

Difference in Differences

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Quantitative Political Methodology

Causality in Regression and Difference in Differences

Road map

Where we have been:

- ▶ What is regression?
- ▶ How to interpret coefficients?
- ▶ Interactions/Dummies

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- ▶ What is regression?
- ▶ How to interpret coefficients?
- ▶ Interactions/Dummies

Today:

- ▶ Using regression for causal inference
- ▶ Using difference-in-differences to make causal claims

Causal inference

- ▶ We will use T to represent a treatment variable.
- ▶ For a categorical treatment

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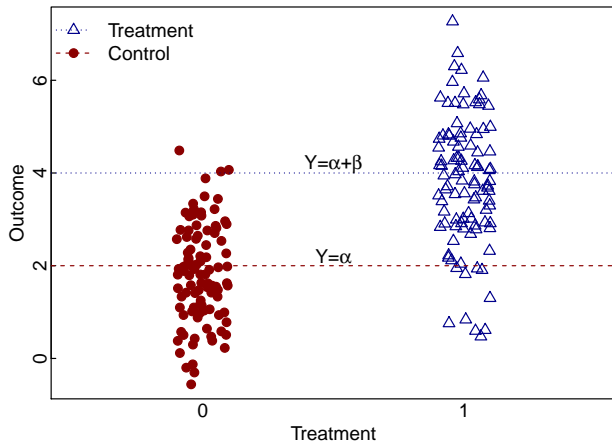
- ▶ We let y_i^1 represent the outcome of the i th unit if the treatment is given.
- ▶ We let y_i^0 represent the outcome of the i th unit if the control is given.
- ▶ One of these is observed, the other is the counterfactual – what would have been observed if the other treatment had been given?

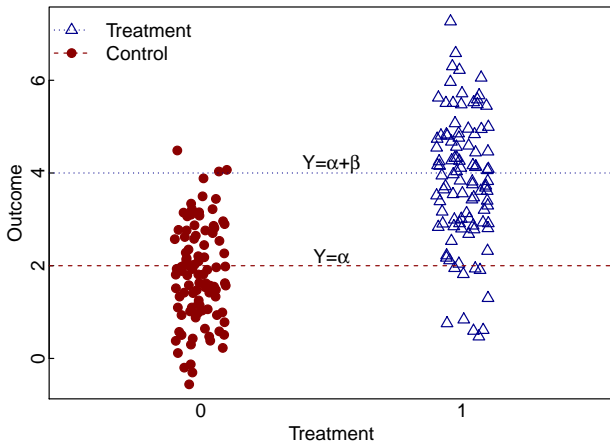
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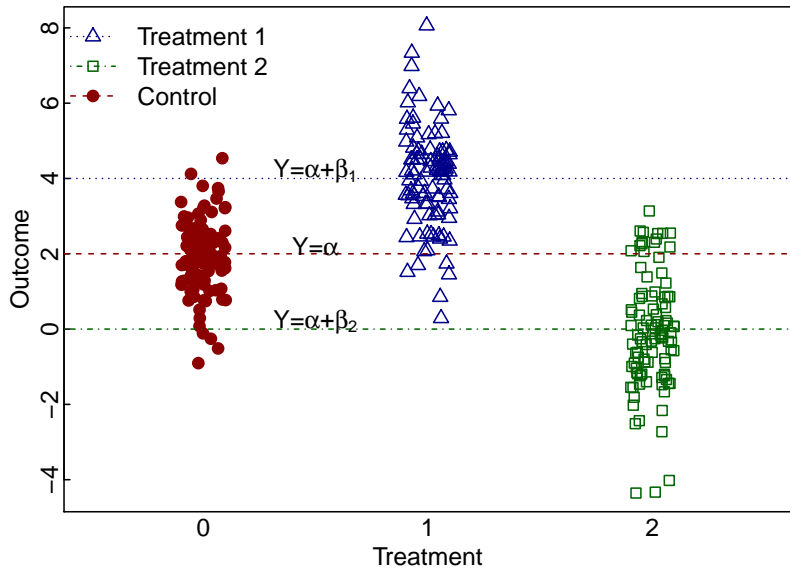
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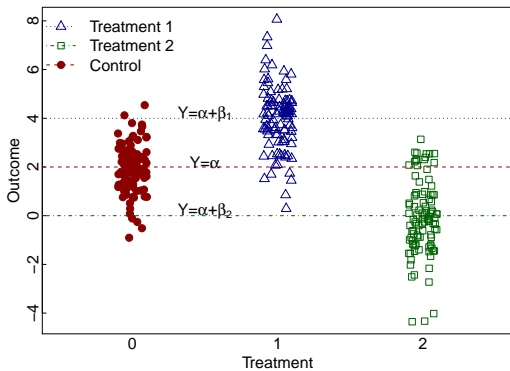
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- ▶ One of these is observed, the other is the counterfactual – what would have been observed if the other treatment had been given?
- ▶ The causal effect of T_i on observation i will then be $y_i^1 - y_i^0$



