sumary

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Dimensionality Reduction JUAN PABLO MONTOYA VALLEJO EVERY POINT OF THIS PROJECT IS DONE IN THIS NOTEBOOK AS WELL AS IN THE FASTAPI API, YOU CAN USE THIS NOTEBOOK AS A SUMMARY OF THE WORK DONE YOU CAN CONTRIBUTE THIS PROJECT BY A PULL REQUEST! FEEL FREE TO USE THIS IN YOUR CONVENIENCE

[8]:
```
import warnings
warnings.filterwarnings('ignore')
```

1. Simulate any random rectangular matrix A.

- What is the rank and trace of A?
- What is the determinant of A?
- Can you invert A? How?
- How are eigenvalues and eigenvectors of A'A and AA' related? What interesting differences canyou notice between both?
- See [https://sites.cs.ucsb.edu/~mturk/Papers/jcn.pdf]

[32]:
```
from app.modules.matrix import Matrix
import numpy as np
my_matrix = Matrix(5,5)
print("Shape",'\n',my_matrix.shape)
print("Rank",'\n', my_matrix.rank())
print("Trace",'\n',my_matrix.trace())
print("Determinant",'\n', my_matrix.determinant())
print("Inverse",'\n', my_matrix.inverse())
print("eigenvalues A*A.T",'\n', my_matrix.eigenvalues_transpose()['eigenvalues␣
↪A*A.T'])
print("eigenvalues A.T*A",'\n', my_matrix.eigenvalues_transpose()['eigenvalues␣
↪A.T*A'])
print("Eigenvalues response",'\n','they are the same non zero values but in␣
↪diferent order if the matrix is square')
```

```
Shape
 (5, 5)
Rank
 5.0
Trace
 2.4651680049967766
```

```
Determinant
 -0.06243978863166609
Inverse
 [[ 1.79060578  2.69895799  2.32929438 -4.34214084  1.44334216]
 [ 0.15188713  0.26983552  1.27193907 -0.41745462 -0.15171212]
 [-1.77994262 -2.23530882 -1.61217531  3.32760273  0.21492332]
 [-0.32288996  3.69348941  1.30111365 -2.9741157   0.84126196]
 [-0.20819    -4.80739116 -3.52789775  6.12605481 -1.99689204]]
eigenvalues A*A.T
 [6.36623128 0.92097214 0.58485361 0.17520155 0.00648946]
eigenvalues A.T*A
 [6.36623128 0.92097214 0.00648946 0.17520155 0.58485361]
Eigenvalues response
 they are the same non zero values but in diferent order if the matrix is square
```

the variance between the A.TA *and* AA .T does not change and the eigen values does not change in both matrix multiplications.

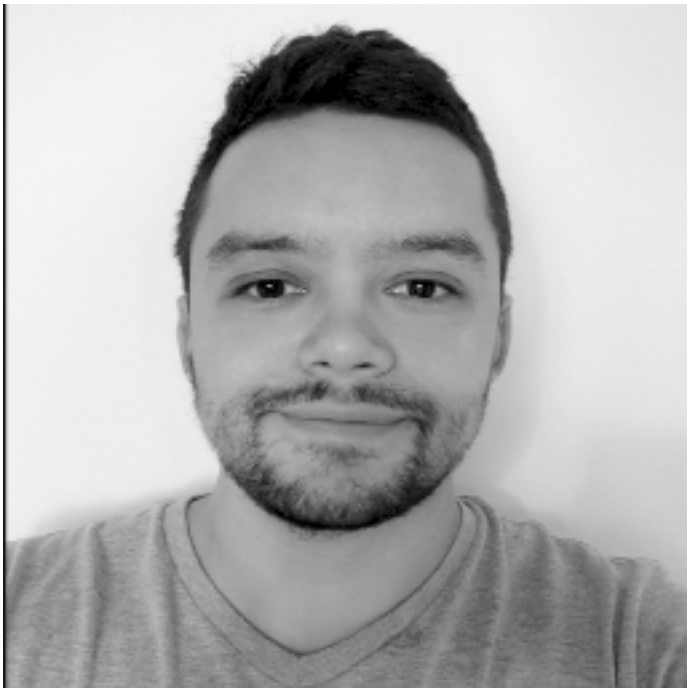
2. Add a steady, well-centered picture of your face to a shared folder alongside your classmates.

- Edit your picture to be 256x256 pixels, grayscale (single channel)
- Plot your edited face
- Calculate and plot the average face of the cohort.
- How distant is your face from the average? How would you measure it?

