Problem Set 1

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Sets

Let $A = \{1, 5, 10\}$ and $B = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

1. Is $A \subset B$, $B \subset A$, both, or neither?
2. What is $A \cup B$?
3. What is $A \cap B$?
4. Partition B into two sets, A and everything else. Call everything else C. What is C?
5. What is $A \cup C$?
6. What is $A \cap C$?
7. A committee is being formed in the Senate, where each committee member is of equal status (there is no hierarchy of chair, vice chair, etc.). How many possible committees of 5 could be formed from 100 senators?
8. If the order in which the 5 senators were inducted into the committee determined their rank within the comittee (e.g. chair, vice-chair, sub-committee chair, sub-committee vice-chair, and member), how many possible committees of 5 could be formed by 100 Senators?
9. Compute $\frac{12!}{7!}$
10. Compute $\frac{5!}{6!}$

Probability

11. If A, B, C, and D are mutually exclussive and collectively exhaustive, what is the joint probability of A, B, C, AND D?
12. If A, B, C, and D are mutually exclussive and collectively exhaustive, what is the joint probability of A, B, C, OR D?
13. Solve what is known as the Monte Hall problem. There are three doors. Behind two of these are goats, while behind the third is a new car. You choose one door. Monte Hall opens one of the other two doors, revealing a goat, and asks if you'd like to stick with the door you have, or switch to the other door he did not open. You get whatever is behind the door you chooose. Should you switch doors? Why or why not?
14. In a certain city, 30% of the citizens are conservatives, 30% are liberals, and 40% are independents. In a recent election, 50% of conservatives voted, 40% of liberals voted, and 30% of independents voted.
    - What is the probability that a person voted?
    - If the person voted, what is the probability that the voter is conservative?
    - If the person voted, what is the probability that the voter is liberal?
15. In rolling two dice labeled X and Y, what is the probability that the sum of their two faces is four, given that either X or Y shows a three?

Use this joint probability distribution:

		X		
		0	1	2
	0	0.10	0.10	0.01
Y	1	0.02	0.10	0.20
	2	0.30	0.10	0.07

16. $p(X < 2)$
17. $p(X < 2|Y < 2)$
18. $\Pr(Y = 2|x \leq 1)$
19. $p(X = 1|Y = 1)$
20. $p(Y > 0|X > 0)$

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21. Assume that 2% of the population of the United States are members of some extremist militia group, ($p(M) = 0.02$). However, members may be unwilling to admit their mempership on a survey. We develope a survey question that is 95% accurate on positive classification $p(C|M) = 0.95$ and 97% accurate on negative classification, $P(C^C|M^C) = 0.97$. Using Bayes' Law, derive the probability that someone positively classified by the survey as being a militia member really is a militia member.

22. Assume A and B are independent events:

a. $\Pr(A \cap B) = \Pr(A)\Pr(B)$. True or False? Explain.
b. $\Pr(A|B) = \Pr(A) + \Pr(A)\Pr(B)$. True or False? Explain.
c. $\Pr(B|A) = \Pr(B)$. True or False? Explain.
d. Let $P(A) = 0.3$ and $P(A \cup B) = .5$. Find $P(B)$. Explain what problems you would run into were A and B not independent?

Properties of distributions

23. Wasserman Exercise 2.9
24. Wasserman Exercise 2.17
25. Suppose that $f_{x,y}(x, y) = \exp(-(x + y))$ for $x, y \geq 0$. Find the marginal distribution for X, $f_X(x)$.
26. If $X_1, \ldots, X_n$ are iid random variables, find the expected value for

$$\frac{1}{n}\sum_{i=1^n}(X_i - \bar{X_n})^2$$

27. Prove theorem 3.27 in Wasserman (see hints in Exercise 3.17).
28. Prove lemma 3.31.1 in Wasserman.
29. Wasserman Exercise 3.5
30. Find the MGF for the Binomial(n,p) distribution.
31. Wasserman Exercise 3.18

Convergence of random variables

32. A Quinnipiac poll from late July sampled 1,048 Texas voters and asked them about their voting preferences for the upcoming 2018 Senate Election. It found that 558 preferred Ted Cruz (R), while 490 preferred Beto O'Rourke (D). Let π be the population proportion of Texas voters who prefer Ted Cruz. Use the central limit theorem to find the approximate distribution of the sample proportion $\hat{\pi}$.

33. Prove the central limit theorem. (See appendix to Chapter 5)
34. Wasserman Exercise 5.5