Lecture 12: T-Test

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Quantitative Political Methodology

Difference in means

Roadmap

Last class

- Now we want to look at two variables
- Our specific aim is to understand if X causes Y

This class:

- Two sample hypothesis tests (large sample)
- Two sample hypothesis tests (small sample)

Midterms

Midterms will be given back at the end of the class

- 34 A's
- 11 A-
- 5 B+
- 2 B's
- 1 Lower

Class business

- PS 4 is due on Wednesday
- Three groups have already scheduled meetings with me on topics. Get on it.

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  - Do women get paid less than men?
  - Do ideological extremists vote at higher rates than moderates?
  - This also allows us to estimate ATE
- Today we will learn how to compare two **independent samples** to estimate the ATE.
  - Large samples
  - Small samples (pooled variance)

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Variable 1 (Outcome or response)	Variable 2 (Explanatory or grouping)
2.1	1
2.4	0
3.0	0
1.79	1
$\vdots$	$\vdots$

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What we want to do (first) is construct a confidence interval for $\mu_2 - \mu_1$. We want to estimate this *difference*.

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Just as before, we use this test statistic to calculate a p-value.

Is this our political world?

We are less extreme than (many of us) think we are

TABLE 1 **Liberal and Conservative Positions, as Perceived by Respondents in Study 1**

	Role of Government		Environmentalism	
	Liberals	**Conservatives**	**Liberals**	**Conservatives**
Actual mean position	3.71 (0.08) (n=444)	5.34 (0.08) (n=342)	3.55 (0.07) (n=342)	4.74 (0.08) (n=445)
Mean estimate by liberal respondents	3.45** (0.08) (n=434)	5.52** (0.08) (n=434)	3.32** (0.07) (n=436)	5.39*** (0.07) (n=435)
Mean estimate by conservative respondents	2.74*** (0.10) (n=305)	5.64** (0.08) (n=306)	2.81*** (0.10) (n=305)	5.12*** (0.08) (n=306)
Mean estimate by moderate respondents	3.63 (0.08) (n=431)	5.16** (0.08) (n=431)	3.60 (0.08) (n=432)	4.89* (0.08) (n=432)

Note: Standard errors in parentheses. Two-sided *t*-tests of the hypothesis that the mean estimate equals the actual mean position. Asterisks denote levels of statistical significance: $^{*}p < .10$; $^{**}p < .05$; $^{***}p < .001$.

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Experimental design:

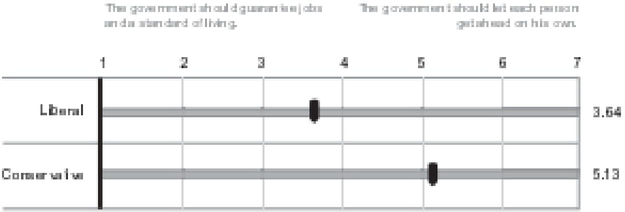
- *Ask*: Individuals are asked to estimate the political views of conservative and liberal voters and then asked about their own political views.
- *Tell*: Individuals are told the political views of conservative and liberal voters and then asked about their own political views.
- *Distort*: Individuals are told that the political views of conservative and liberal voters are *more* polarized than they are and then asked about their own political views.

Source: Douglas J. Ahler. 2014. "Self-Fulfilling Misperceptions of Public Polarization." *Journal of Politics* 76 (3): 607-620.

TELL

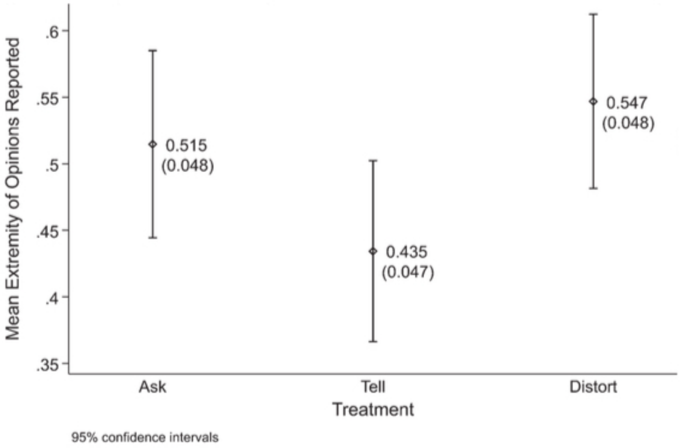
Respondents to a recent national survey were asked the following question: "Some people feel the government in Washington should see to it that every person has a job and a good standard of living. Suppose these people are at one end of a scale, at point 1. Others think the government should just let each person get ahead on their own. Suppose these people are at the other end, at point 7. And, of course, some other people have opinions somewhere in between, at points 2, 3, 4, 5, or 6. Where would you place yourself on this scale?"

The average positions taken by people who call themselves "liberal" and people who call themselves "conservative" are shown below:



Results

Figure 3 Mean Extremity of Political Opinion Reported by Study 2 Participants, by Experimental Condition



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We are going to do the exact same thing as we did before except that:

- We are going to use the t-distribution instead of the normal (recalling that we have to use degrees of freedom)
- We are going to use a slightly different calculation for standard errors.

Comparing groups with small samples

We will calculate the standard error as:

$$\hat{\sigma}_{\bar{y_2}-\bar{y_1}} = \sqrt{\frac{\hat{\sigma}^2}{n_1} + \frac{\hat{\sigma}^2}{n_2}} = \hat{\sigma}\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

where $\hat{\sigma}^2$ is the **pooled variance** calculate as:

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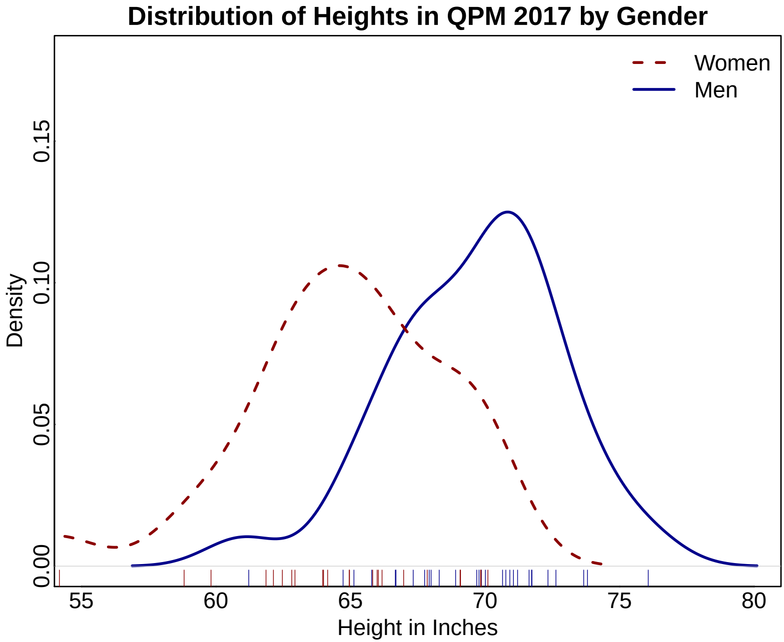

Distribution of Heights in QPM 2017 by Gender
Women
Men
Density
0.00
0.05
0.10
0.15
55
60
65
70
75
80
Height in Inches

Example: Height of men and women WashU

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