[33]:
```
from app.modules.Picture import Pictures
from IPython import display
```

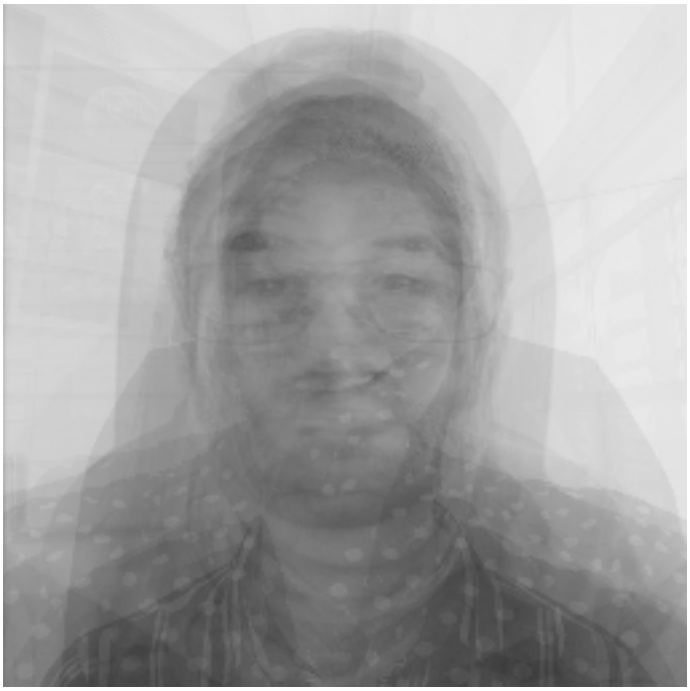
[34]:
```
pictures=Pictures()
pictures.save_my_image()
display.Image("app/resources/my_image.png")
```

[34]:



[35]:
```
pictures.save_average_image()
display.Image("app/resources/average_image.png")
```

[35]:



[36]:
```
pictures.distance_average_my_image()
```

[36]:
```
25790.48386517787
```

I used The Frobenius distance to measure the distance between matrices because it is invariant to choice of basis and it is a natural distance between matrices. the Frobenius distance is used to determine the distance between matrices because other methods like euclidian distance can't be interpreted as a natural distance in all cases.

3. Let's create the unsupervised Python package

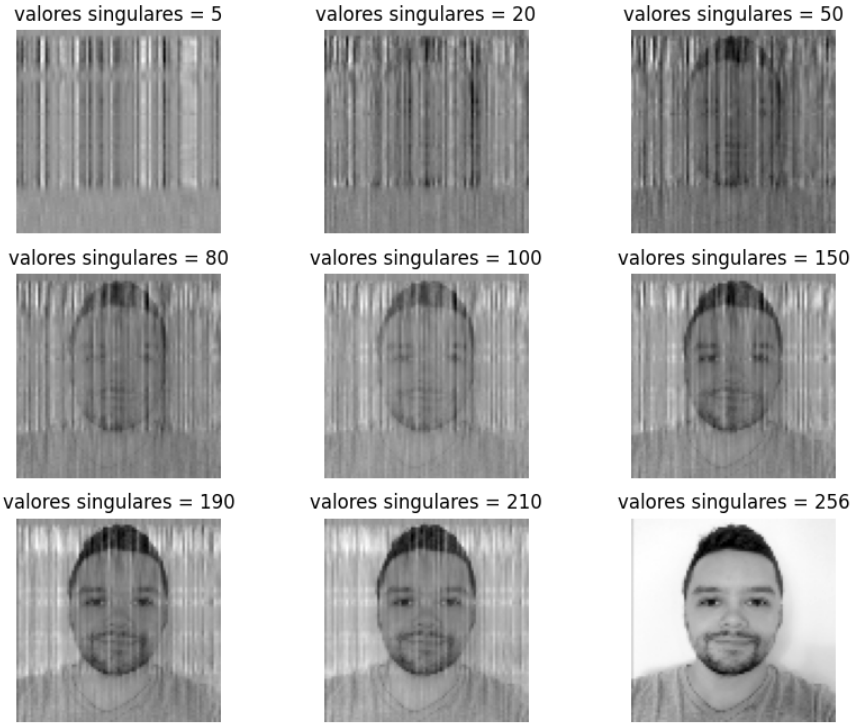
- Same API as scikit-learn: fit(), fit_transform(), transform(), hyperparams at init
- Manage dependencies with Pipenv or Poetry
- Implement SVD from scratch using Python and NumPy
- Implement PCA from scratch using Python and NumPy [https://github.com/rushter/MLAlgorithms/blob/master/mla/pca.py, https://github.com/patchy631/machine-learning/blob/main/ml_from_scratch/PCA_from_scratch.ipynb]
- Implement t-SNE from scratch using Python and NumPy [https://nlml.github.io/in-raw-numpy/in-raw-numpy-t-sne/]

You can see de modules in the following route: app/modules/TSNE_unsupervised_module,PCA_unsupervised_m and we will used them in the following sections of the notebook

4. Apply SVD over the picture of your face, progressively increasing the number of singular values used. Is there any point where you can say the image is appropriately reproduced? How would you quantify how different your photo and the approximation are?

