On the Ropes: An Experimental Investigation of Rope Extension

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I collected and analyzed data regarding the extension of different varieties of ropes that could potentially be used for rock climbing. The ultimate goal of this project was to identify the type of rope (subject to safety considerations) that stretches the most when weight is added to it. Ultimately, I ran two experiments, obtaining data on how a rope's extension is affected by the rope's diameter and the material out of which it is made. In the first experiment, I implemented a 4^2 factorial design, testing four rope materials and four rope diameters. The results indicated that cotton ropes stretch far more than do ropes made of any other material tested. The results also showed that the effect of increasing a rope's diameter varies between rope materials and is a much less important factor than the material out of which the rope is made. Since the regression results regarding the effect of diameter were less significant than those regarding the effect of material, it seemed that there might be room to clarify the diameter-related results by obtaining additional data. Having identified cotton as the rope material that stretches the most, I conducted a follow-up experiment, using only cotton ropes, in order to gather additional information on the effect of rope diameter. The results of the second experiment were very similar to those obtained in the first. They indicated that, for cotton ropes, there is a slight decrease in stretch that comes with increasing the rope's diameter. Based on the results of both experiments, it seems that cotton ropes with lower diameters are preferable, but that the effect of the rope's material is much stronger than that of the rope's diameter. Given the opportunity to redo the experiment, I would attempt to introduce more precision into my measurements and more variety into the factors being tested. In particular, I might utilize a greater number of diameters in order to assess whether the material-specific trends continue outside the range of diameters tested.[1]

1 EXPERIMENTAL DESIGN

When rock climbing, a rope is usually attached to the climber's harness, in order to protect the climber from injury, should he or she fall. The choice of rope can dramatically affect the shock that a climber experiences upon falling. One often wants one's ropes to stretch moderately when weight is applied to them because, when one falls, a more flexible rope will make the end of that fall softer (because the rope stretching will make the stop at the bottom of the fall less sudden). I conducted an experiment in order to examine how the rope's material and diameter affect the degree to which it stretches. This information was obtained in two experiments.

The first experiment was conducted using ropes made of four different materials: cotton, nylon, manila hemp, and polypropylene. For each material, four diameters of rope ($\frac{1}{4}$ inches, $\frac{3}{8}$ inches, $\frac{1}{2}$ inches, and $\frac{5}{8}$ inches) were used. All ropes were obtained from the same manufacturer. In the second experiment only cotton ropes (chosen based on the results of the first experiment) were used. The cotton ropes employed in the second experiment were of the same diameters as those used in the first experiment.

I attempted to obtain a wide enough range of diameters that, if a relationship between diameter and rope stretching exists, it could be clearly identified. Thus, I picked some ropes that were slightly outside the range of

[1]Additional suggestions for improving precision and potential additional factors to include are discussed in the conclusion of this report.

widths normally used for climbing (which are generally closer to $\frac{1}{2}$ inches than to $\frac{1}{4}$ inches). The rope materials used in this experiment were also selected with an eye toward breadth. For this reason, I chose two natural rope materials and two synthetic ropes materials. Within those categories, I chose one fiber known for its durability (manila hemp for the natural group and polypropylene for the synthetic group) and one known for its flexibility (cotton for the natural group and nylon for the synthetic group). Only ropes that are known to not repeatedly stretch and recoil (i.e., bounce) when weight is added were chosen. For safety reasons, one generally wants to avoid a large amount of repeated extension and recoil, so that the climber's position stabilizes quickly after a fall. It is for this reason that ropes are not usually made of materials such as bungee cord. While finding a balance between rope stretch and rope recoil is an interesting avenue of exploration, doing so would have been a more significant undertaking than could be accomplished with the resources available. As a result, I elected to limit the rope materials to those known to not have that undesirable quality, even if they might stretch more than some of the ropes included in the experiment.

To measure how much the ropes stretched when weight was attached to them, I cut three-foot units of rope and marked off the middle foot before applying any weight. That is, one marking was one foot from the top of the rope and the other marking was one foot from the bottom of the rope. One end of the unit of rope was attached to a beam; a 10-pound weight was attached to the other end. Then the distance between the markings on the rope (which were separated by one foot when there was no weight attached) was measured and the distance greater than the initial one foot span was recorded. Measurements were taken to the nearest sixteenth of an inch on a standard yardstick. I refer to the distance greater than one foot as the "extension" of the rope.

Before conducting the experiment, I expected that different rope materials would stretch to different degrees. This seemed logical, given the different uses that these rope materials often have, some materials being preferred when stretching is necessary and others not so. The cotton rope seemed likely to be the most stretchable, since cotton ropes are often used in applications in which that quality is prized (home goods, pet products, etc.). On the other hand, the manila hemp rope seemed least likely to exhibit significant stretch, because the material is often used in situations where stretching might be disadvantageous, (such as ship lines and fish nets). For similar reasons, I anticipated that nylon would stretch slightly less than cotton and that polypropylene would stretch slightly more than manila hemp. Additionally, I anticipated that thicker ropes would stretch more than thinner ropes, because thicker ropes have more fibers out of which slack could be taken, when weight is applied.

2 Analysis of Experimental Data

In the first experiment, I tested all 16 rope material/diameter combinations four times. This replicated 4^2 factorial design was chosen for efficiency, as I was interested equally in the effects of both factors. Each of the sets of tests occurred at a different time. I treated each of the four sets of tests as a separate block, in an effort to control for the potential effects of temperature, humidity, and other time-dependent environmental factors. Within each block, the rope materials and diameters were tested in a random order, chosen via a random number generator. Below, I present analyses of the data obtained during the experiment. In the first subsection, I analyze the data purely based on its appearance in various graphs. Then, I present statistical results, including an analysis of variance (ANOVA) in the second subsection and a linear regression in the third subsection. (All figures and tables can be found in the appendix.[2])

In the follow-up experiment, I collected 20 data points on cotton ropes of different diameters, obtaining five data points per diameter.[3] I did not include a block variable, because all observations in the follow-up experiment were obtained in the span of an hour. In the fourth subsection, I present a regression analysis using the data from the follow-up experiment. That analysis regresses the log of extension on rope diameter. The hope was that removing block-to-block variation (which was present in the first experiment) and including a slightly higher number of observations would lead to clearer regression results. No data from the first experiment was used in this regression, because the first experiment was the basis for the selections made in conducting the second. As a result, including data from the first experiment in the more focused regression might invalidate the inferences I am otherwise able to make. All of my analyses treated both rope material and diameter as fixed effects, since the ropes used in the experiment are similar to those used for climbing.

[2]Data collected during the first experiment is presented in Table 1 of the Appendix.
[3]Data collected during the follow-up experiment is presented in Table 4 of the appendix.

2.1 Graphical Analyses of Factorial Experiment

The box plot of rope extension by material (Figure 1) appears to indicate relatively large variation between materials. Cotton appears to have both the highest median extension and the largest range. Nylon has a median extension that is approximately half that of cotton's, while manila hemp and polypropylene both have similarly small median extensions. All materials except cotton seem to have reasonably small ranges. In contrast, the box plot of rope extension by diameter (Figure 2) does not show nearly the same differences between factor levels. All rope diameters seem to have approximately the same median and range.

