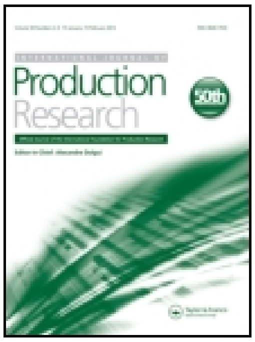

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An approach to establishing a method for calculating inventory

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In periodic review inventory systems, inventory is classified into cycle stock and safety stock. Cycle stock is defined as inventory that absorbs differences between supply and demand frequencies. It can be calculated without deficiency or excess because a method has been established for ensuring that the minimum on-hand inventory during a periodic review is zero. Safety stock is defined as inventory that absorbs various differences between supply and demand. Unlike for cycle stock, a method for calculating safety stock without deficiency or excess remains to be established. An approach is proposed to establishing a method for calculating inventory in which inventory is classified on the basis of the holding purpose and the calculation factors indicate solutions. This approach was applied to inventory held to absorb, on the basis of fluctuations in demand, the difference in terms of time and quantity between supply and demand. Stock held for this purpose is referred to as 'fluctuation stock'. The objective is to establish a method for calculating fluctuation stock so that the minimum on-hand inventory during a periodic review is zero and to clarify the relationship between fluctuation stock and safety stock.

Keywords: fluctuation stock; periodic review system; safety stock

1. Introduction

In periodic review inventory systems, inventory is classified into cycle stock and safety stock. Cycle stock and safety stock are calculated separately, and the target level is set as their sum. Cycle stock is defined as inventory that absorbs differences between supply and demand frequencies and is given by

$$cS = \bar{d} * (T + L) \quad (1)$$

$\bar{d}$: Average demand

T : Review period

L : Lead time.

Use of this equation ensures that cycle stock is calculated without inventory deficiency or excess because the minimum on-hand inventory during a periodic review period is zero. An increase in the supply frequency leads to a reduction in inventory, which is quantified by this equation. The reduction in inventory depends not only on supply frequency but also on average demand because the average demand variable is included in this equation.

Safety stock is defined as inventory that absorbs various differences between supply and demand. These differences result from changes in the timing of customer purchases, differences between average demand and actual demand, fluctuations in demand, machine breakdowns, employee absenteeism, material shortages, product defects, etc. Traditionally, safety stock is given by

$$sS = z * \sigma_t * \sqrt{T + L}$$

z : Safety factor

σ_t : Standard deviation of demand per unit time. (2)

If the minimum on-hand inventory during a periodic review period is positive, there is an overstock situation. If the minimum on-hand inventory is negative, there is a shortage situation. That is, if the results of the calculation lead to the minimum on-hand inventory being zero, the calculation method is one that gives inventory without deficiency or excess. With the equation above, unlike for cycle stock, the minimum on-hand inventory of safety stock is not zero. A method for calculating safety stock without inventory deficiency or excess remains to be established.

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Safety stock has also been assumed to be the inventory held to avoid inventory shortages caused by uncertainty. Holding a larger inventory imposes higher carrying costs, while holding a smaller inventory imposes higher shortage costs. An optimal policy minimises production cost for safety stock. This policy is briefly described in the following section. As mentioned above, safety stock is defined as inventory that absorbs various differences between supply and demand. In other words, safety stock is inventory with mixed holding purposes.

To establish a calculation method for safety stock, the holding purposes should be identified in the same manner as they are for cycle stock. To the best of our knowledge, there has been no previous research that identified the various purposes of safety stock and established a calculation method for each one. This is the approach we believe needs to be taken.

Many different methods have been proposed for inventory problem solving, a branch of science and technology. Technology is the use of science in industry, engineering, etc. to invent useful tools or to solve problems. For example, an optimal policy leading to inventory reduction or cost reduction is a form of technology. Science, on the other hand, is the pursuit and application of knowledge and understanding of the natural and social worlds. One area where scientific research has been lacking is manufacturing. One possible reason for this is a lack of understanding by researchers in this area of what is not known and what should be known. This, however, is not entirely correct.

