Lecture 8: Hypothesis Testing 1

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Quantitative Political Methodology

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Roadmap

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Next class:

- Hypothesis tests with small samples
- Types of errors

Hypothesis testing: The big picture

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To test a hypothesis, we take our data and conduct a *significance test*. Does the data support my hypothesis?

The five steps of hypothesis testing

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- Examples?

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Large sample significance testing for means

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Step 2: State hypotheses

- $H_0 : \mu = \mu_0$ (e.g., $\mu_0 = 0$)
- This is a "two-sided test", but it may be a "one-sided."

Step 3: Calculate a test statistic

- $TS = \frac{\bar{Y} - \mu_0}{\sigma_{\bar{Y}}}$
- TS is our "Test statistic"
- Just as before, this comes from the sampling distribution of $\bar{Y}$

Step 4: P-Value

- $p = Pr(Z \geq |\frac{\bar{Y}-\mu_0}{\sigma_{\bar{Y}}}|) + Pr(Z \leq -|\frac{\bar{Y}-\mu_0}{\sigma_{\bar{Y}}}|)$
- $= 2 \times Pr(Z \geq |\frac{\bar{Y}-\mu_0}{\sigma_{\bar{Y}}}|)$
- We use both tails, because we want to find the probability of error in both directions.

Step 5: Draw a conclusion

- If $p \leq \alpha$ we conclude that the evidence supports H_a.
- If $p > \alpha$ we say that "we cannot reject the null hypothesis."

Example: Females "like" the Democratic party

2004 ANES asked all respondents to rate the Democratic party on a feeling thermometer (0-100). We are just going to use data from female respondents. We are going to assume, for the purposes of this class, that any thermometer rating above 58 means you "like" the Democratic party. Test the research hypothesis that women are not neutral towards the Democratic party (they either like or dislike them). A score of exactly 58 means the respondent is "neutral."

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- Is that good enough?
- Why are we using a two-sided test?

Large sample test of proportions

Castenedat v. Partida

- The true number of Mexican-Americans was 79.1% of the population.
- Individuals were selected for jury participation using the "key man" system.
  - Key men in the area provide a list of possible jurors
  - Jurors are selected at random from the list
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Research hypothesis: The "key man" system produces lists that significantly under-represent Mexican-Americans.

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Jury Example:

- *Type of data*: Nominal data
- *Population distributions*: No assumptions needed
- *Sample size*: Large enough for the Central Limit Theorem
- *Sampling method*: Jury members are selected at random

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Jury example:

- *Null hypothesis*: The proportion of Mexican-Americans provided by key men is the same as the proportion of Mexican-Americans in the district.
- $H_0 : \pi > 0.791$
- *Alternative hypothesis:* The proportion is less than that.
- $H_a : \pi \leq 0.791$

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In this case, Z is measuring the number of standard deviations the observed data is from the population mean and standard deviation assumed by the null hypothesis.

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- Why don't we ask $Pr(Z = -6.46)$?

Step 5 of 5: Draw a Conclusion

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- Science!