AEP Assignment 2

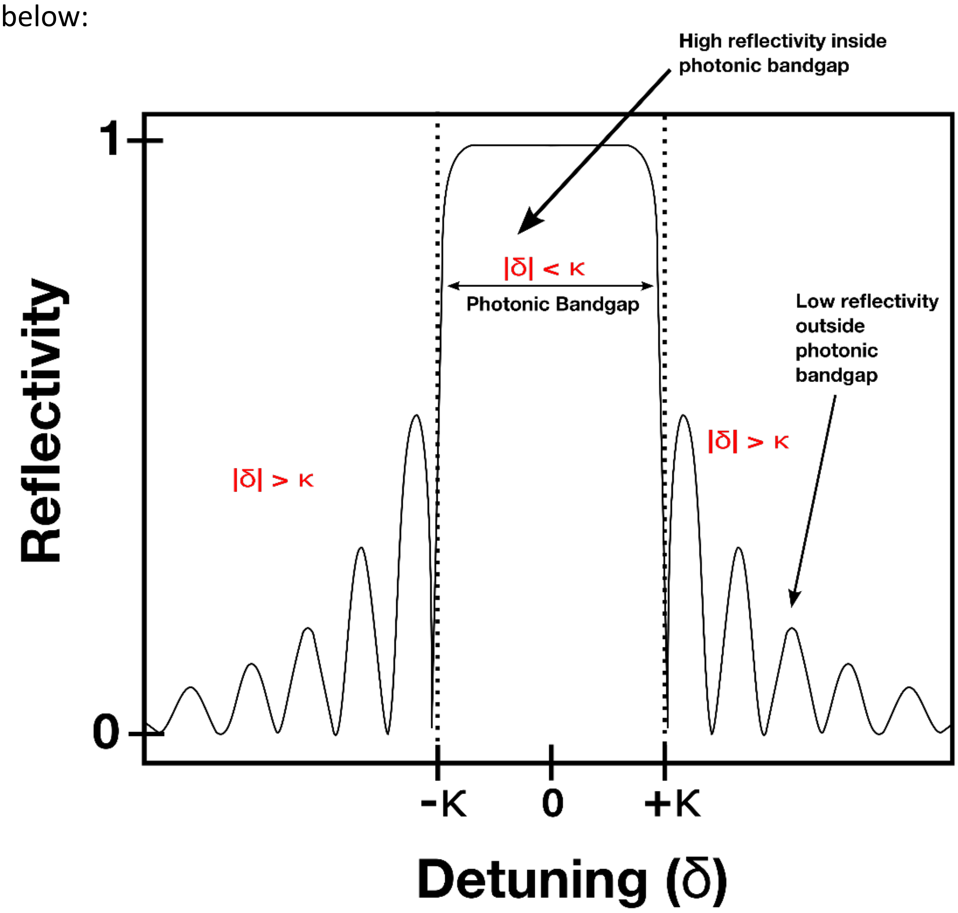
SID: 490423699

Question 1

(a) Discuss the nature of the solutions for the envelopes of the electric field as a function of position z along the grating:

For the case where the frequency detuning $|\delta| < \kappa$, α is real, and therefore the forward propagating and backward propagating envelopes are described by combinations of hyperbolic functions. This in turn describes the evanescent decay of the electric field for frequencies within the stop-band. Physically, this corresponds to the case where there are no propagating solutions in the grating – rather, the incident field undergoes Bragg reflection and the resulting envelope of the electric field decays within the grating.

When $|\delta| > \kappa$, which corresponds to frequencies far detuned from the Bragg resonance (outside of the bandgap), α becomes imaginary and the solutions are oscillatory (combinations of trigonometric functions), corresponding to propagating solutions. Accordingly, light at these frequencies are transmitted by the Bragg grating rather than being reflected. In the context of the reflection spectrum we expect to measure for such a Bragg grating, large detuning from the Bragg resonance ($|\delta| > \kappa$) will result in low reflectivity as frequencies outside the band-gap are transmitted, while small detunings within the photonic bandgap ($|\delta| < \kappa$) are expected to yield large reflectivity as per the diagram below:



