Lecture 5: Sampling Distributions

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Quantitative Political Methodology

Lecture 5

Class business

- Problem set 2 is available via the syllabus.
- Groups will be assigned this week.

Roadmap

Last time:

- Understand core concepts of probability
- Understand concept of a “parameter”
- Introduce some probability distributions

This time:

- Understanding the concept of a sampling distribution
- Understand the concept of a standard error
- See how the CLT allows us to know the distribution of certain sample statistics

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We use probability theory to derive a **distribution for a statistic**, which allows us (eventually) to make inferences about **population parameters**.

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- As $n \rightarrow \infty$, the standard error is going to get smaller and smaller.
- **This** is why the normal distribution is very important.
- Usually n=30 is “good enough”, but it will depend on the distribution.

It works for EVERYTHING

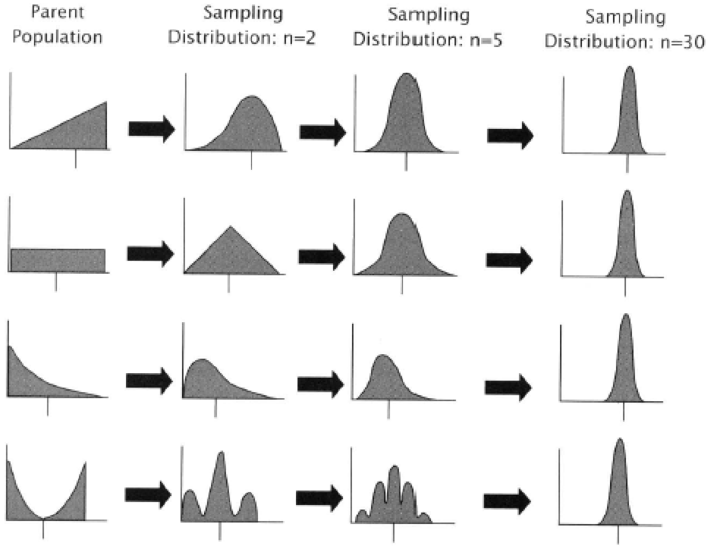
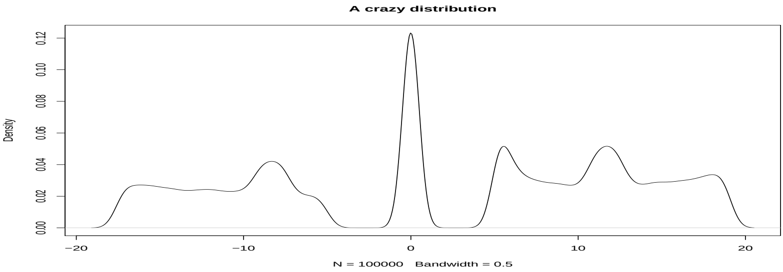


FIGURE 4.33 Given sufficient sample size the sampling distribution of the mean approaches normal shape irrespective of the variable's distributional shape.

Let's do our own experiment in R

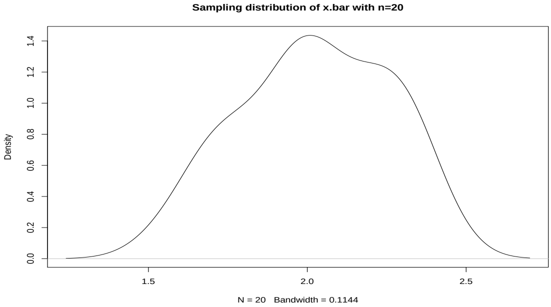
```r
x<-runif(100000,min = -1, max=1)
## Crazy transformation of the data
x<-sqrt(1+x)+2*x^3-23*x+abs(log(abs(x)))+2*(x>.5)+-2*(x< -.5)
## Adding a big whole in the middle with a point mass at zero
x[x>-5&x<5]<-0
plot(density(x, bw = .5), main="A crazy distribution")
```



```r
# We are going to take 1000 random samples
n.samples<-1000
# The sample size is 20
sample.size<-20
# We are going to use something called a "for loop"
# First we make an empty vector to store all of our
# sample statistics

# Create a vector filled with NA (missing data)
# vector is of length 20
x.bars<-rep(NA, sample.size)
# Now we are going to "loop" over the vector 1, 2, ..., 20
# in each iteration the variable "i" will increment up on value
for(i in 1:sample.size){
  # Draw a random sample
  this.sample<-sample(x, size = n.samples, replace=F)
  # Calculate the mean and add it to the vector
  x.bars[i]<-mean(this.sample)
}
```

```r
plot(density(x.bars),
     main="Sampling distribution of x.bar with n=20")
```

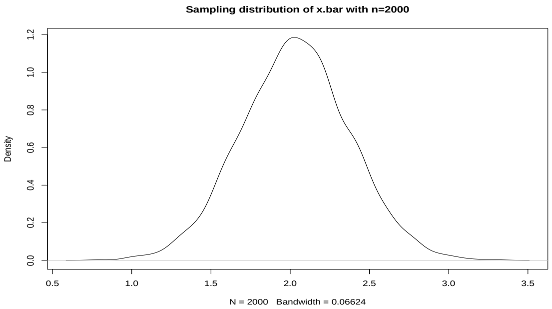


Now with a larger sample size

```
# We are going to take 1000 random samples
n.samples<-1000
# The sample size is 2000
sample.size<-2000
# We are going to use something called a "for loop"
# First we make an empty vector to store all of our
# sample statistics

# Create a vector filled with NA (missing data)
# vector is of length 2000
x.bars<-rep(NA, sample.size)
# Now we are going to "loop" over the vector 1, 2, ..., 2000
# in each iteration the variable "i" will increment up on value
for(i in 1:sample.size){
  # Draw a random sample
  this.sample<-sample(x, size = n.samples, replace=F)
  # Calculate the mean and add it to the vector
  x.bars[i]<-mean(this.sample)
}
```

```r
plot(density(x.bars),
     main="Sampling distribution of x.bar with n=2000")
```



Sampling Distribution of $\bar{y}$

A key common statistic is $\bar{y} = \frac{1}{n}\sum y_i$ for a single sample, where n is the sample size. How is this statistic distributed?

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A key common statistic is $\bar{y} = \frac{1}{n} \sum y_i$ for a single sample, where n is the sample size. How is this statistic distributed? The mean of the distribution is known to be μ (the population mean). What about the spread?

Standard error

The standard deviation of the sampling distribution of $\bar{y}$, denoted $\sigma_{\bar{y}}$, is called the standard error of $\bar{y}$, and is equal to $\dfrac{\sigma}{\sqrt{n}}$.

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Under certain circumstances we can safely assume that $\bar{y} \sim N(\mu, \frac{\sigma}{\sqrt{n}})$.

Class business

- Next class we are going to learn how to calculate probabilities for a known distribution
- Then we pull it all together to make our first true inference