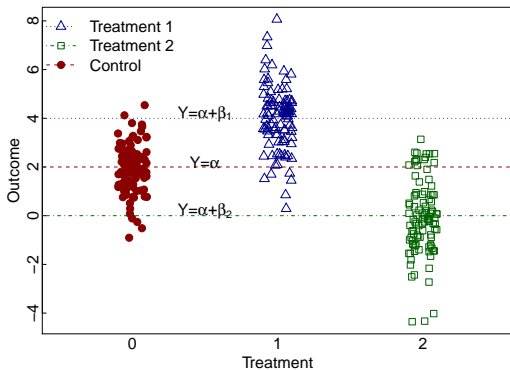


- ▶ $ATE = mean(y_i^1) - mean(y_i^0)$
- ▶ $ATE = (\alpha + \beta) - (\alpha)$
- ▶ $ATE = \beta$





► $Y = \alpha + T_1\beta_1 + T_2\beta_2$

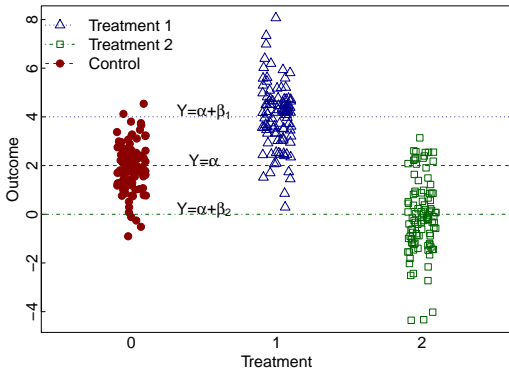


► $Y = \alpha + T_1\beta_1 + T_2\beta_2$

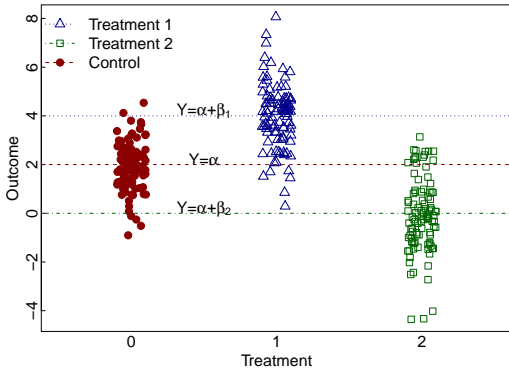
► Control: $Y = \alpha$

► Treatment 1: $Y = \alpha + \beta_1$

► Treatment 2: $Y = \alpha + \beta_2$

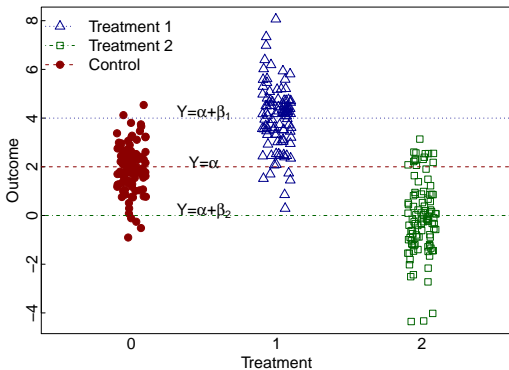


- ▶ $Y = \alpha + T_1\beta_1 + T_2\beta_2$
- ▶ $ATE_1 = mean(y_i^1) - mean(y_i^0)$
- ▶ $ATE_1 = (\alpha + \beta_1) - (\alpha)$
- ▶ $ATE_1 = \beta_1$