Figure 3, which charts average rope extension by material and diameter, and Figure 4, which plots all data points by material and diameter, are consistent with the box plots. Cotton shows the highest extension, across all diameters. Nylon is consistently second. And, manila hemp and polypropylene both have relatively low extension across diameters. However, Figures 3 and 4 do seem to suggest that some variation in extension may be associated with changes in the rope's diameter. A statistical analysis is necessary to test whether an association actually exists. Particularly, it is interesting that, for both cotton and nylon, there seems to be a decrease in extension between $\frac{1}{4}$ inches and $\frac{3}{8}$ inches, but then an increase in extension between $\frac{3}{8}$ inches and $\frac{1}{2}$ inches. It is possible that there is something unique about $\frac{3}{8}$ inch ropes, but it is much more likely that the fluctuations are simply attributable to experimental randomness. That theory is reaffirmed by the extreme data points recorded for cotton ropes with diameters of $\frac{3}{8}$ inches and $\frac{1}{2}$ inches. These points seem to suggest relatively high variance in the physical process that generated the data.

2.2 Analysis of Variance for Factorial Experiment

As is clear from Figure 4, the data points from different rope materials and diameters have quite different variances. This is consistent with what one might expect from such an experiment. The physics is such that smaller data points come from ropes less prone to stretching and ropes less prone to stretching are unlikely to exhibit as much variability in extension between runs. Due to this observed difference in variance, all analyses are performed on the log of the output variable (extension), rather than the raw data, in order to avoid statistical problems with heteroscedasticity. Additionally, it seems logical that the factors, especially diameter, might have a multiplicative effect, rather than an additive one. That is, rather than each additional fraction of an inch of diameter contributing X inches of extension, the marginal increase may be related to how much the rope stretches already. This theory reaffirms the appropriateness of a log-linear model.

After taking the log of extension, I first performed an ANOVA in which I included rope material, rope diameter, a material-diameter interaction term, and the block in which an observation was recorded as independent variables. Here, I treated diameter as a categorical variable, with the intent of understanding the magnitude of its effect, absent any assumptions about that effect's linearity. (The next subsection reviews a regression model that tests whether or not a linear relationship exists between diameter and the log of extension.) The results of the ANOVA are presented in Table 2. Consistent with the graphical analysis, the ANOVA results indicate that the material out of which a rope is made is very influential in determining the extent to which the rope stretches. Per the decomposition of variance, a rope's material explains about 84% of the variance in the log of extension. The rope's diameter also has a statistically significant effect, but only explains about 3% of variance. The material-diameter interaction term is not significant; however, it's p-value is only around 0.14, which comes close to conventional significance thresholds. The block variable, in contrast, is not significant at all, with a p-value of nearly 0.85; it explains almost none of the variance.

Having obtained results from the ANOVA, it is important to check that the model's assumptions are fulfilled by our data set. (If the assumptions do not hold, the ANOVA's p-values are suspect.) In running the ANOVA, it is assumed that the model is a reasonable description of the data and that any experimental error (for which we use residuals as a proxy) is normally and independently distributed, with mean 0 and constant variance. Per Figure 5, the residuals appear to be roughly normal, as most points fall on or near the line that marks the quantiles that would be expected if the residuals were from a normal distribution. There are a few data points further from that line, at the more extreme quantiles; however, because the ANOVA is reasonably robust to moderate departures from normality, this is not a cause for significant concern. Figure 6 plots residuals by the order in which the associated data point was obtained within its block. There does not seem to be any decisive trend over the course of a block, nor any indication of correlation between residuals of data points obtained within the same block. Similarly, the plot of fitted values against residuals (Figure 7) is in line with expectations. The residuals appear generally unrelated to the fitted values. There is a slight decrease in variance for the residuals at higher fitted values. This trend is not decisive enough to cause suspicion that the ANOVA is invalid. Finally, the residuals by rope material and rope diameter are plotted in Figures 8 & 9. There does appear to be some heteroscedasticity in both plots (even though the output is logged). However, the heteroscedasticity is sufficiently mild that it does not raise concern that the model is truly inaccurate.

2.3 Regression for Factorial Experiment

I also performed a linear regression in order to assess the direction of change in rope extension associated with the factor levels. All variables that were used in the ANOVA, even those that did not turn out to be significant, were also used in the linear regression in order to avoid invalidating the p-values and confidence intervals obtained. Regression results are reported in Table 3. In general, I am more interested in the direction and relative size of the effect of each factor level than I am in the number of inches of extension associated with it, because the latter will ultimately depend on the weight attached to the rope.

Binary variables for each rope material were created to analyze the materials' effects. I used polypropylene as the baseline material. Consistent with the graphs reviewed earlier, all material-related variables, except for the one associated with manila hemp have coefficients that are large and positive. That is, both cotton and nylon ropes achieve much greater extension than polypropylene and manila hemp ropes. The variables identifying cotton and nylon ropes are both highly significant, with p-values of $2.16 \text{x} 10^{-9}$ and $4.98 \text{x} 10^{-5}$, respectively. Manila hemp, on the other hand, does not stretch in a way that is significantly different from polypropylene.

In the ANOVA discussed previously, diameter is treated as a categorial variable with four levels. However, I am also interested in whether or not a monotonic relationship exists between diameter and extension. Such information might indicate whether ropes smaller or larger than those included in the experiment ought to be investigated. (E.g., if rope extension is an increasing function of diameter, one might attempt to find the rope of the largest diameter that could be reasonably used.) If it appears that no monotonic relationship exists, one might simply choose the rope diameter with the largest average extension. A statistical test of this relationship is particularly critical, given the fluctuations in extension across diameters of cotton and nylon ropes. As a result, in the regression, diameter is treated as a continuous variable that records the number of sixteenths of an inch in the rope's diameter. (E.g., the variable is equal to 4 for all $\frac{1}{4}$ diameter ropes, because there are 4 sixteenths of an inch in $\frac{1}{4}$ inches.) Additionally, recall that the output variable has been log-transformed, so small coefficients can be interpreted as percent increases in extension. Thus, the coefficient for the diameter variable can be interpreted as the percent change in extension per $\frac{1}{16}$ inch increase in diameter.

Per the regression results, it seems that there is, in fact, a relationship between rope extension and diameter. The diameter variable is significant at the 0.01 level and indicates that there is about a 12% increase in rope extension associated with each additional sixteenth of an inch of diameter. The variables denoting cotton and nylon ropes both have a statistically significant interaction with the diameter variable. Nylon's interaction with diameter makes the effect of diameter roughly neutral. I.e., the diameter variable indicates that there is about a 12% increase in rope extension per sixteenth of an inch of diameter and the interaction term indicates that the extension of nylon ropes decreases about 12% per sixteenth of an inch of diameter. Cotton's interaction with diameter indicates that, for cotton ropes, the positive effect of increasing diameter is actually reversed. Between the main effect of diameter and the interaction between diameter and cotton, cotton ropes see about a 3% decrease in extension per sixteenth of an inch of additional diameter.