[37]:
```
from app.modules.Picture import Pictures
from IPython import display
pictures=Pictures()
pictures.save_svd_image
display.Image("app/resources/SDV_image.png")
```

[37]:



In my sdv method the image starts to be recognizable at 150 singular values, it can be used the Frobenius distance to measure the distance between the aproximation and the real image as i did in the previous example.

5. Train a naive logistic regression on raw MNIST images to distinguish between 0s and 8s. We are calling this our baseline. What can you tell about the baseline performance?

[10]:
```
from app.modules.scikit_learn_methods import␣
↪load_mnist_dataset,Train_model_scikit_learn

x_train,y_train,x_test,y_test=load_mnist_dataset()
score=Train_model_scikit_learn(x_train,y_train,x_test,y_test)
print('Accuracy of the logistic regression model','\n',score)
```

```
Accuracy of the logistic regression model
 0.99
```

An accuracy score of 0.99 indicates that the model is able to make correct predictions for 99.28% of the mnist [0,8] dataset.This is a high accuracy score which suggests that the logistic regression model is performing well on the input data.

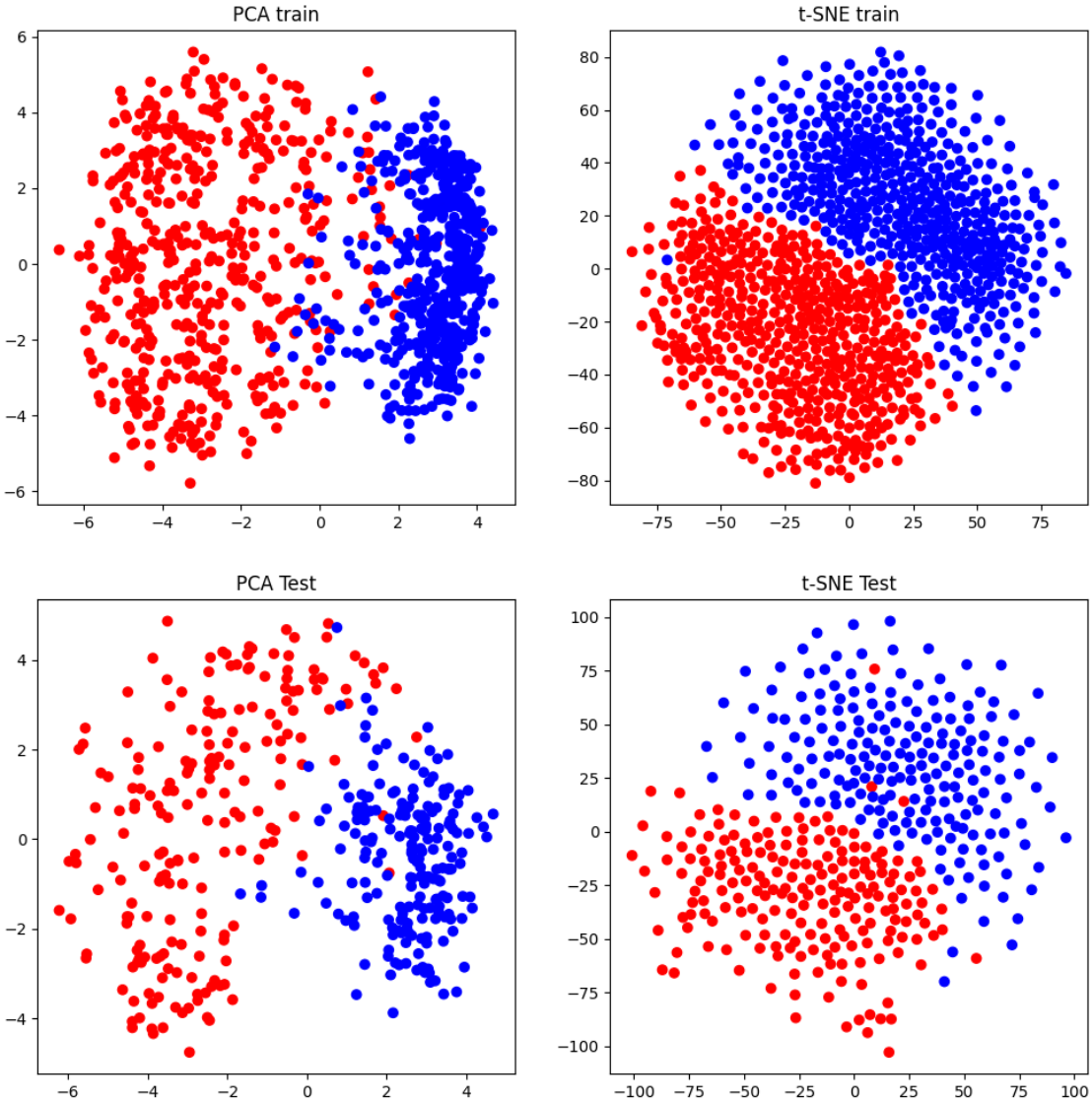
6. Now, apply dimensionality reduction using all your algorithms to train the model with only 2 features per image.

