Inference for regression

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Quantitative Political Methodology (L32 363)

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Overview

Last time:

- How to think about regression

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- How to think about regression
- "Best" parameters for drawing a line through data

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- Conditional variance
- Confidence intervals and hypothesis testing with regression
- Reading regression tables

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$$\hat{\alpha} = \bar{Y} - \hat{\beta}\bar{X}$$

Just one more statistic

Conditional standard deviation

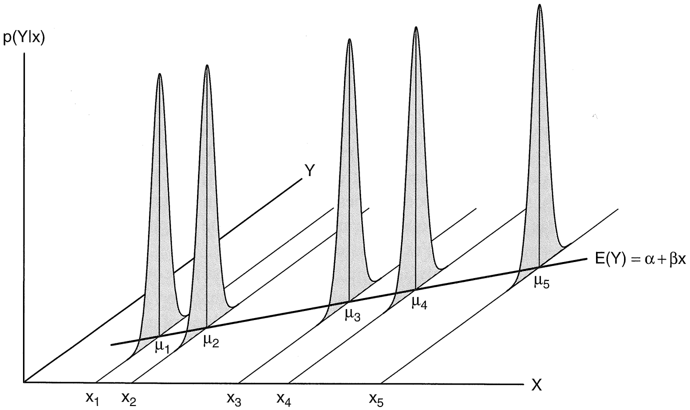
$$\hat{\sigma} = \sqrt{\frac{SSE}{n-2}} = \sqrt{\frac{\sum(Y_i - \hat{Y}_i)^2}{n-2}}$$

Unconditional standard deviation

$$\hat{\sigma}_Y = \sqrt{\frac{\sum(Y_i - \bar{Y})^2}{n-1}}$$

Conditional variance will be smaller. Why is there a difference?

Conditional variance gives a measure of spread of the residuals.



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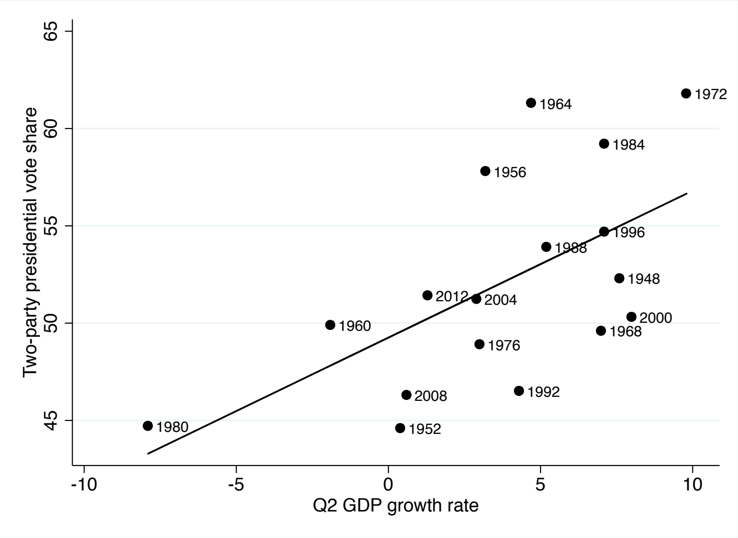

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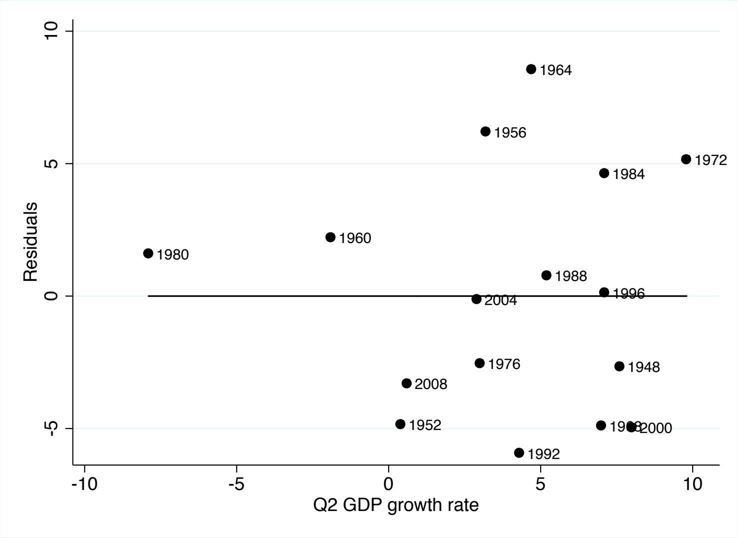

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Hypothesis testing:

1. Stating assumptions
2. Specifying hypotheses
3. Calculating a test statistic
4. Calculating a p-value
5. Drawing conclusions

Assumptions

- Random sample from population

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State hypotheses

We have parameter estimates $(\hat{\beta}, \hat{\alpha})$. So we can make some hypotheses.

$$H_0 : \beta = 0$$
$$H_a : \beta \neq 0$$

Or, for the intercept:

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What do all of these mean?

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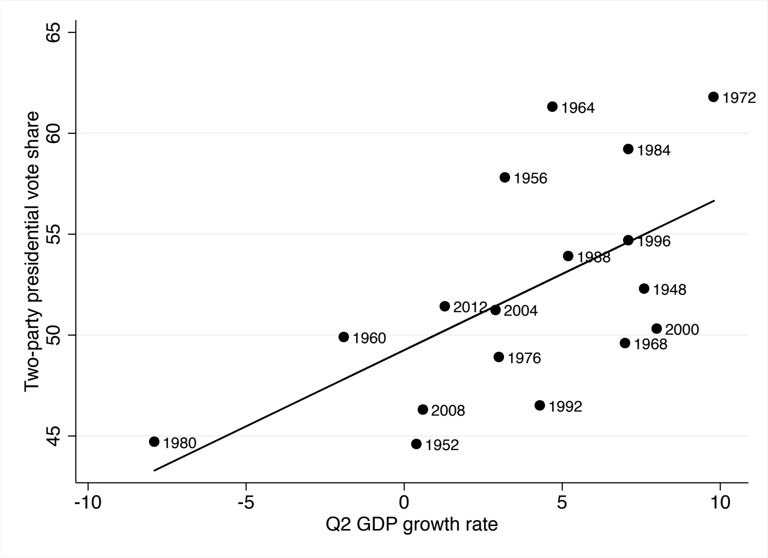
GDP Growth and Presidential Elections



R output

```
Call:
lm(formula = vote ~ q2gdp, data = Abram)

Residuals:
   Min     1Q Median     3Q    Max
-6.002 -3.409  0.084  2.078  8.496

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  49.2560     1.4411  34.179 1.21e-15 ***
q2gdp         0.7549     0.2578   ?  ?
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 4.481 on 15 degrees of freedom
Multiple R-squared:  0.3637,Adjusted R-squared:  0.3213
F-statistic: 8.573 on 1 and 15 DF,  p-value: 0.01039
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With your team, calculate the p-value for q2gdp.

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summary(lm(formula = vote ~ q2gdp, data = Abram))

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---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

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- Not all tables will include t-statistics