Lecture 5: Sampling Distributions

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Quantitative Political Methodology

Lecture 5

Class business

- Problem set 2 will be distributed today via the syllabus
- Groups will be assigned this week.

Roadmap

Last time:

- Understand core concepts of probability
- Understand concept of a "parameter"
- Introduce some probability distributions

This time:

- Understanding the concept of a sampling distribution
- Understand the concept of a standard error
- See how the CLT allows us to know the distribution of certain sample statistics.

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We use probability theory to derive a **distribution for a statistic**, which allows us (eventually) to make inferences about **population parameters**.

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It works for EVERYTHING

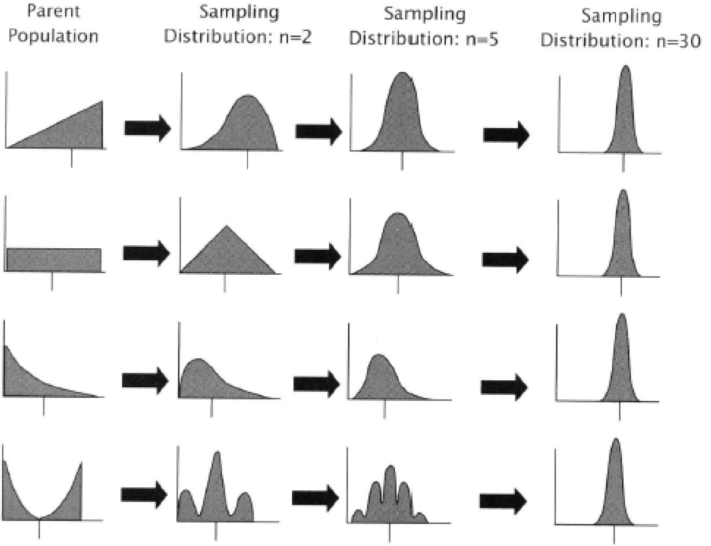
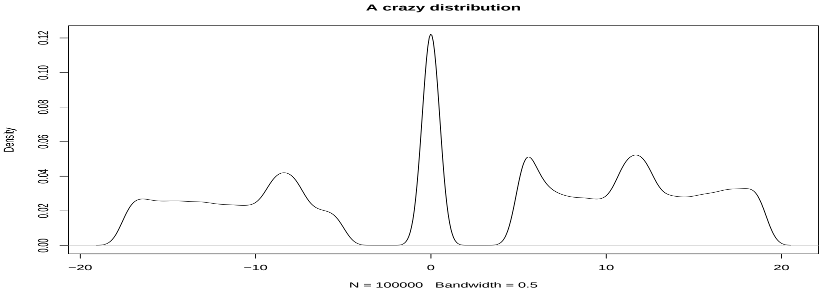


FIGURE 4.33 Given sufficient sample size the sampling distribution of the mean approaches normal shape irrespective of the variable's distributional shape.

Let's do our own experiment in R

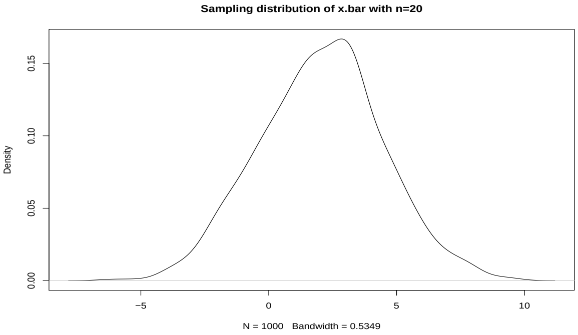
```
x<-runif(100000,min = -1, max=1)
## Crazy transformation of the data
x<-sqrt(1+x)+2*x^3-23*x+abs(log(abs(x)))+2*(x>.5)+-2*(x< -.5)
## Adding a big whole in the middle with a point mass at zero
x[x>-5&x<5]<-0
plot(density(x, bw = .5), main="A crazy distribution")
```



```r
# We are going to take 500 random samples
n.samples<-1000
# The sample size is 20
sample.size<-20
# We are going to use something called a "for loop"
# First we make an empty vector to store all of our
# sample statistics

# Create a vector filled with NA (missing data)
# vector is of length 200
x.bars<-rep(NA, n.samples)
# Now we are going to "loop" over the vector 1, 2, ..., 200
# in each iteration the variable "i" will increment up on value
for(i in 1:n.samples){
  # Draw a random sample
  this.sample<-sample(x, size = sample.size, replace=F)
  # Calculate the mean and add it to the vector
  x.bars[i]<-mean(this.sample)
}
```

```r
plot(density(x.bars),
     main="Sampling distribution of x.bar with n=20")
```



Now with a larger sample size

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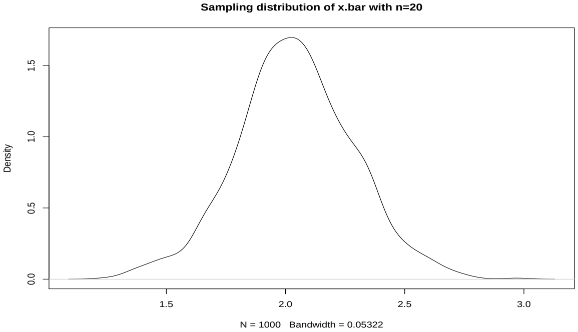

Sampling Distribution of $\bar{y}$

A key common statistic is $\bar{y} = \frac{1}{n} \sum y_i$ for a single sample, where n is the sample size. How is this statistic distributed?

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A key common statistic is $\bar{y} = \dfrac{1}{n}\sum y_i$ for a single sample, where n is the sample size. How is this statistic distributed? The mean of the distribution is known to be μ (the population mean). What about the spread?

Standard error

The standard deviation of the sampling distribution of $\bar{y}$, denoted $\sigma_{\bar{y}}$, is called the standard error of $\bar{y}$, and is equal to $\frac{\sigma}{\sqrt{n}}$.

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Under certain circumstances we can safely assume that $\bar{y} \sim N(\mu, \frac{\sigma}{\sqrt{n}})$.

Class business

- Next class we are going to learn how to calculate probabilities for a known distribution
- Then we pull it all together to make our first true inference