- ▶ $ATE_2 = mean(y_i^2) - mean(y_i^0)$
- ▶ $ATE_2 = (\alpha + \beta_2) - (\alpha)$
- ▶ $ATE_2 = \beta_2$

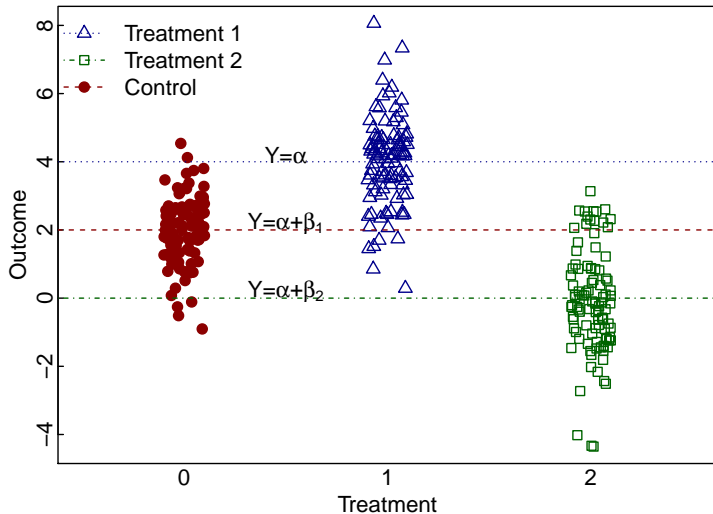
Is blue Different than green?



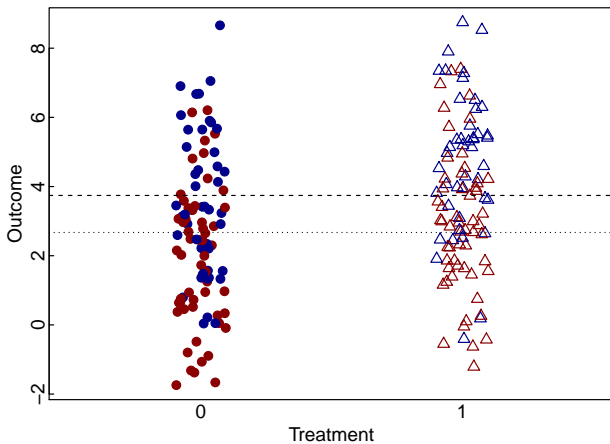
- ▶ $ATE_{2vs.1} = mean(y_i^2) - mean(y_i^1)$
- ▶ $ATE_{2vs.1} = (\alpha + \beta_2) - (\alpha + \beta_1)$
- ▶ $ATE_{2vs.1} = \beta_2 - \beta_1$

Is blue Different than green?

A much easier way to do this is to change what is the “control” and leave that out.

