Lecture 13: Contingency Tables

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Quantitative Political Methodology

Contingency tables

Roadmap

- **Before**: Comparing two independent samples
- **Today** we will learn how to see if two variables are dependent.
  - How to display the data informatively
  - Chi-squared test of independence
  - Standardized residuals

Comparing populations with categorical outcomes

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Variable 1 (Outcome or response)	Variable 2 (Explanatory or grouping)
1	1
2	0
3	1
5	2
3	2
2	0
4	0
$\vdots$	$\vdots$

Cross-tabs: The basics

Assume we have two variables that are nominal.

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We will use a contingency table, which is usually (at least by me) referred to as a cross-tabulation.

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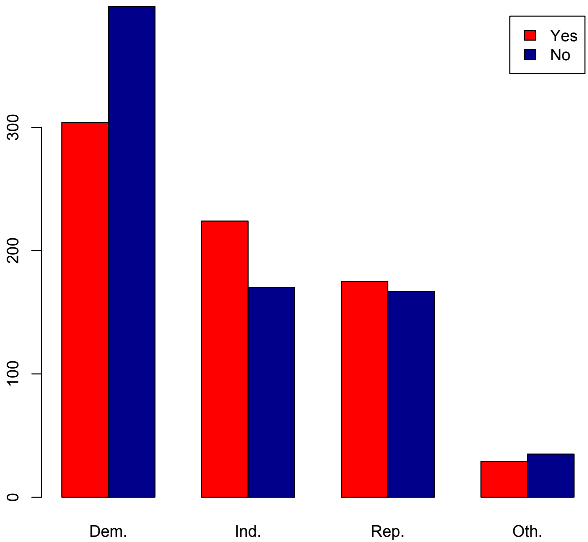
	Yes	No	Total
Democrats	304	398	702
Independents	224	170	394
Republicans	175	167	342
Other	29	35	64
Total	732	770	1502

Plot of data: 1972 GSS



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H_0: The variables are statistically independent.

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If H_0 is true, then we would expect $f_{observed} = f_{expected}$

Chi-square statistic: The calculations

$$\chi^2 = \sum \frac{(f_0 - f_e)^2}{f_e}$$

Chi-square test: Example

"Please tell me whether or not you think it should be possible for a pregnant woman to obtain a legal abortion if the family has a very low income and cannot afford any more children?"

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	$f_e = 192.12$	$f_e = 201.98$	
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$$\chi^2 = \sum \frac{(f_o - f_e)^2}{f_e} = \frac{(304 - 342.12)^2}{342.12} + \frac{(398 - 359.88)^2}{359.88} + \ldots \approx 19.79$$

Calculating p-values for Chi-squared tests

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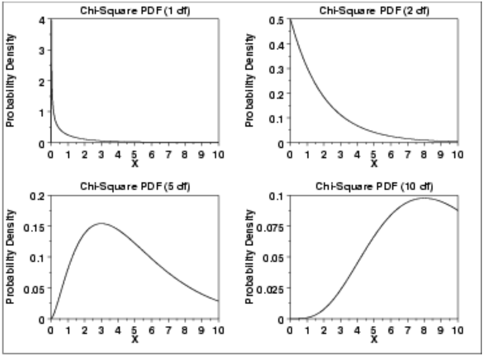

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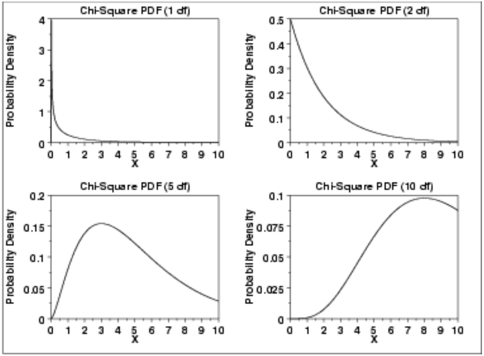

- We are going to need to calculate the degrees of freedom.
- This is skewed right and strictly positive.
- $\sum Z^2 \sim \chi^2$
- Always use the upper-tail (no $\times$ 2).

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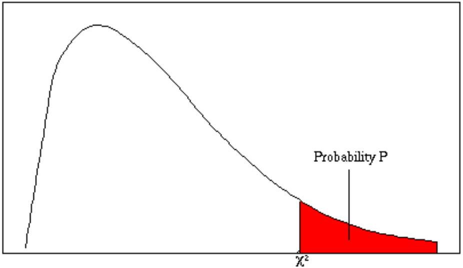

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p-value = `pchisq(19.78999, df=3, lower.tail=F) = 0.00019`

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- We need to find the **adjusted residual**:

$$z = \frac{f_{observe} - f_{expected}}{se} = \frac{f_{observe} - f_{expected}}{\sqrt{f_e(1 - \text{row prop.})(1 - \text{column prop.})}}$$

- The denominator is the standard error of the quantity $f_o - f_e$ under the null hypothesis

Example: Calculating standardized residuals

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$$z_{11} = \frac{304 - 342.12}{\sqrt{342.12(1 - \frac{702}{1502})(1 - \frac{732}{1502})}} \approx -2.395$$

2008 GSS

"Please tell me whether or not you think it should be possible for a pregnant woman to obtain a legal abortion if the family has a very low income and cannot afford any more children?""

	Yes	No	Total
Democrats	222	225	447
Independents	201	277	478
Republicans	113	223	336
Other	18	12	30
Total	554	737	1291

Challenging jurors

- North Carolina Racial Justice Act of 2009
- Act specifically identified the kind of evidence that could be considered
- Defendant must prove that race was a significant factor in the imposition of the death penalty
- The evidence before you is exactly what was used to commute the sentence of Marcus Raymond Robinson
- Repealed by legislator in 2012