Least Absolute Shrinkage and Selection Operator (LASSO)

Bryant Moy

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Outline

Motivation and Intuition

- Variable Selection

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- Too Many Covariates in the Model

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- Shrinkage Method to Push β's towards zero

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- Variable Selection
- Too Many Covariates in the Model
- Shrinkage Method to Push β's towards zero
- But Why?

LASSO Estimation

- The Lasso coefficients $\hat{\beta}^L_\lambda$ minimizes the following equation:

$$\sum_{i=1}^{n}(y_i - \beta_0 - \sum_{i=1}^{P}\beta_j x_{ij})^2 + \lambda\sum_{j=1}^{P}|\beta_j|$$

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- L 1 Norm Penalty
- Ridge Regression is $L2$ Norm $\rightarrow \beta^2$

LASSO vs. Ridge

LASSO

$$\sum_{i=1}^{n}(y_i - \beta_0 - \sum_{i=1}^{P} \beta_j x_{ij})^2 + \lambda \sum_{j=1}^{P} |\beta_j|$$

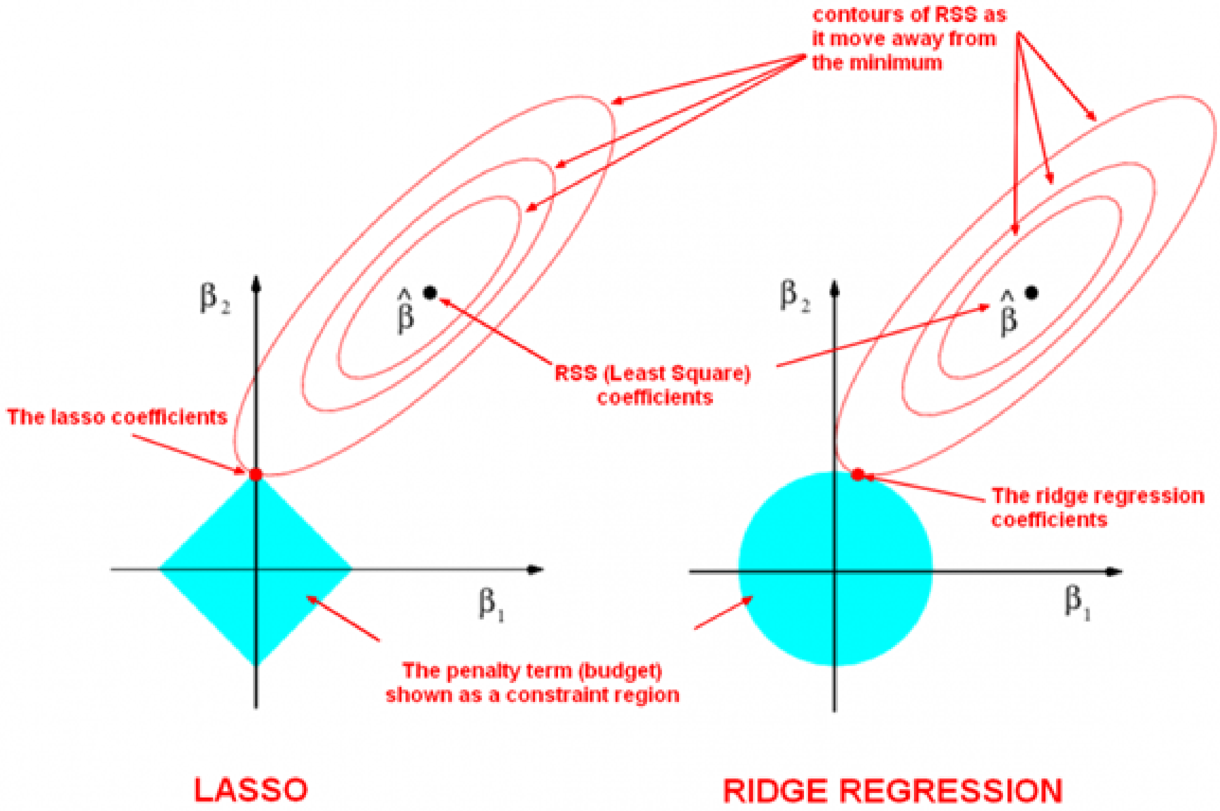
$$RSS + \lambda \sum_{j=1}^{P} |\beta_j|$$

