
4. Difference-in-differences (Dummies with interactions)
5. Regression discontinuity
6. Instrumental variables

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  4. rhetoric,
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