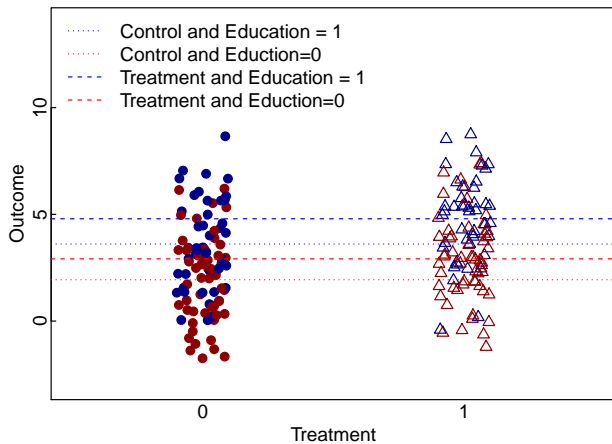


Control variables



$$Y = \alpha + T\beta_1 + X\beta_2$$

Control variables

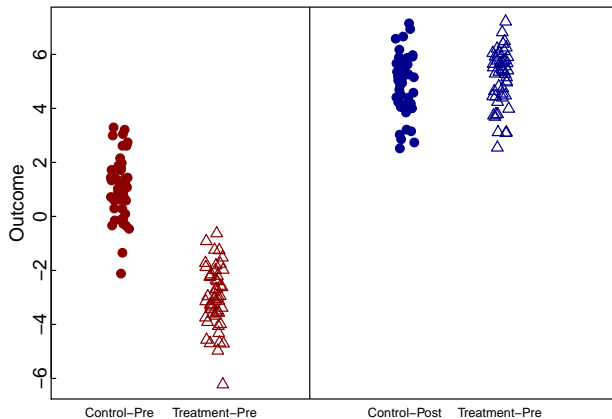


$$Y = \alpha + T\beta_1 + X\beta_2$$

Adding control variables improves precision

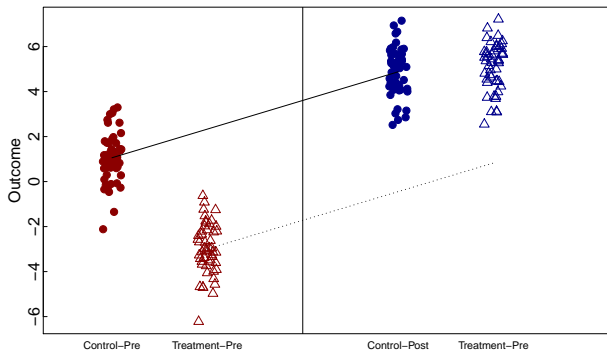
	No Control	Control
Intercept	2.67 (0.22)	1.89 (0.24)
Treatment	1.43 (0.31)	1.42 (0.29)
Covariate		1.78 (0.29)
R^2	0.056	0.136

Difference in differences

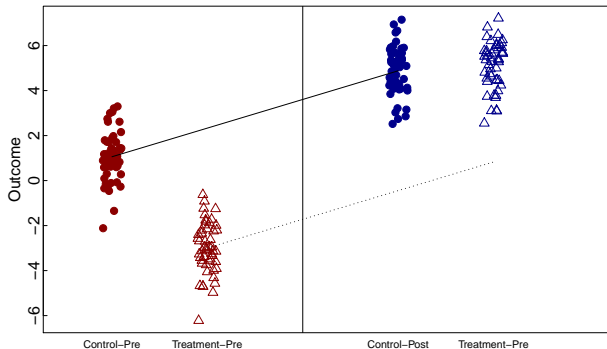


$$Y = \alpha + Treatment\beta_1 + Pre/Post\beta_2 + (T \times PP)\beta_3$$

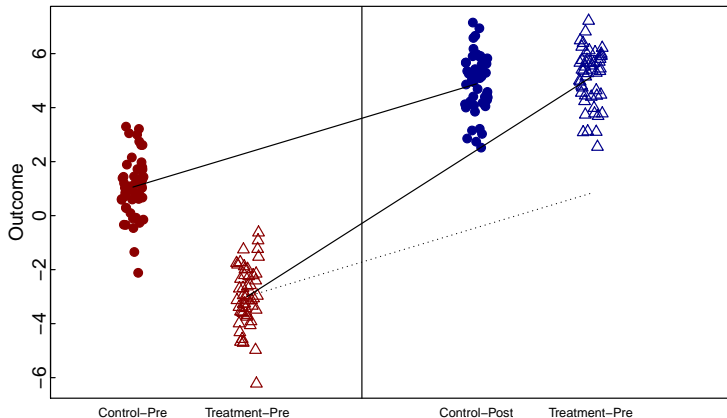
Difference in differences



Difference in differences



- ▶ Assume parallel paths
- ▶ The treatment group would have moved in parallel to the control group in the absence of the intervention.
- ▶ We can estimate the effect of the treatment on the treated



- ▶ We need to model both what happened and what would have happened
- ▶ $Y = \alpha + Treatment\beta_1 + Pre/Post\beta_2 + (T \times PP)\beta_3$