Ridge Regression

$$\sum_{i=1}^{n}(y_i - \beta_0 - \sum_{i=1}^{P} \beta_j x_{ij})^2 + \lambda \sum_{j=1}^{P} \beta_j^2$$

$$RSS + \lambda \sum_{j=1}^{P} \beta_j^2$$

LASSO vs. Ridge

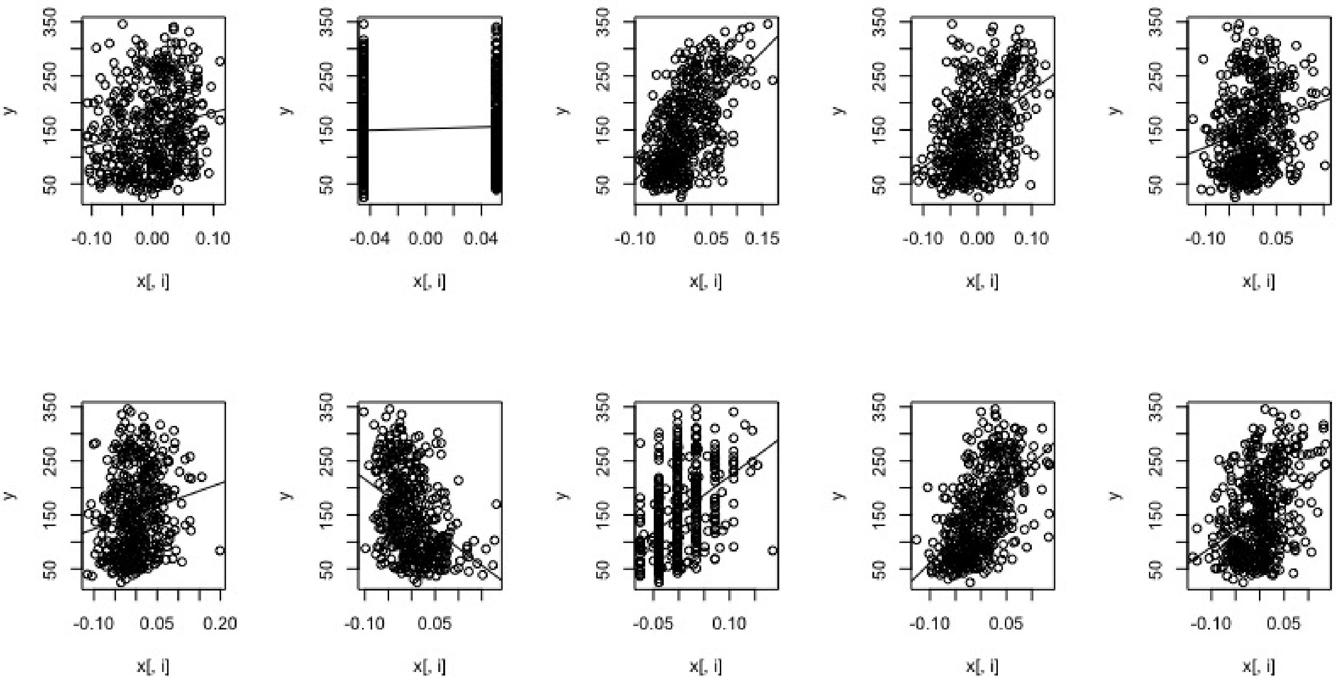


Code and Example

```
library(lars)
library(glmnet)
data(diabetes)
```