I also performed model adequacy checks for the regression. Like the ANOVA, the linear regression assumes normally distributed errors with mean 0 and constant variance. Per Figure 10, the residuals are close to normally distributed. Some of the residuals at the lower quantiles appear to be a bit inconsistent with normality (falling away from the line of expected quantiles), but not sufficiently so to discredit the regression. I also examined the residuals by block order (Figure 11). It does not appear that residuals change in a patterned way over the course of a block; nor is there much indication that residuals from the same block are similar. The graph of regression residuals against their fitted values (Figure 12) does not indicate any discernible pattern. It seems as though there might be a very slight decrease in the variance of the residuals, as the fitted values increase.

However, this change is not large enough to be cause for concern. I also examined the influence of each observation, by graphing the DFFITS for each data point. Only a few points appear to have uniquely high influence (i.e., high absolute DFFITS). These points (e.g., observations 2 and 47) are manila hemp or polypropylene ropes with higher-than-usual levels of extension. I elect not to exclude them from the regression, both because their influence is not so great as to be truly worrisome and because those data points likely provide substantive information regarding the variation of the physical process from which they came.

2.4 Regression for Follow-Up Experiment

As described previously, I collected additional data in a follow-up experiment with only cotton ropes. Using only the new data, I performed a simple linear regression of the log of extension against the continuous diameter variable (i.e., diameter in sixteenth's of an inch). The results of the regression are reported in Table 5. The output shows a very similar relationship between extension and diameter as the one identified in the initial experiment. Per the regression results, for cotton ropes an additional sixteenth of an inch of diameter results in about a 5% decrease in that rope's extension. The relationship is significant at the 0.05 level, with a p-value of 0.03. While this does not provide any new information about the effect of diameter on a rope's extension, it does at least confirm the findings of the initial experiment.

I performed the same model adequacy checks on this regression as on the previous one. Per Figure 14, the residuals do appear roughly normally distributed. Figure 15 graphs the regression's residuals against the corresponding fitted values. There does not appear to be a pattern to the residuals. While the third group of residuals does appear slightly lower than the others, that appearance seems to be largely due to one particularly small residual (rather than the group being truly different). Figure 16 displays the residuals by the order in which the observation was taken. There does seem to be a slight upward trend over the course of the block. However, the appearance of an upward trend is largely due to one particularly small residual and is not decisive enough to be convincingly nonrandom. Finally, I graph the DFFITS of each observation, in Figure 17, and note that none of them seem to have a uniquely strong influence on the regression. Thus, I do not implement any adjustments to the regression.

3 Conclusions

A few conclusions can be drawn from both the initial experiment and the follow-up. First and foremost, it seems that the material out of which a rope is made accounts for the overwhelming majority of variation in how much that rope stretches. Consistent with expectations, cotton ropes clearly stretch more than any other rope that was tested. As a result, I can comfortably conclude that I would choose cotton ropes for climbing. Secondarily, (and in contrast to my initial expectations) the extension of cotton ropes decreases with diameter. If one were trying to maximize rope extension, one should attempt to obtain the lowest-diameter, cotton rope that would be safe. Generally, it is interesting to know that the effect of diameter changes with rope material. It is possible that a climber may, for various reasons (cost, concerns about durability, etc.), prefer a rope made of a material other than cotton. Based on the results of this experiment, if that individual opts for polypropylene or manila hemp ropes, for example, he/she may want to choose a higher-diameter rope in order to maximize stretch.

As mentioned earlier, there are a few improvements that could be made if a similar experiment were to be performed in the future. First, one ought to explore a wider range of diameters, in order to assess whether the trends observed are present outside of the diameter range tested. Second, additional precision in measurement might be useful. In order to make the results more precise, one might find a yardstick with smaller intervals between ticks or a piece of technology which could provide very finite measurements. Finally, it might be informative to look at more factors that could affect rope extension. For example, one might test the ropes under different conditions, e.g., wet ropes vs. dry ropes, in order to identify whether the weather conditions one might affect which rope choice would be optimal. Additionally, it could be informative to test the ropes with additional weight (closer to the weight of the human they might be used to hold, in practice). It's possible that different results might appear under different weight settings.

4 APPENDIX

Table 1: Experimental Data from 4^2 Factorial Experiment[4]

Material	$\frac{1}{4}$ inches	$\frac{3}{8}$ inches	$\frac{1}{2}$ inches	$\frac{5}{8}$ inches
Cotton: Block 1	1.125	0.625	1.000	0.875
Cotton: Block 2	1.375	0.750	1.250	1.125
Cotton: Block 3	1.500	1.750	0.938	1.000
Cotton: Block 4	1.375	0.938	1.688	0.938
Manila Hemp: Block 1	0.125	0.063	0.125	0.250
Manila Hemp: Block 2	0.063	0.125	0.125	0.125
Manila Hemp: Block 3	0.125	0.125	0.125	0.188
Manila Hemp: Block 4	0.188	0.063	0.188	0.250
Nylon: Block 1	0.625	0.375	0.750	0.438
Nylon: Block 2	0.563	0.500	0.500	0.500
Nylon: Block 3	0.750	0.500	0.625	0.688
Nylon: Block 4	0.563	0.313	0.563	0.500
Polypropylene: Block 1	0.188	0.250	0.125	0.188
Polypropylene: Block 2	0.125	0.063	0.313	0.188
Polypropylene: Block 3	0.063	0.063	0.250	0.250
Polypropylene: Block 4	0.063	0.188	0.188	0.125

Table 2: Analysis of Variance Results from 4^2 Factorial Experiment
Note: Output variable is logged.

Source	DF	Sum of Sq.	Mean Sq.	F-Value	P-Value
Block	3	0.11	0.035	0.269	0.84760
Material (A)	3	51.78	17.259	131.814	$<2 \times 10^{-16}$
Diameter (B)	3	1.68	0.561	4.282	0.00965
AB	9	1.88	0.209	1.599	0.14464
Residuals	45	5.89	0.131		

Table 3: Regression Results from 4^2 Factorial Experiment
Note: Output variable is logged.

Variable	Estimate	Std. Error	T-Value	P-Value
Intercept	-2.79171	0.31711	-8.804	5.95×10^{-12}
Block 2	-0.04963	0.13196	-0.376	0.70832
Block 3	0.06455	0.13196	0.489	0.62674
Block 4	0.01238	0.13196	0.094	0.92561
Cotton	3.12151	0.43365	7.198	2.16×10^{-9}
Manila	0.04076	0.43365	0.094	0.92546
Nylon	2.20372	0.43365	5.082	4.98×10^{-6}
Diameter	0.12129	0.04173	2.907	0.00532
Cotton:Diameter	-0.15598	0.05901	-2.643	0.01078
Manila:Diameter	-0.02138	0.05901	-0.362	0.71860
Nylon:Diameter	-0.12797	0.05901	-2.168	0.03463
R-Squared: 0.8796 **Adjusted R-Squared:** 0.8569 **F-Statistic:** 38.73 **– P-Value for F-Statistic:** $<2.2 \times 10^{-16}$				

[4] Numbers are rounded to 3 decimal places.

