Problem Set 4

QPM II - Fall 2018

10/11/2018

1. Consider $X_1, ...X_n$ which are distributed $N(\mu, \sigma^2)$. Using a $N(\mu_0, \sigma_0^2)$ prior for μ, find the posterior distribution. Find the point estimate for μ and calculate a 95% CI, given the data

$$X_1...X_n = \{5.35, 5.07, 4.62, 6.67, 4.86, 5.78, 5.75, 3.10, 6.12, 3.82\}$$

The Pareto Distribution

Let $X_1, X_2, \ldots, X_n$ be a simple random sample of a Pareto random variables with density

$$f(x|\theta) = \frac{\theta}{x^{\theta+1}} \text{ for } x > 1$$

.

2. A conjugate prior for θ is a gamma distribution

$$\propto \theta^{\alpha-1} e^{-\beta\theta}$$

   Find the posterior distribution of θ.

3. Calculate a 95% posterior credible interval and 95% HPD, given $\alpha = \beta = 15$, and the data

$$X_1...X_n = \{3.59, 1.23, 1.47, 0.32, 2.09\}$$

4. Re-do the above using the uninformative prior

$$\pi(\theta) = \frac{1}{\theta}$$

   .

The Negative Binomial Distribution

In the so-called negative binomial experiment we continue a Bernoulli trial with parameter θ until we obtain s successes, where x is fixed in advance. We assume that s is known. Let x be the number of trials needed. The pdf is then

$$f(x|\theta, s) = \binom{x-1}{s-1} \theta^s (1-\theta)^{x-s}$$

Assume that we observe the following observations where $s = 1$:

$$23, 14, 24, 17, 4, 40, 17, 13, 31, 24$$

5. In Bayesian methods, we can calculate the posterior distributions using only numerical integration methods.

$$E(\theta) = \frac{\int \theta\pi(\theta)L(\theta)d\theta}{\int \pi(\theta)L(\theta)d\theta}$$

   Use the **integrate** function in R to find the EAP estimate of θ. Use a beta distribution for the prior with parameters $\alpha = \beta = 1$.

6. Now execute a similar method to find the posterior variance of θ.

7. Show that these results are sensitive to choices of α and β. Plot different priors, calculate the posterior mean and sd using the numerical method above, and explain the changes.

8. Perform a non-parametric bootstrap to find the standard error of the MLE.

Jackknife & Non-parametric Bootstrap

Using the vector

$$x = (2, 4, 5, 3, 8, 10, -2, 1, -1, 2, 5, 5)$$

9. Find the bootstrap SE for the median.
10. Find the jackknife SE for the median.
11. Estimate the absolute error $E\left(|\hat{\theta} - \theta|\right)$ for the median and the mean using the bootstrap.

Lowess smoothing

12. Replicate the lowess smoother at the end of lecture 9 (on non-parametrics).
13. Use the bootstrap method (assuming iid) to estimate the 95% CI for the curve.
14. Add the CI to the plot.
15. Re-do this for several different values of delta (both smaller and bigger than 0.2).
16. What is driving this result.