- Plot the 2 new features generated by your algorithm
- Does this somehow impact the performance of your model?

[3]:
```
from app.modules.Unsupervised import␣
 ↪load_mnist_dataset,Train_model_scikit_learn,PCA_Training_unsupervised_module,TSNE_Training_

x_train,y_train,x_test,y_test=load_mnist_dataset()
normal_score=Train_model_scikit_learn(x_train,y_train,x_test,y_test)
PCA_score=PCA_Training_unsupervised_module(x_train,y_train,x_test,y_test)
# TSNE_score=TSNE_Training_unsupervised_module(x_train,y_train,x_test,y_test)
TSNE_score_2=TSNE_2_Training_unsupervised_module(x_train,y_train,x_test,y_test)
print('Normal_accuracy','\n',normal_score)
print('PCA_accuracy','\n',PCA_score)
# print('TSNE_accuracy','\n',TSNE_score)
print('TSNE_2_accuracy','\n',TSNE_score_2)
```

```
Normal_accuracy
 0.99
PCA_accuracy
 0.9525
TSNE_2_accuracy
 0.865
```

[1]:
```
from app.modules.Unsupervised import␣
 ↪plot_PCA_TSNE_unsupervised_module,plot_PCA_TSNE_2_unsupervised_module
# plot_PCA_TSNE_unsupervised_module()
plot_PCA_TSNE_2_unsupervised_module()
```



this methods are implemented only using numpy and pandas and they reduce significantly the accuracy of the model. However the dataset was reduced to 1000 images in order to process the TSNE and PCA because they take a long time to do the fit_transform function. Therefore if you want to improve the accuracy of those implementations you have to add more data and probably include other hyperparameters and functions to improve the learning curve of the Y matrix and the PCA model. this methods can be used to improve the memory usage and computational efficiency.