(b)

b) $r = \frac{\mathcal{E}_-(0)}{\mathcal{E}_+(0)}$ $\qquad \mathcal{E}_+(0) = 1$

$$= \frac{i\mathcal{E}_+(0)K\sinh(\alpha L)}{\alpha\cosh(\alpha L) - i\delta\sinh(\alpha L)} \times \frac{\alpha\cosh(\alpha L) - i\delta\sinh(\alpha L)}{\alpha\cosh(\alpha L) - i\delta\sinh(\alpha L)}$$

$$= \frac{i\mathcal{E}_+(0)K\sinh(\alpha L)}{\alpha\cosh(\alpha L) - i\delta\sinh(\alpha L)} \quad \text{but } \mathcal{E}_+(0) = 1$$

$$r = \frac{iK\sinh(\alpha L)}{\alpha\cosh(\alpha L) - i\delta\sinh(\alpha L)}$$

$$R = |r|^2 = \frac{K^2\sinh^2(\alpha L)}{\alpha^2\cosh^2(\alpha L) + \delta^2\sinh^2(\alpha L)}$$

At peak reflectivity, $\delta = 0$, so:

$$R_{peak} = \frac{K^2\sinh^2(\alpha L)}{\alpha^2\cosh^2(\alpha L)} \qquad \text{also, } \alpha^2 = K^2 - \delta^2, \ \alpha^2 = K^2, \ \alpha = K$$

$$= \frac{\sinh^2(\alpha L)}{\cosh^2(\alpha L)}$$

$$R_{peak} = \tanh^2(\alpha L)$$

$$= \tanh^2(KL) \quad \text{as } \alpha = K \text{ for } \delta = 0$$

Now for small KL, we can Taylor expand:

let $x = KL$:

$$\tanh(x) = x - \frac{1}{3}x^3 + \frac{2}{15}x^5 \ldots$$

$$\tanh^2(x) = \left(x - \frac{1}{3}x^3 + \frac{2}{15}x^5 + \ldots\right)\left(x - \frac{1}{3}x^3 + \frac{2}{15}x^5 + \ldots\right)$$

$$= x^2 - \frac{1}{3}x^4 + \frac{2}{15}x^6 - \frac{1}{3}x^4 + \frac{1}{9}x^6 - \frac{2}{45}x^8 + \frac{2}{15}x^6 - \frac{2}{45}x^8 + \frac{4}{225}x^{10} + \ldots$$

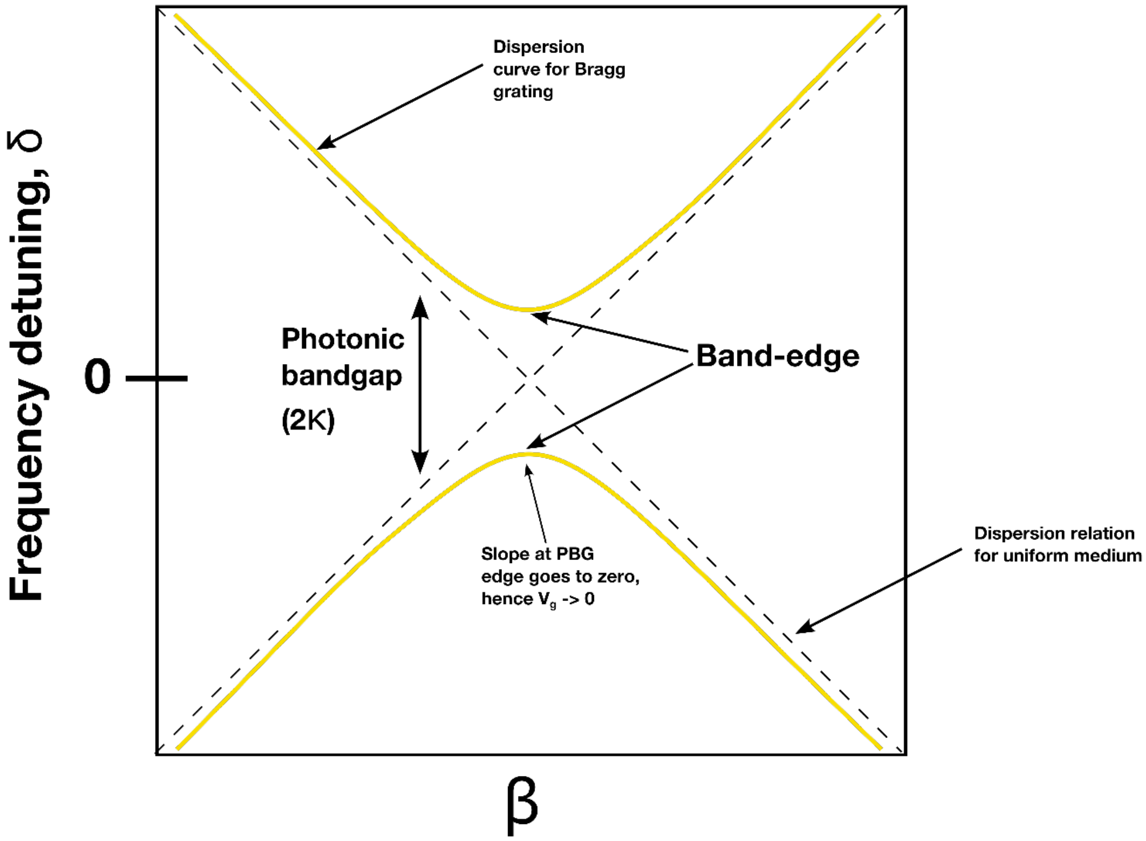
∴ Dropping high-order terms which are negligible, we have:

$$\tan^2h(KL) \sim (KL)^2 \text{ for } KL \ll 1$$

$$\rightarrow R = |r|^2 \sim (KL)^2$$

(c) Explain briefly using schematic diagrams the origin of dispersion and slow light in Bragg gratings when they are operated in transmission.

At frequencies outside of the stop-band, even though light is transmitted, it is still affected by dispersion due to the grating. Bragg gratings exhibit strong dispersion at frequencies close to the photonic band-gap (PBG) edge. This in turn results in a significant reduction in the group velocity of incident light (pulses propagate at much less than the speed of light) at these frequencies, giving rise to a phenomenon known as ‘slow light’. The origin of slow-light is elucidated by the photonic band-gap picture as per the diagram below. Here, the x-axis corresponds to wave number and the y-axis corresponds to frequency. The orange curves correspond to the dispersion relations for a Bragg grating: we note a band-gap for frequencies for which there are no propagations solutions for the coupled mode equations (evanescent solutions). However, for frequencies near the PBG, the slope of the dispersion curve (which corresponds to the group velocity v_g) approaches zero, resulting in a significant reduction in the group velocity of light. For a propagating pulse, this slow-light phenomenon can result in a significant delay as it propagates through the Bragg grating.



(d) Explain how the anisotropy in this structure affects the optical characteristics (e.g., optical transmission spectrum) as measured by probing from outside the structure.

As one changes the orientation of the crystal, the photonic bandgap can also change as a result of the changing . When probing the optical characteristics from outside the structure, these anisotropies can result in a shifting of peaks and dips in the transmission spectrum, reflecting the change in the photonic bandgap (and thus the allowed/forbidden frequencies) as a function of angle. Thus, the optical properties, namely the transmission spectrum, change as a function of angle of the incident light due to the anisotropy of the crystal.

(e) Explain "Complete photonic bandgap" in the context of two-dimensional photonic bandgap materials. Under what conditions can we expect a complete photonic bandgap.

A complete photonic bandgap is one in which light of certain frequencies cannot propagate through the 2D photonic crystal, regardless of its direction (angle in the context of a two-dimensional photonic bandgap material) and polarisation. That is, a range of frequencies for which light cannot propagate, common to all directions/angles of incidence in 2D. We expect a complete photonic bandgap when there is a large index contrast between the high-index inclusions and the low-index background. This is because the size of the band-gap is proportional to the index contrast Δn, and as result, a broader range of frequencies are reflected.

Question 2

(a) Discuss the importance of dispersion in achieving efficient frequency conversion with specific mention of the processes that rely on quadratic nonlinearities and cubic nonlinearities.