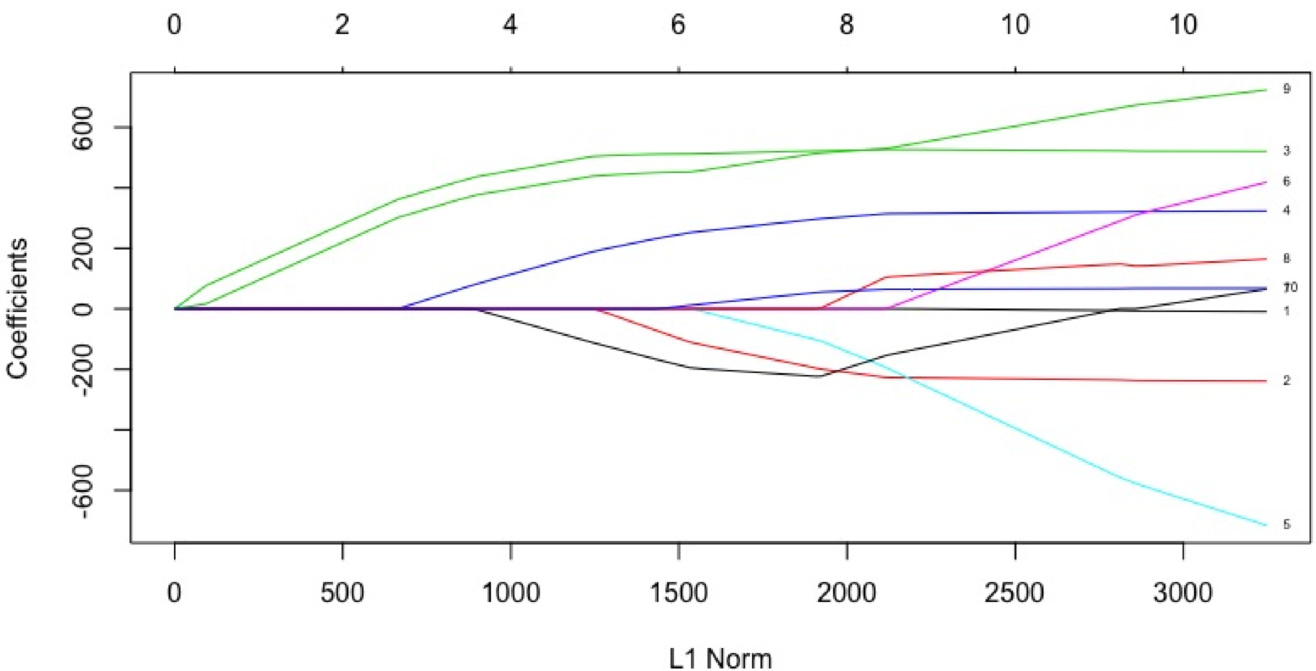
- age
- sex
- bmi
- map
- tc
- ldl
- hdl
- tch
- ltg
- glu

```r
for(i in 1:10){
  plot(x[,i], y)
  abline(lm(y~x[,i]))
  }
```



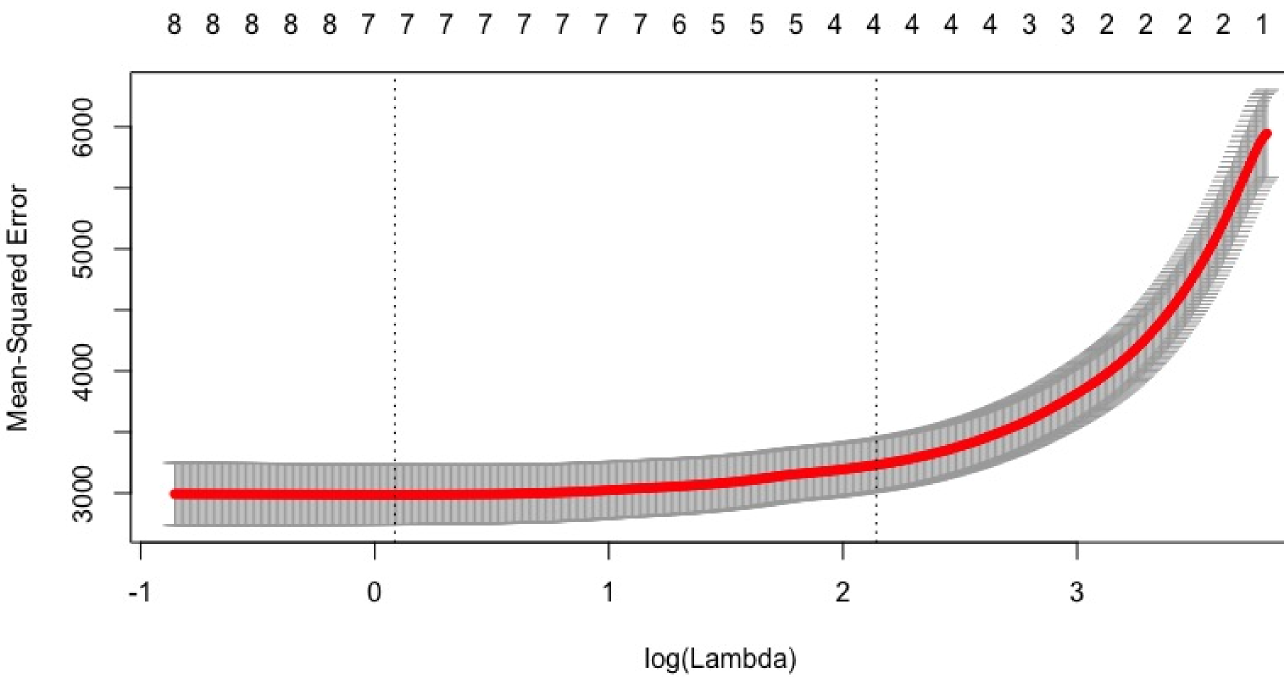
Code and Example

```
model_lasso <- glmnet(x, y)
plot.glmnet(model_lasso, xvar = "norm", label = TRUE)
```