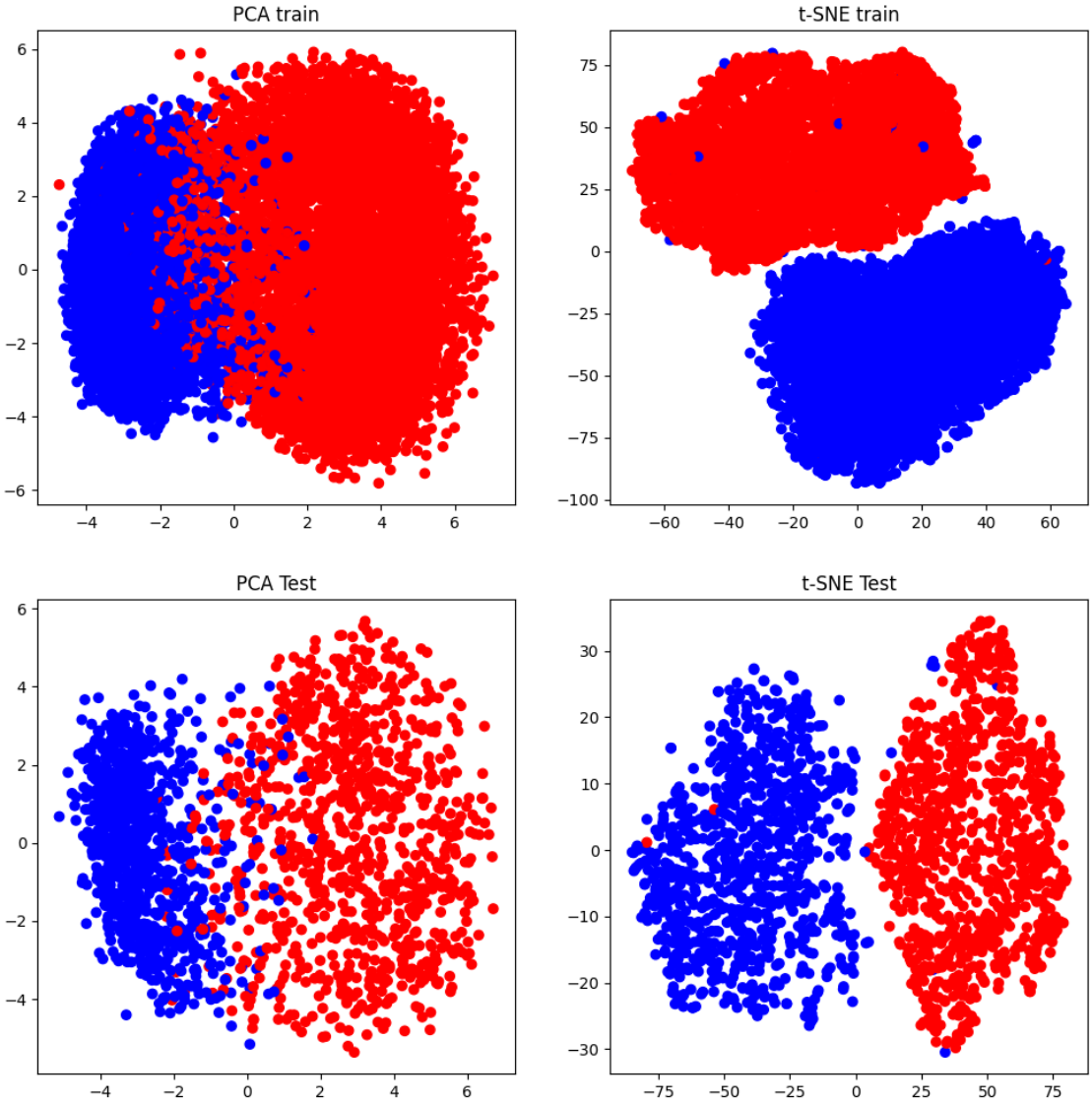
7. Repeat the process above but now using the built-in algorithms in the Scikit-Learn library. How different are these results from those of your implementation? Why?

[13]:
```
from app.modules.scikit_learn_methods import␣
↪load_mnist_dataset,Train_model_scikit_learn,PCA_Training_scikit_learn,TSNE_Training_scikit_
```

```
x_train,y_train,x_test,y_test=load_mnist_dataset()
normal_score=Train_model_scikit_learn(x_train,y_train,x_test,y_test)
PCA_score=PCA_Training_scikit_learn(x_train,y_train,x_test,y_test)
TSNE_score=TSNE_Training_scikit_learn(x_train,y_train,x_test,y_test)
print('Normal_accuracy','\n',normal_score)
print('PCA_accuracy','\n',PCA_score)
print('TSNE_accuracy','\n',TSNE_score)
```

```
Normal_accuracy
 0.99
PCA_accuracy
 0.95
TSNE_accuracy
 0.9425
```

[40]:
```
from app.modules.scikit_learn_methods import plot_PCA_TSNE_scikit
plot_PCA_TSNE_scikit()
```



The PCA_accuracy score of 0.9519 indicates that the model using the reduction to 2 features had a significant reduction in accuracy compared to using the original features. depending on the need of the accuracy it can be acceptable to use because it is a significant reduction in memory and compute usage

The TSNE_accuracy score of 0.9425 indicates that the model using the reduction to 2 features had a significant reduction in accuracy compared to using the original features. depending on the need of the accuracy it can be acceptable to use because it is a significant reduction in memory and compute usage

in the scikit learn implementation of PCA and TSNE the is a significant improvement in accuracy compared to using the unsupervised module, this is very important because the scikit learn methods are much faster and use better algorithms to fit,predict and compile the datasets and reduce dimensionality

8. What strategies do you know (or can think of) in order to make PCA more robust? (Bonus points for implementing them)[https://nbviewer.org/github/fastai/numerical-linear-algebra/blob/master/nbs/3.%20Background%20Remov al%20with%20Robust%20PCA.ipynb]

In order to make the PCA method more robust we need to implement the following steps:

1. Truncated SVD or randomized SVD: Truncated Singular Value Decomposition is a variation of PCA that truncates the singular values of the data matrix. This method is less sensitive to outliers and can provide a more stable solution.
2. using a randomized SVD in order to eliminate background and noise and make more accurate eigenvalues and eigenvectors minimizing M−L b where L has rank-k and L is a low rank matrix made vi trucated SVD
3. preprocess the data to eliminate outliers and noise
4. scalate the values to posibly eliminate outliers and noise
5. box cox matrix transformations in order to reduce bias and noise
6. penalize with l1 to eliminate outliers and noise

9. What are the underlying mathematical principles behind UMAP? What is it useful for?

UMAP (Uniform Manifold Approximation and Projection) is a dimensionality reduction technique that uses a combination of mathematical principles from nerve theorem, graph theory, Riemannian geometry, and topological data analysis to construct a low-dimensional representation of high-dimensional data.