In order to achieve efficient frequency conversion for many processes that rely on quadratic non-linearities such as second harmonic generation (SHG), and three wave mixing (TWM), as well as third-order nonlinearities such as third harmonic generation (THG) and four-wave mixing (FWM), the phase-matching condition must be satisfied so that generated waves constructively interfere. For example, in the case of SHG, if the refractive index of the fundamental frequency does not match the refractive index of the doubled light, chromatic dispersion will cause a non-zero phase mismatch between the interacting waves and thus the phase-matching condition will no longer be met. This in turn results in the efficiency of conversion being diminished and a lower overall second harmonic output power. To properly manage dispersion so that phase-matching and therefore efficient frequency conversion occurs, dispersion engineering can be used. For example, birefringent materials have different refractive indices depending on polarisation – by rotating the birefringent crystal, one can tune the refractive indices so that the fundamental and second harmonic frequencies have equivalent refractive indices and thereby achieve phase matching.

(b) Achieving broadband frequency (say over an octave frequency range) conversion can be particularly challenging. Suggest two schemes for achieving frequency conversion over 1 octave frequency range based on (1) quadratic and then (2) cubic nonlinearities.

One scheme for achieving broadband conversion based on quadratic nonlinearities is to leverage second-harmonic generation (SHG) and sum frequency generation (SFG) along with the appropriate measures to ensure phase-matching. If two input frequencies are sent through a crystal with a large second order nonlinearity, this can result in the generation of light at the second harmonic of each of these input frequencies, as well as the sum or difference of the two. Thus, by tuning the two input frequencies, over 1 octave frequency range conversion can be achieved. In order to compensate for phase-mismatch and thereby enable efficient frequency conversion, quasi-phase matching (QPM) can be used.
In the case of cubic nonlinearities, supercontinuum generation can be used to achieve broadband frequency conversion over 1 octave frequency range. An effective approach would be to use a tapered chalcogenide fibre with a high gamma factor and therefore high degree of nonlinearity, allowing for dramatic spectral broadening over short device lengths and for relatively low peak powers (low energy threshold for SC generation). A femto-second laser would be used to generate the pulses for this system.

(c) Supercontinuum generation in photonic crystal fibres was first reported by Ranka in 2000, see the figure below. The key to this experiment was the special design of a silica fibre that enabled dramatic spectral broadening of the curve.

(i) Summarise key attributes of the fibre that allowed for this supercontinuum process.

To produce a supercontinuum, some of the key factors in designing an appropriate photonic crystal fibre include (1) reduced waveguide dimension to obtain enhanced nonlinearity and fine-tuned dispersion via tapering (small core diameters) (2) tailoring the geometry of the PCF via the arrangement and size of the air-holes to achieve a honeycomb-like structure (3) using an air-silica microstructure which exhibits anomalous dispersion which contributes to dramatic spectral broadening.

(ii) What limits the overall achievable bandwidth in this supercontinuum process?

Chromatic dispersion is what limits the maximum amount of spectral broadening possible. This is because dispersion can limit the efficiency of the underlying processes which contribute to supercontinuum generation, such as SPM and FWM. Further, the amount of nonlinearity in the material (as indicated by the gamma factor) will also be a limiting factor for the amount of spectral broadening achieved. The properties of the pulse itself (e.g., pulse duration and peak power) being launched into the fibre can also affect the achievable bandwidth.

(d) Nonlinearity and dispersion play a critical role in the design of long-haul optical fibre communication links.

(i) Why were optical solitons considered for optical communication systems in the first place?

Optical solitons were considered for optical communication systems because they exhibit unique properties such as robustness to pulse broadening, whereby the balance of dispersive and nonlinear effects allow the wave-packets to maintain their shape as they propagate without degradation over long distances (compared to conventional optical pulses). This makes optical solitons an ideal candidate as a carrier of information in long-haul optical fibre communication systems.

