Lecture 7: Confidence intervals

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Quantitative Political Methodology

Lecture 7: Confidence intervals

Class business

- We will be working on CIs today and hypothesis tests on Wednesday
- Next Monday we will talk about posters and do some catch up
- PS2 due next class
- PS3 due on 10/9
- Midterm on 10/11

Learning objectives

- Defining: Estimates, confidence intervals, confidence levels
- Calculating confidence intervals
- Place confidence intervals int the larger story of this class

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- $\bar{y} \sim N\left(\mu, \frac{\sigma}{\sqrt{n}}\right)$
- This will be our first true statistical inference.

Estimation basics

A ***point estimate*** *is a sample statistics that gives a good guess about a population parameter.*

- **Example**: Point estimation for population mean ($\hat{\mu}$)
  - $\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i$
  - $med(y_1, y_2, \ldots, y_n)$

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  - $med(y_1, y_2, \ldots, y_n)$
- **Example**: Point estimate for population standard deviation ($\hat{\sigma}$)
  - $S = \sqrt{\frac{\sum(y_i - \bar{y})^2}{n-1}}$

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- Consistent (as $n \to \infty$ then $E(\hat{\mu}) \to \mu$)

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Notes:

- Sometimes there are tradeoffs between these (e.g., median)
- **This** is why there is such a funny equation for S.

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Note: We are not assuming that the population is normal. We are just assuming that our real goal is to find a good estimate of μ and that n is large.

Class discussion

You are the campaign manager for a candidate who is deciding whether or not to publish a new deficit reduction proposal. You commission a poll of voters in the district to find out whether they approve or disapprove of this proposal. Which of the following statements would you find most useful from your pollster?

Class discussion

You are the campaign manager for a candidate who is deciding whether or not to publish a new deficit reduction proposal. You commission a poll of voters in the district to find out whether they approve or disapprove of this proposal. Which of the following statements would you find most useful from your pollster?

1. We can be 25% confident that between 54 and 55 percent of voters approve of the plan.
2. We can be 95% confident that between 48.5 and 59.5 percent of voters approve of the plan.
3. We can be 99% confident that between 45.75 and and 62.25 percent of voters approve of the plan.
4. We can be 100% confident that between 0 and 100 percent of voters approve of the plan.

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- $0.95 \rightarrow 95\%$ confidence interval
- $0.70 \rightarrow 70\%$ confidence interval

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We can use the sampling distribution of $\bar{y}$ (assuming a large sample) to calculate a confidence interval for the population mean.

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- Remember that $\sigma_{\bar{y}} = \frac{\sigma}{\sqrt{n}}$ and $\hat{\sigma}_{\bar{y}} = \frac{S}{\sqrt{n}}$

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- Now we have an **estimated** sampling distribution, $N(\bar{y}, \hat{\sigma}_{\bar{y}})$
  - We use our knowledge of the normal distribution to find a CI
  - E.g., we want 2.5% of the probability to be outside of our interval on each side.

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5. Use these values to calculate $\bar{y} \pm Z \times \hat{\sigma}_{\bar{y}}$

Exercise: If $\bar{y} = 9.6$, $n = 100$, and \$S=4, what is the 99% confidence interval for μ?

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Answer: $\bar{y} \pm 1.96 \times \hat{\sigma}_{\bar{y}}$

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Answer: $\bar{y} \pm 1.96 \times \hat{\sigma}_{\bar{y}} = 9.6 \pm 1.96 \times \frac{4}{10} = [8.816, 10.384]$

Reprise:

How to calculate a confidence interval:

1. Calculate $\bar{y}$ and $\sigma_{\bar{y}} = \dfrac{S}{\sqrt{n}}$
2. How much area do we need under the curve to the left?
   - Example: For a 95% confidence interval we need .025 under the curve.
   - (confidence coefficient)/2
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Team time!