Table 4: Experimental Data from Follow-Up Experiment[5]

Diameter	Extension
1/4"	1.125
1/4"	1.500
1/4"	1.063
1/4"	1.813
1/4"	1.313
3/8"	0.750
3/8"	0.938
3/8"	1.375
3/8"	1.063
3/8"	1.313
1/2"	1.063
1/2"	1.563
1/2"	0.875
1/2"	1.188
1/2"	1.250
5/8"	0.813
5/8"	0.938
5/8"	0.875
5/8"	1.250
5/8"	0.938

Table 5: Regression Results from Follow-up Experiment

Note: Output variable is logged.

Variable	Estimate	Std. Error	T-Value	P-Value
Intercept	0.46472	0.15531	2.992	0.00782
Diameter	-0.04938	0.02114	-2.336	0.03124
R-Squared: 0.2114 Adjusted R-Squared: 0.2327 F-Statistic: 5.458 – P-Value: 0.03124				

[5]Numbers are rounded to 3 decimal places.

Figure 1: Box Plot of Rope Extension, by Rope Material

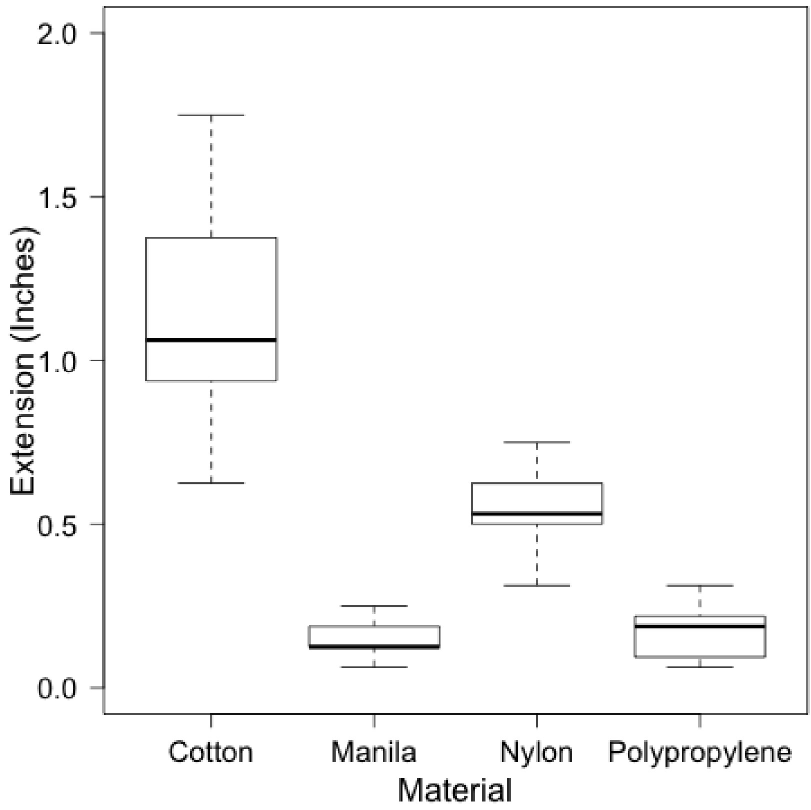

Figure 2: Box Plot of Rope Extension, by Rope Diameter

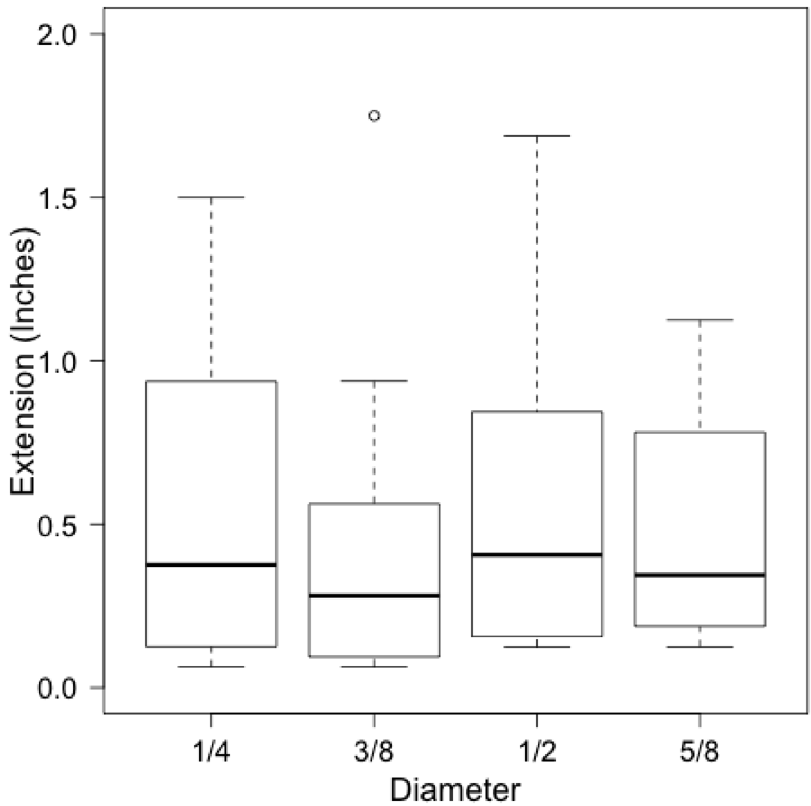

Figure 3: Average Rope Extension, by Rope Material and Rope Diameter