Code and Example

```
cv_fit <- cv.glmnet(x=x, y=y, alpha = 1, nlambda = 1000)
plot.cv.glmnet(cv_fit)
cv_fit$lambda.min # 1.069371
```



Code and Example (OLS)

```
summary(model_ols)
Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept)  152.133      2.576  59.061  < 2e-16 ***
xage         -10.012     59.749  -0.168 0.867000
xsex        -239.819     61.222  -3.917 0.000104 ***
xbmi         519.840     66.534   7.813 4.30e-14 ***
xmap         324.390     65.422   4.958 1.02e-06 ***
xtc         -792.184    416.684  -1.901 0.057947 .
xldl         476.746    339.035   1.406 0.160389
xhdl         101.045    212.533   0.475 0.634721
xtch         177.064    161.476   1.097 0.273456
xltg         751.279    171.902   4.370 1.56e-05 ***
xglu          67.625     65.984   1.025 0.305998
---
Residual standard error: 54.15 on 431 degrees of freedom
Multiple R-squared:  0.5177,Adjusted R-squared:  0.5066
F-statistic: 46.27 on 10 and 431 DF,  p-value: < 2.2e-16
```

Code and Example (LASSO)

```
fit <- glmnet(x=x, y=y, alpha = 1, lambda=cv_fit$lambda.min)
fit$beta
10 x 1 sparse Matrix of class "dgCMatrix"
            s0
age          .
sex -193.51972
bmi  521.56400
map  294.92639
tc   -98.53276
ldl          .
hdl -222.65684
tch          .
ltg  511.53755
glu   52.60584
```

Why the LASSO is Uniquely Important: Causal Estimates

Lasso adjustments of treatment effect estimates in randomized experiments

Adam Bloniarz[a,1], Hanzhong Liu[a,1], Cun-Hui Zhang[b], Jasjeet S. Sekhon[a,c], and Bin Yu[a,d,2]

[a]Department of Statistics, University of California, Berkeley, CA 94720; [b]Department of Statistics and Biostatistics, Rutgers University, Piscataway, NJ 08854; [c]Department of Political Science, University of California, Berkeley, CA 94720; and [d]Department of Electrical Engineering and Computer Science, University of California, Berkeley, CA 94720

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$$\hat{\sigma}^2_{e^{(a)}} = \frac{1}{n_A - df^{(a)}} \sum_{i \in A} \left(a_i - \bar{a}_A - (\mathbf{x}_i - \bar{\mathbf{x}}_A)^T \hat{\boldsymbol{\beta}}^{(a)}_{\text{Lasso}} \right)^2,$$

$$\hat{\sigma}^2_{e^{(b)}} = \frac{1}{n_B - df^{(b)}} \sum_{i \in B} \left(b_i - \bar{b}_B - (\mathbf{x}_i - \bar{\mathbf{x}}_B)^T \hat{\boldsymbol{\beta}}^{(b)}_{\text{Lasso}} \right)^2,$$

where $df^{(a)}$ and $df^{(b)}$ are degrees of freedom defined by the following:

$$df^{(a)} = \hat{s}^{(a)} + 1 = \left\| \hat{\boldsymbol{\beta}}^{(a)}_{\text{Lasso}} \right\|_0 + 1; \; df^{(b)} = \hat{s}^{(b)} + 1 = \left\| \hat{\boldsymbol{\beta}}^{(b)}_{\text{Lasso}} \right\|_0 + 1.$$

Define the variance estimate of $\sqrt{n}(\widehat{ATE}_{\text{Lasso}} - ATE)$ as follows:

$$\hat{\sigma}^2_{\text{Lasso}} = \frac{n}{n_A} \hat{\sigma}^2_{e^{(a)}} + \frac{n}{n_B} \hat{\sigma}^2_{e^{(b)}}. \qquad \textbf{[20]}$$

Thanks!