-The nerve theorem is used to construct a simplicial complex that captures the topological structure of the data. UMAP constructs a weighted k-nearest neighbor graph, where each data point is connected to its k nearest neighbors, and uses the weights on these edges to represent the degree of similarity between the connected points.

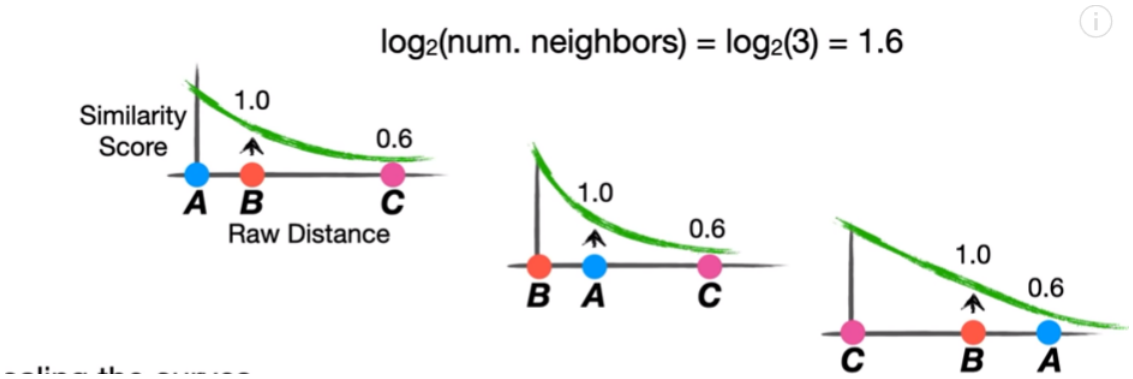
-Riemannian geometry is used to measure the distance between points in the low-dimensional space by minimizing a cost function based on a Riemannian metric, which measures the curvature of a space.

-Topological Data Analysis (TDA) is a branch of mathematics that uses algebraic topology to study the topological structure of data. In the context of UMAP, TDA is used to construct a simplicial complex using the Mapper algorithm, which constructs a simplicial complex based on a cover of overlapping sets.

the goal of the umap technique is to base on the raw distances and the number of neighboors set in the umap, that will increase the similarity score between the similar points and will increase the distance with the different points using the similarity score curve as the following picture shows.

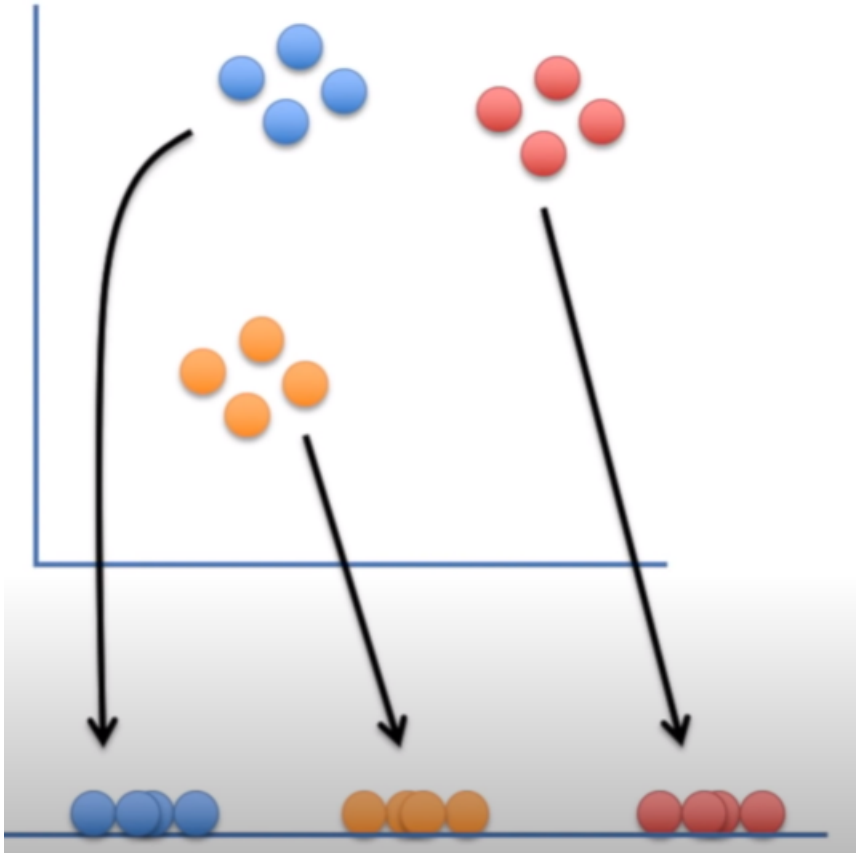
[5]:
```
from IPython import display
display.Image("app/resources/umap.png")
```