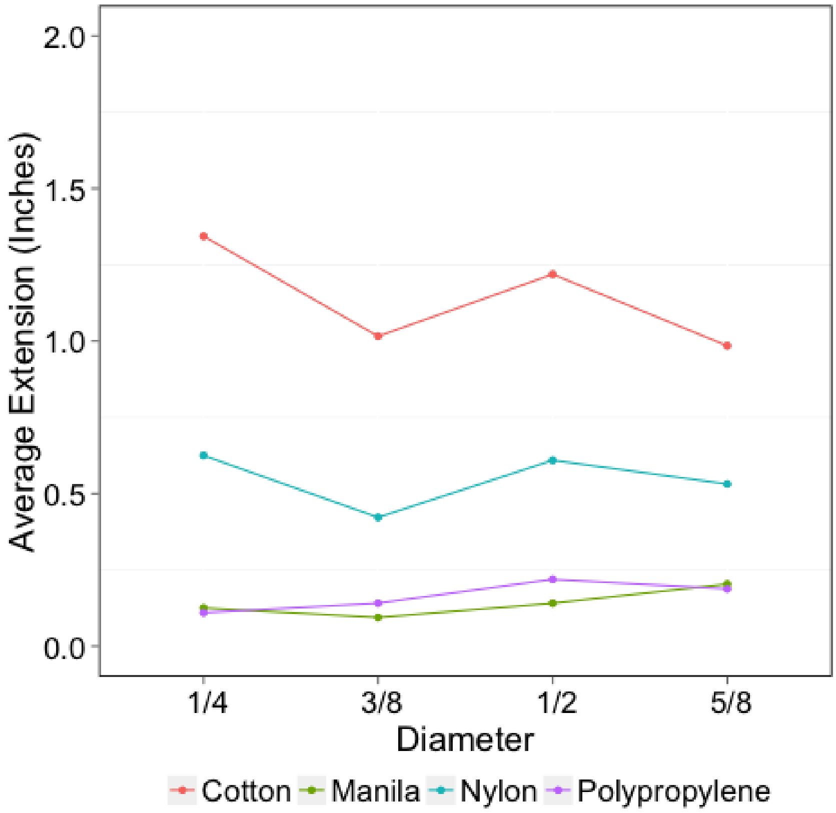

Figure 4: Rope Extension, by Rope Material and Rope Diameter

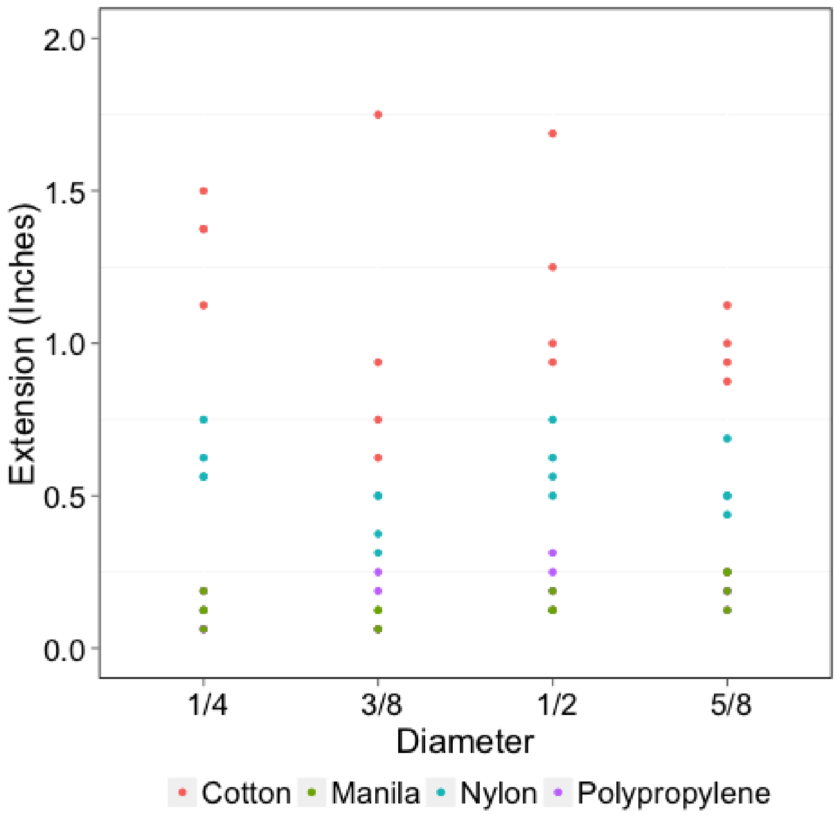

Figure 5: Normal Probability Plot of ANOVA Residuals

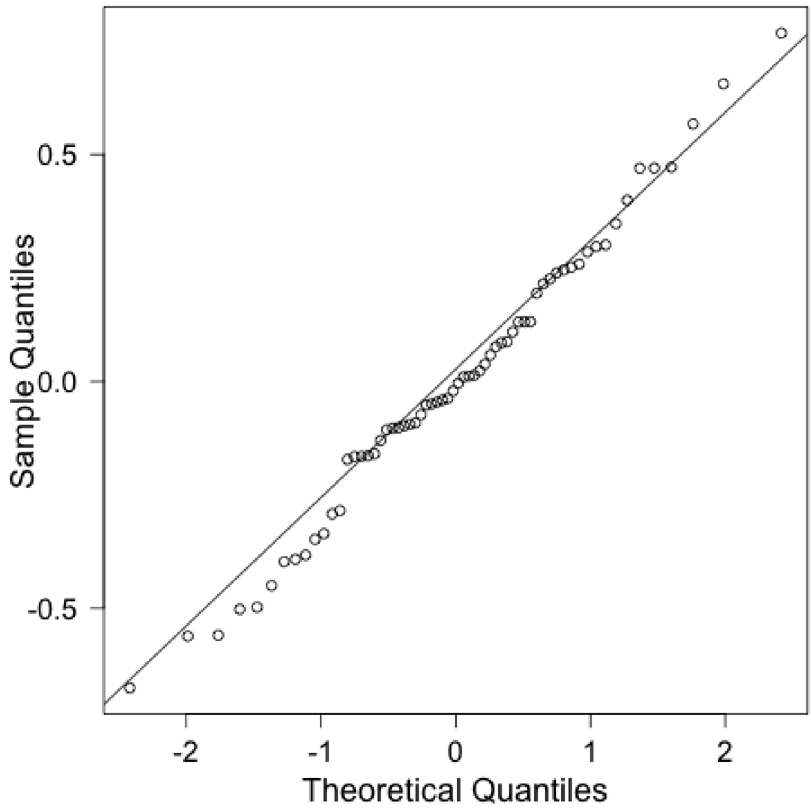

Figure 6: ANOVA Residuals by Block Order

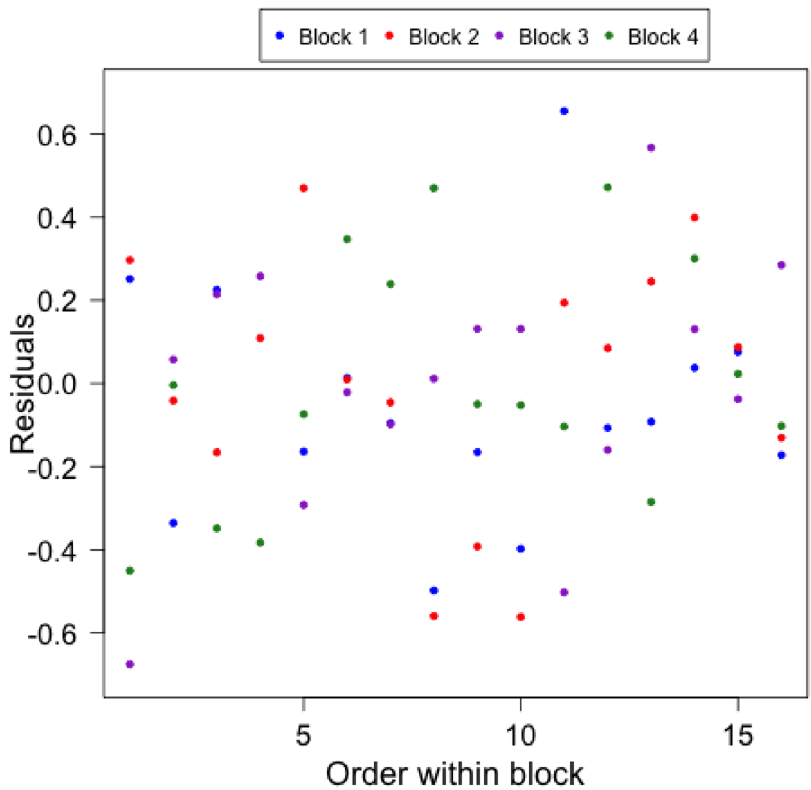

Figure 7: ANOVA Residuals vs. Fitted Values

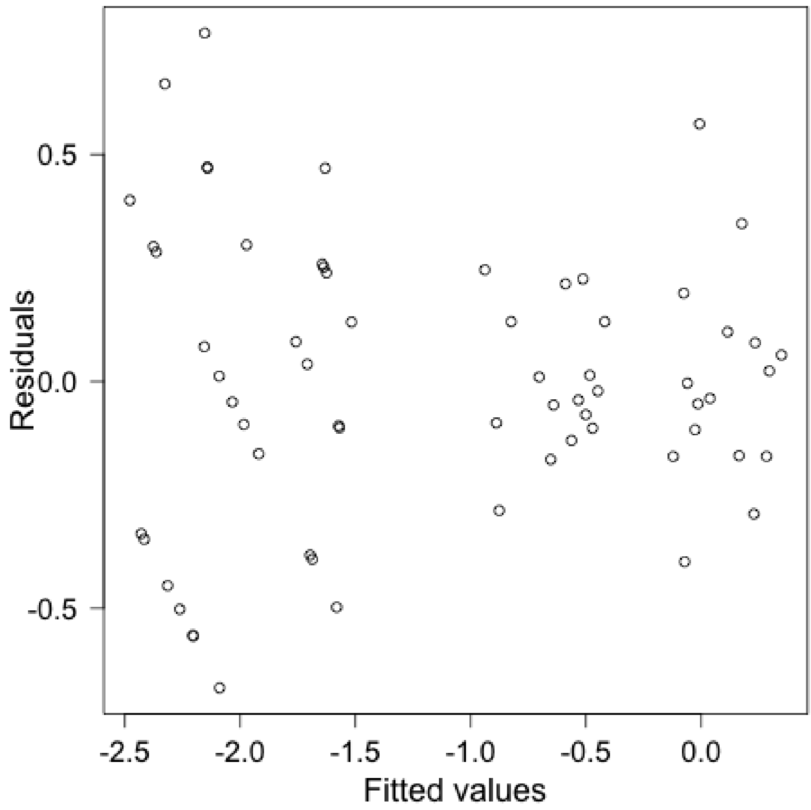

Figure 8: ANOVA Residuals by Rope Material