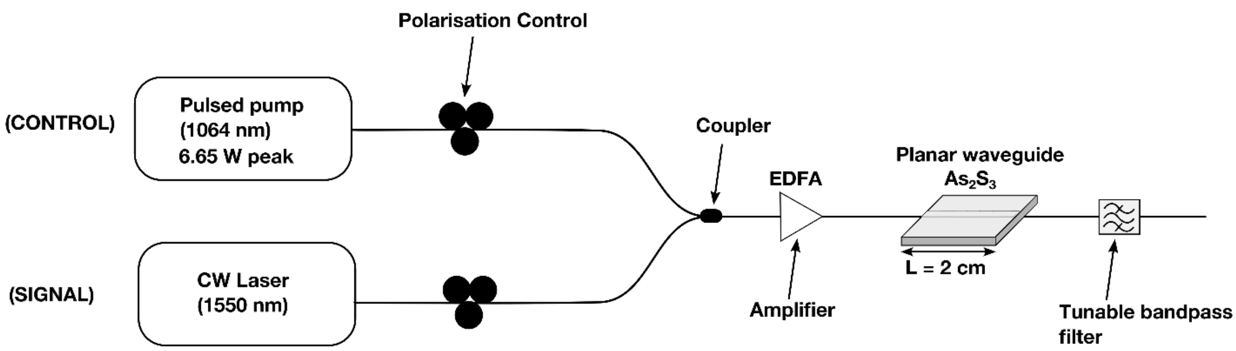
(ii) What happens to the shape of an optical soliton as it propagates along a transmission fibre due to the attenuation of the fibre?

As the soliton propagates along a lossy transmission fibre, the soliton peak power (amplitude) decreases. This in turn reduces the nonlinear effects (which are dependent on intensity), affecting the fine balance between non-linear effects and dispersion required for the soliton to maintain its shape. Correspondingly, the effects of fibre chromatic dispersion will begin to dominate as it is no longer supressed by nonlinear effects, and so the pulse will start to spread, resulting in an increase in its width that is in proportion to the amount of loss. Therefore, the amplitude of the optical soliton is reduced, and its width is increased as a result of attenuation. To account for fibre losses, optical amplifiers are typically introduced in soliton systems.

Question 3

An all-optical phase modulator would be a very useful component in an ultrafast communication system. Design and explain a system for modulating the phase of an optical beam using cross phase modulation at 1550nm in a chalcogenide planar waveguide chip of length L = 2cm. The mode area of the chip is 1 micron squared. The modulator is controlled by light from a pulsed laser of wavelength 1064nm. The chalcogenide has a refractive index of n = 2.5 and a nonlinear refractive index of $n_2 = 2\times10^{-18}$ m^2/W. Estimate the optical power of the controlling light (at 1064nm) that is necessary for modulating the phase of the controlling light by a π phase shift.

The signal at 1550nm and the control light at 1064nm are both coupled into the chalcogenide waveguide where the cross-phase modulation is mediated. Here, the intensity of the control light is pulsed to encode digital data (in the form of 0s and 1s): for example, if the bit is 0, then the control pulse power is set to a value which does not induce a phase shift in the signal light via XPM, while if the bit is 1, the control pulse power is set to a level that causes a π phase shift in the signal light (via XPM). Thus, by using XPM, one can encode information in the phase of an optical beam by pulsing the controlling light. Given the high nonlinearity of the chalcogenide waveguide, rapid phase modulation can be achieved (with ~ps optical pulses), facilitating ultrafast communication rates. A high-level schematic of one way to realise such as system is included below:



Estimation of optical power required:

$$\Delta\Phi_{\omega1.XPM} = 2\gamma P_2 L$$

$$P_2 = \frac{\Delta\phi}{2\gamma L} = \frac{\Delta\phi A_{eff}}{2Lk_0 n_2}$$

Using $\Delta\phi = \pi, A_{eff} = 10^{-12}m^2, n_2 = 2*10^{-18}m^2W^{-1}, L = 0.02\ m, k_0 = \left(\frac{2\pi}{\lambda_0}\right)$:

$$P_2 = \frac{\pi * 10^{-12}}{2 * 0.02 * 2 * 10^{-18} * (\frac{2\pi}{1064 * 10^{-9}})} = 6.65\ W$$

Therefore, approximately 6.65 W of optical power is necessary.