Lecture 4: Probability

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Quantitative Political Methodology

Lecture 4

CLass business

- PROBLEM SET 1 IS DUE RIGHT NOW
- Problem set 2 will be distributed today via the syllabus

Facebook and survey

- Sign up for our Facebook group: `https://www.facebook.com/groups/1071702902960687/`
- Take the class survey! Can't assign teams until you all do.

`https://wustl.az1.qualtrics.com/jfe/form/SV_6rpSYD3xxmbRe5v`

Roadmap

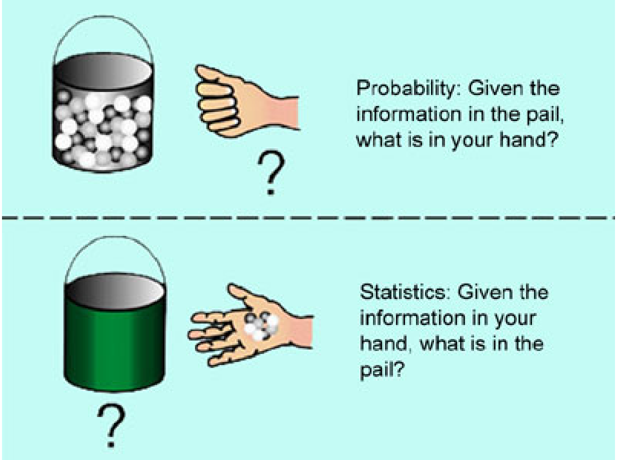
Last time:

- Visualizing data
- Measures of central tendency and spread

This time:

- Understand core concepts of probability
- Understanding concept of a "parameter"
- Introduce some probability distributions

Why are we studying this?



Probability defined

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AF p. 73: "For a particular possible outcome for a random phenomenon, the probability of that outcome is the proportion of times that the outcome would occur in a very long sequence of observations."

Example

Imagine you were rolling two six-sided dice.

1. Write down all possible scores.
2. Calculate the probability of each score
    - What is the probability of rolling a 2?

36 possible outcomes for the two dice:

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

How many outcomes will generate a total score of 2?

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

$\text{P(roll=2)} = \frac{1}{36} = 0.028.$

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- If I flip a coin three times, what is the probability that I will get exactly two heads?
- If I roll two dice, what is the probability of getting a two?
- If I take a random sample of 100 Wash U students, what is the probability that less than 40% of the sample will be male?

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- $$\sum_{k=1}^{K} p(y_k) = 1$$

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11	$2/36$
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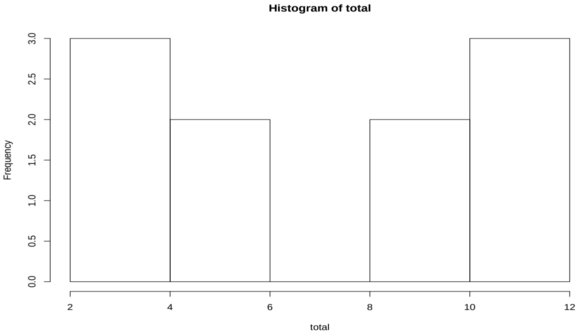
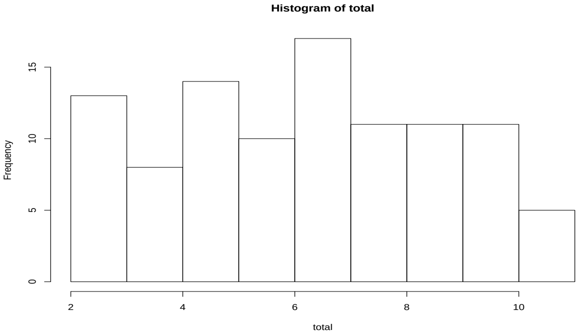
$\sigma^2 = E(Y - \mu)^2$ requires extra calculations

A little simulation

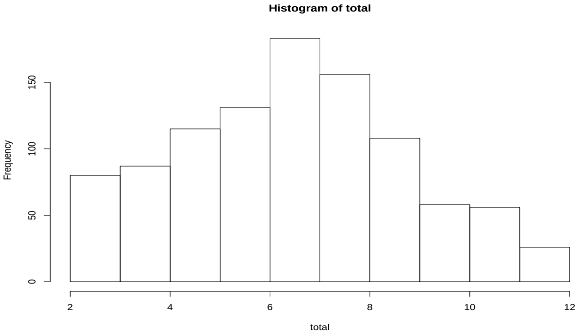
```
posVal<-c(1,2,3,4,5,6)
numRoll<-10
die1<-sample(x = posVal, size=numRoll, replace=TRUE)
die2<-sample(x = posVal, size=numRoll, replace=TRUE)
total<-die1+die2
hist(total)
```



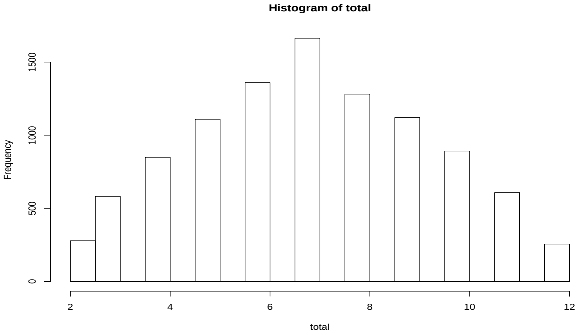
```r
posVal<-c(1,2,3,4,5,6)
numRoll<-100
die1<-sample(x = posVal, size=numRoll, replace=TRUE)
die2<-sample(x = posVal, size=numRoll, replace=TRUE)
total<-die1+die2
hist(total)
```



```r
posVal<-c(1,2,3,4,5,6)
numRoll<-1000
die1<-sample(x = posVal, size=numRoll, replace=TRUE)
die2<-sample(x = posVal, size=numRoll, replace=TRUE)
total<-die1+die2
hist(total)
```



```r
posVal<-c(1,2,3,4,5,6)
numRoll<-10000
die1<-sample(x = posVal, size=numRoll, replace=TRUE)
die2<-sample(x = posVal, size=numRoll, replace=TRUE)
total<-die1+die2
hist(total)
```



End of part 1

- What is a probability?
- What is a frequency distribution?
- What are the two most important parameters for characterizing a distribution?