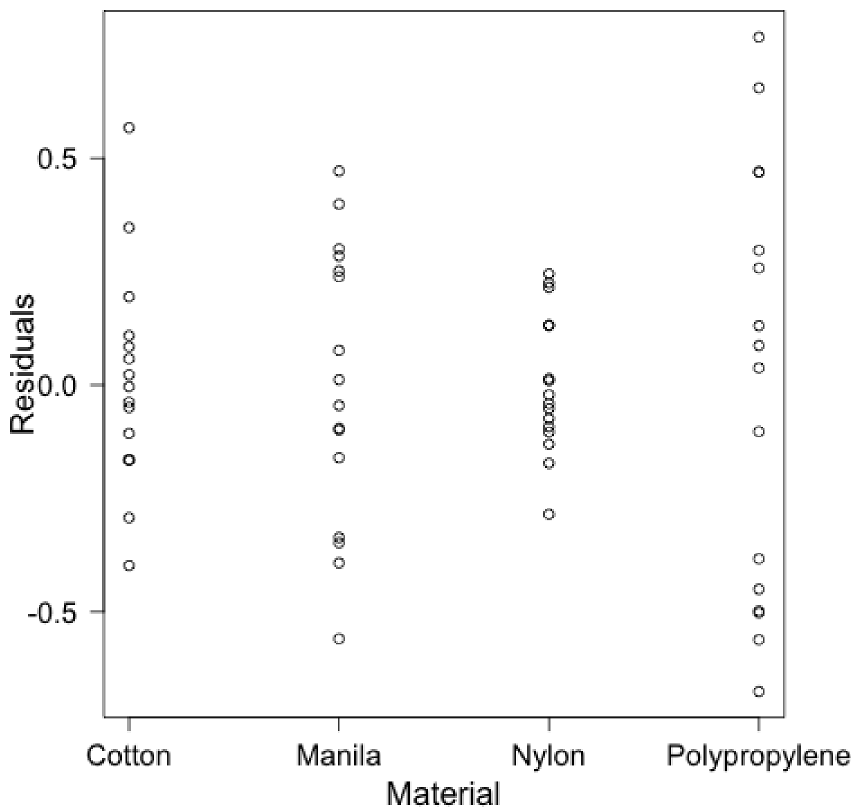

Figure 9: ANOVA Residuals by Rope Diameter

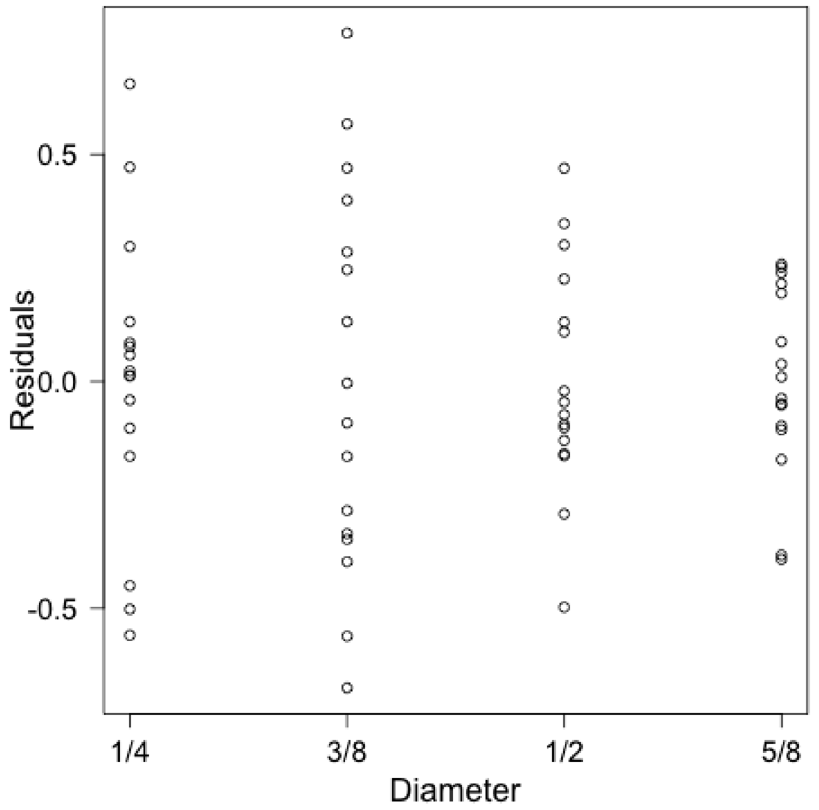

Figure 10: Normal Probability Plot of Primary Regression Residuals

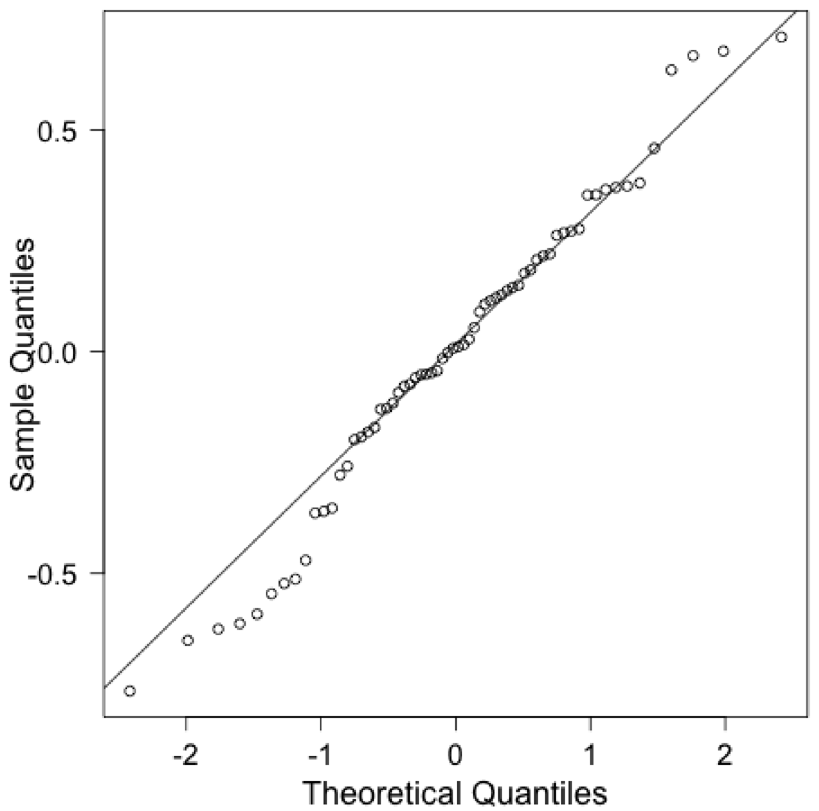

Figure 11: Primary Regression Residuals by Block Order

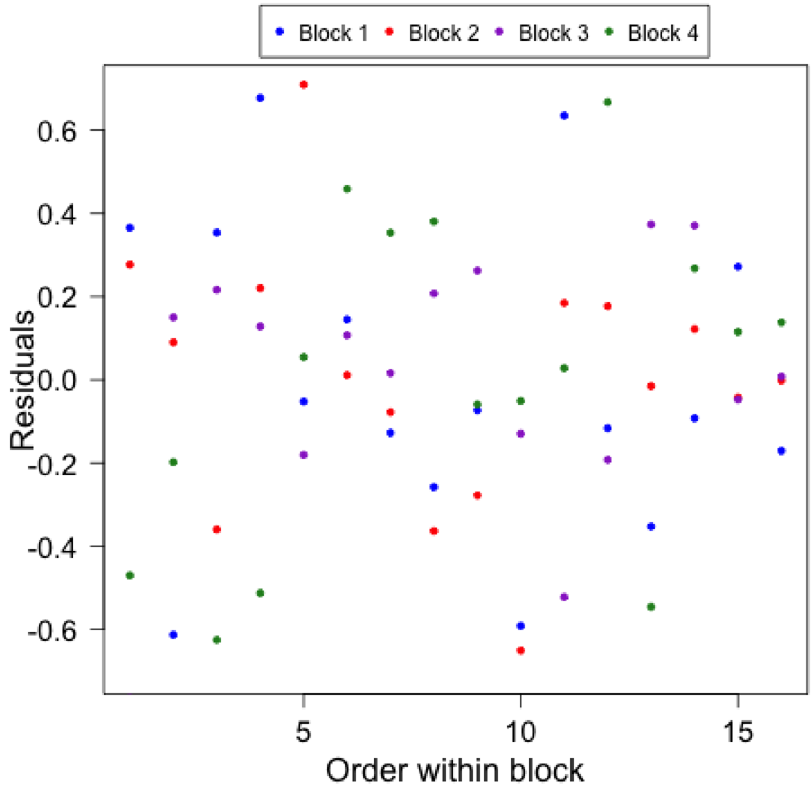

Figure 12: Primary Regression Residuals vs. Fitted Values

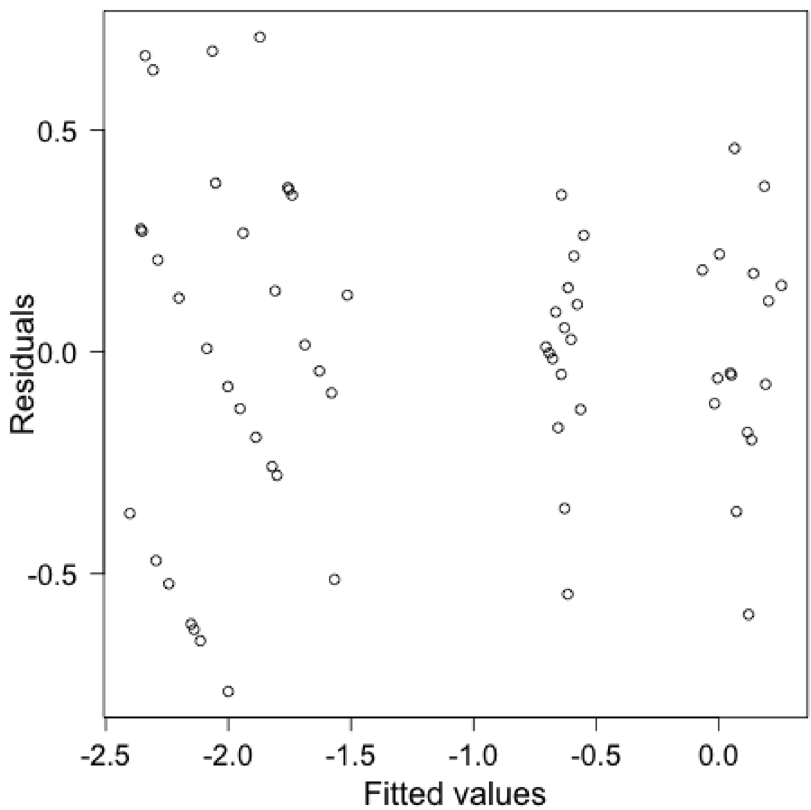