Diff-in-diff model

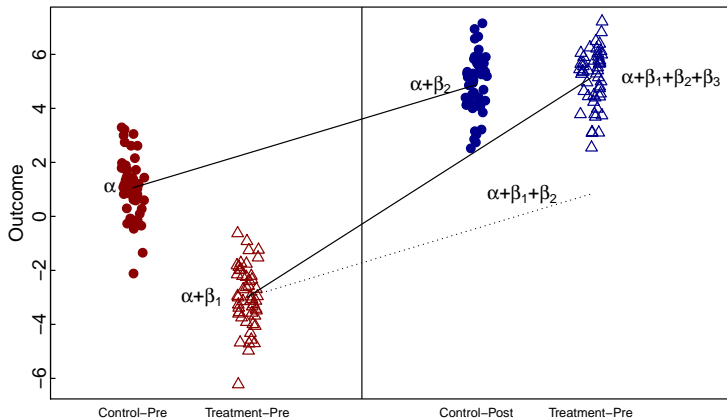
	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	1.0591	0.1547	6.84	0.0000
Assignment	-4.0522	0.2188	-18.52	0.0000
PrePost	3.8202	0.2188	17.46	0.0000
Assignment:PrePost	4.2994	0.3095	13.89	0.0000

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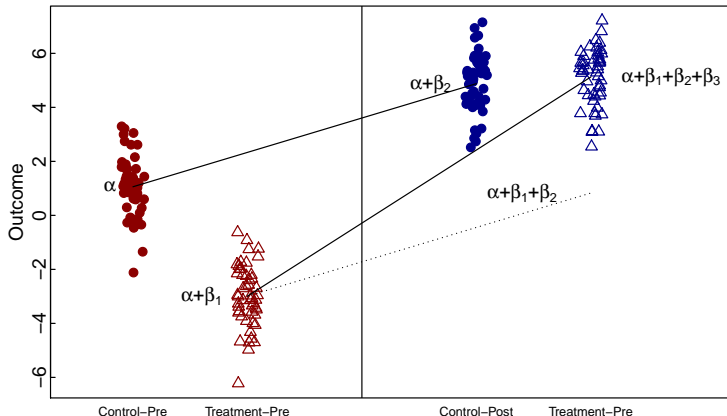
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Let's write down the prediction equation on the board.

Difference in differences



Difference in differences



The difference between what we observed ($\alpha + \beta_1 + \beta_2 + \beta_3$) and what we would have observed ($\alpha + \beta_1 + \beta_2$) is

$$(\alpha + \beta_1 + \beta_2 + \beta_3) - (\alpha + \beta_1 + \beta_2) = \beta_3$$

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Difference in difference: Discussion

- ▶ Despite being tricky to think about it, it is simple to estimate.
- ▶ Does *not require* that the treatment and control group be similar, only that they would have had parallel paths.
- ▶ You can (and should) also control for other factors, which will help improve your estimates.

Applied Example: Minimum wage laws

What is the effect of increasing the minimum wage on low-wage workers?

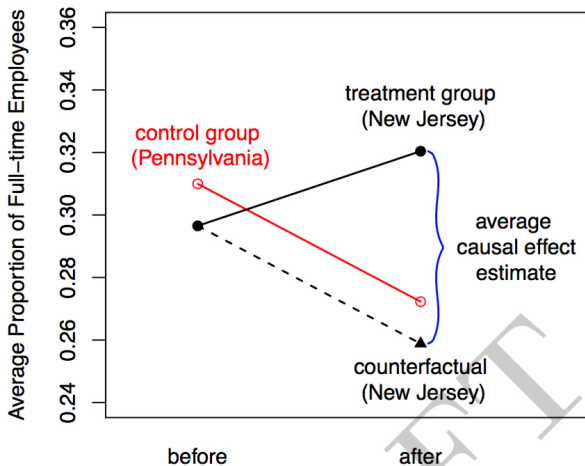
- ▶ Side 1?
- ▶ Side 2?

Study basics:

- ▶ In 1992, New Jersey increased the minimum wage from \$4.25/hour to \$5.05/hour.
- ▶ Researchers collected information about the number of full time employees (as a proportion of total number of employees) at various fast food chains before and after the reform.

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- ▶ Researchers collected information about the number of full time employees (as a proportion of total number of employees) at various fast food chains before and after the reform.
- ▶ Researchers *also* collected the same information from the same chains in Pennsylvania to serve as a counterfactual.
- ▶ Are New Jersey and Pennsylvania perfect counterfactuals for each other?



- ▶ Divide up into your groups.
- ▶ Interpret this plot. What was the effect of the wage increase in New Jersey? Be ready to answer.