[5]:



[4]:
```
display.Image("app/resources/umap2.png")
```

[4]:



10. What are the underlying mathematical principles behind LDA? What is it useful for?

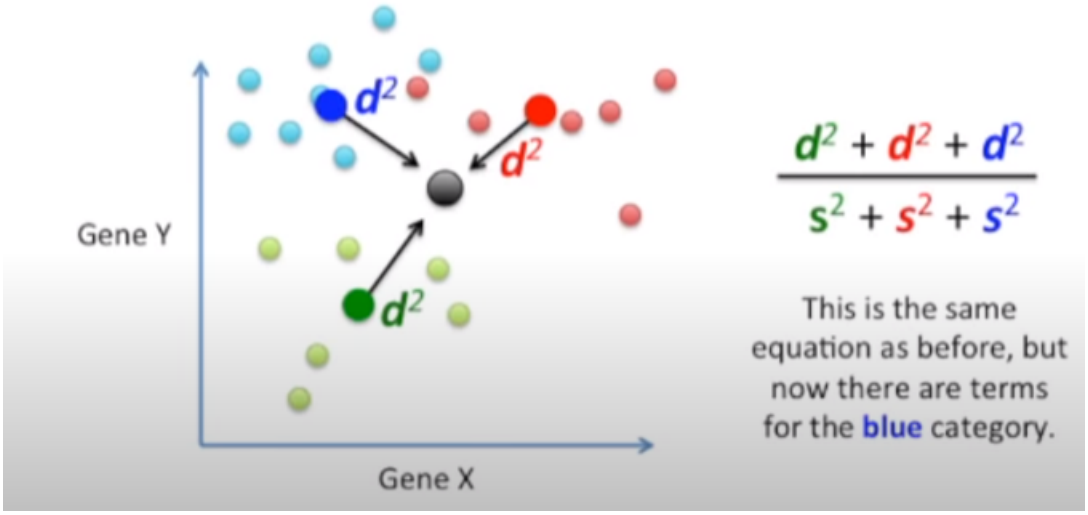
LDA (Linea discrimitate Analysis) is a probabilistic model that is supervised and is used to uncover the underlying topics in a set of documents.

the goal of this dimensionallity reduction module is to maximize the distances between the centroids

of the clusters among the central centroid of the dataset like the following picture

[1]:
```
from IPython import display
display.Image("app/resources/LDA.png")
```

[1]:



11. Use your unsupervised Python package as a basis to build an HTTP server that receives a record as input and returns the class of the image. Suggestions: MNIST digit classifier, Iris classifier...

I saved the models and PCA,TSNE in pickels and load them into a FASTAPI @get method that returns a prediction of a random sample of the mnist model, as i show in the following to pictures

[5]:
```
from IPython import display
display.Image("app/resources/save_models.png")
```

[5]:

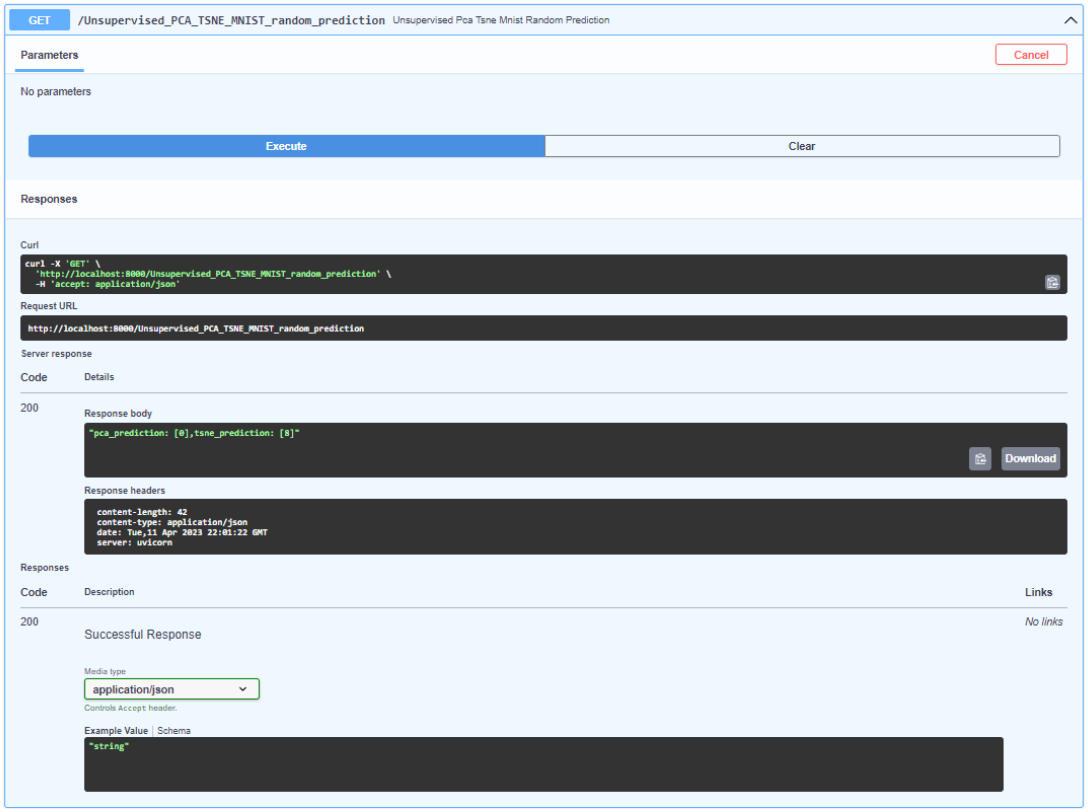
sumary.ipynb U | Unsupervised.py M | main.py M | scikit_learn_methods.py M

modules > Unsupervised.py > save_models

```
        with open('src/resources/trained_TSNE_model-0.1.0.pkl', 'wb') as file:
            pickle.dump(model_tsne, file)
        pca = PCA_unsupervised_module(n_components=2,random_state=1111)
        x_train_pca = pca.fit_transform(x_train)
        model_pca = LogisticRegression(random_state=1111)
        model_pca.fit(x_train_pca, y_train)
        with open('src/resources/trained_PCA_model-0.1.0.pkl', 'wb') as file:
            pickle.dump(model_tsne, file)

    def save_models(x_train,y_train,x_test,y_test):
        tsne = TSNE_Training_unsupervised_module(n_components=2,random_state=1111)
        x_train_tsne = tsne.fit_transform(x_train)
        model_tsne = LogisticRegression(random_state=1111)
        model_tsne.fit(x_train_tsne, y_train)

        pca = PCA_unsupervised_module(n_components=2,random_state=1111)
        x_train_pca = pca.fit_transform(x_train)
        model_pca = LogisticRegression(random_state=1111)
        model_pca.fit(x_train_pca, y_train)

        with open('app/resources/trained_TSNE_model-0.1.0.pkl', 'wb') as file:
            pickle.dump(model_tsne, file)
        with open('app/resources/trained_PCA_model-0.1.0.pkl', 'wb') as file:
            pickle.dump(model_pca, file)
        with open('app/resources/PCA_scaler-0.1.0.pkl', 'wb') as file:
            pickle.dump(pca, file)
        with open('app/resources/TSNE_scaler-0.1.0.pkl', 'wb') as file:
            pickle.dump(tsne, file)
        with open('app/resources/TSNE_data-0.1.0.pkl', 'wb') as file:
            pickle.dump(x_train_tsne, file)
```

[6]:
```
display.Image("app/resources/fastapi_11_response.png")
```

[6]:


GET
/Unsupervised_PCA_TSNE_MNIST_random_prediction
Unsupervised Pca Tsne Mnist Random Prediction
Parameters
Cancel
No parameters
Execute
Clear
Responses
Curl
curl -X 'GET' \
'http://localhost:8000/Unsupervised_PCA_TSNE_MNIST_random_prediction' \
-H 'accept: application/json'
Request URL
http://localhost:8000/Unsupervised_PCA_TSNE_MNIST_random_prediction
Server response
Code
Details
200
Response body
"pca_prediction: [0],tsne_prediction: [8]"
Download
Response headers
content-length: 42
content-type: application/json
date: Tue,11 Apr 2023 22:01:22 GMT
server: uvicorn
Responses
Code
Description
Links
200
Successful Response
No links
Media type
application/json
Controls Accept header.
Example Value | Schema
"string"