Figure 13: DFFITS for Primary Regression

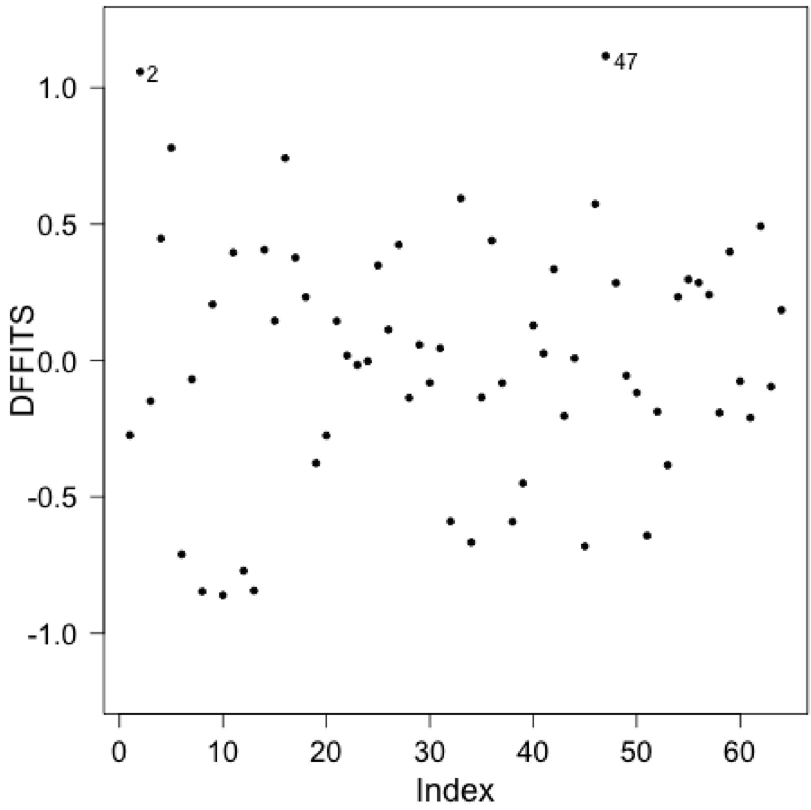

Figure 14: Normal Probability Plot of Follow-Up Regression Residuals

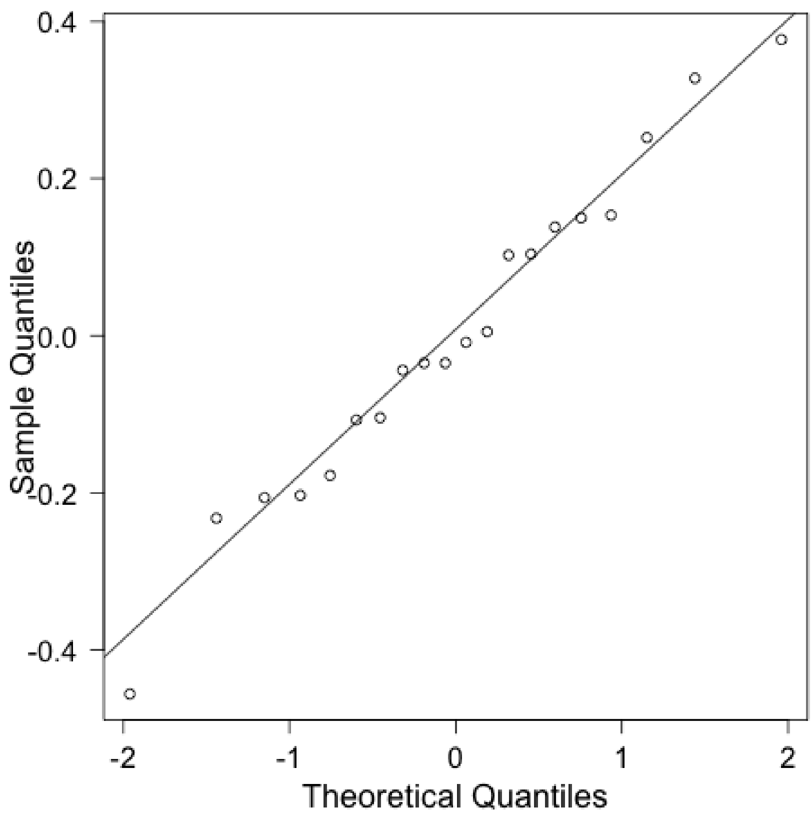

Figure 15: Follow-Up Regression Residuals vs. Fitted Values

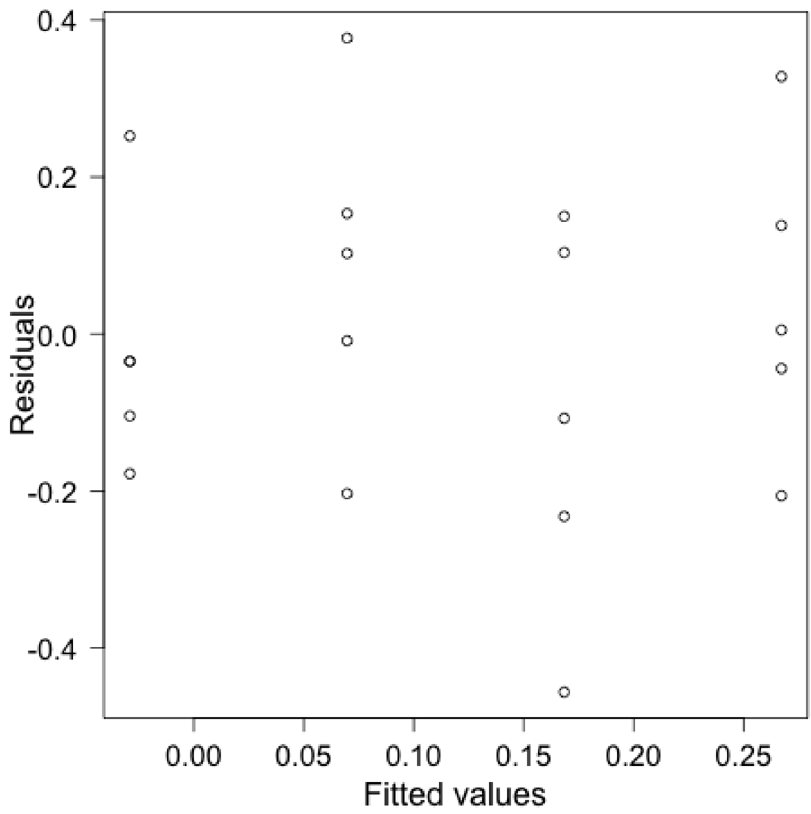

Figure 16: Follow-Up Regression Residuals by Block Order

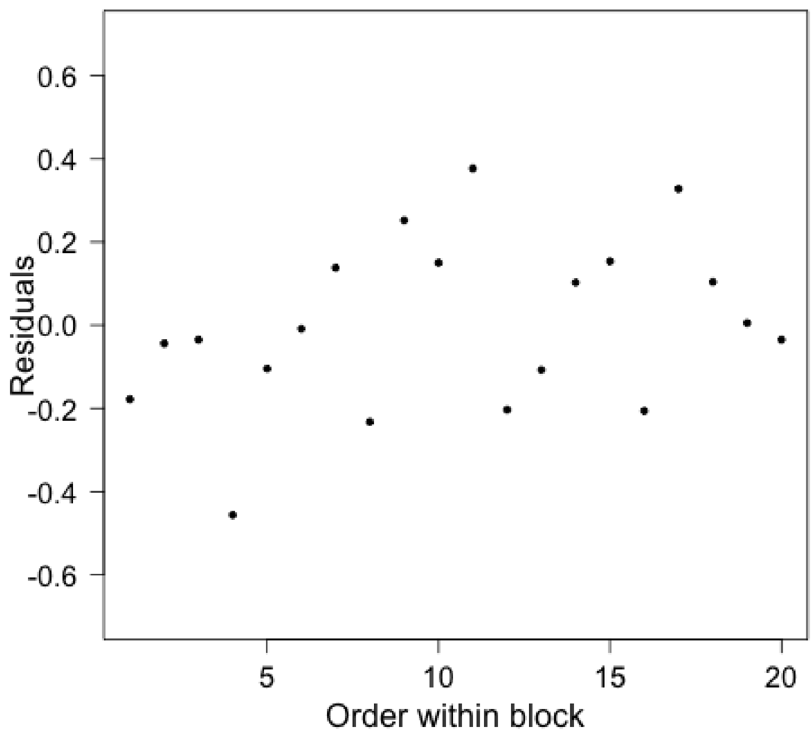

Figure 17: DFFITS for Follow-Up Regression