The objective of most previous research on inventory problem solving has been cost minimisation because this problem can be approached in terms of technology. To our knowledge, there has been no previous research in which the problem was approached in terms of science. For example, while a method for calculating safety stock so as to reduce cost has been researched and reported, there has been no reported research on the purpose of safety stock or on the composition of safety stock. Such research is in the realm of science. Since technology is based on science, there is a need to conduct such research.

A system is defined as an organised, purposeful structure that consists of interacting, interrelated or interdependent components. Clarifying a system means that the purpose of the system is defined, all components are derived and the relationships between all components are revealed. These same actions are needed to establish the purpose for holding inventory. If the inventory system is clarified, the inventory quantity required to satisfy the holding purpose can be calculated and the optimal policy can be derived under the correct conditions. That is, inventory system clarification is knowledge required for researching inventory problems, and the pursuit of such clarification can be considered as scientific investigation. However, few types of inventory systems (e.g. a system consisting of cycle stock) have been clarified. One possible reason for this is, as mentioned above, a lack of understanding by researchers in this area of what is not known and what should be known. It is thus necessary to present an inventory system that has not been discussed. Furthermore, the need for clarifying an inventory system before the optimal policy is discussed can be understood by presenting the features of the inventory system.

We define stock that absorbs, on the basis of fluctuations in demand, the differences between supply and demand in terms of time and quantity as ‘fluctuation stock’ and establish a method for calculating fluctuation stock. Fluctuation stock is closely related to safety stock, which is held to avoid shortages caused by uncertainty, as described in Section 3.

This paper differs from previous papers in its composition. Previous papers on this topic described the problem to be solved in a section like ‘Model Description’. In this section, various constraint conditions (e.g. production capacity, emergency replenishment and product lifetime) of the model were given, and an optimal policy was presented. This paper establishes a method for calculating fluctuation stock that is equivalent to finding the calculation factors, which are similar to the conditions in an optimal policy. That is, the constraint conditions were assumptions in previous papers, and calculation factors indicate solutions in this paper. The calculation factors are presented in Section 4.1. Because the formulation of the calculation method presented in Section 4.2 was found in an intuitive and heuristic way through trial and error, it cannot be easily described. If the process of finding the formulation could be sufficiently explained, it could be applied to other types of inventory comprising safety stock. This is a worthwhile topic for future research.

Previous papers discussed the performance of the proposed optimal policy in a section like ‘Numerical Analysis’. This section compared the inventory level and/or production cost calculated using the proposed optimal policy with various condition values. The inventory level or production cost did not show causation but correlation with the conditions in the model. It was not necessary to select the conditions with which inventory level or production cost showed causation; the selected conditions only had to affect inventory level or production cost in the optimal policy. The conditions were arbitrarily derived, and the actual inventory situation, which was complex, was simplified and modelled. Thus, the qualitative estimation of a condition was given. In this paper, the inventory level calculated using the proposed method showed causation with the calculation factors. This paper finds the factors that are the components of an inventory system. There is only one set of factors in each inventory system, so the quantitative evaluation of a factor is given in this paper. The calculated results, which are given in Section 6, are obvious if the results are evaluated qualitatively, but this is the first presentation of a quantitative evaluation.

Previous papers described the effectiveness of the proposed optimal policy in a section like ‘Computational Results’. The proposed policy was compared with a previous policy in terms of production cost, etc. Since an optimal policy was being proposed, this section should have explained how much improvement could be obtained by using the proposed policy. The new inventory system proposed in this paper cannot be compared with an optimal policy based on a traditional inventory system. The objective here is not the optimisation of the inventory level or a reduction in production cost. The objective is to show the effectiveness of the proposed approach through which the new inventory system is defined and the calculation method is established. Thus, there is no section like 'Computational Results' in this paper.

The contributions of this paper are threefold. First, a method for calculating other types of inventory included in safety stock is considered through the approach presented in this paper. Second, various optimal policies based on the inventory system proposed in this paper are considered. Finally, the relationship between fluctuation stock and safety stock is clarified.

2. Literature review

There are various streams of research that are related to inventory policies for periodic review systems. Here we outline four of them. All of them have the development of an inventory policy in common. Previous research modelled the actual complex situation by adding or combining various conditions.

The first stream of research focused on a disappointed model, letting the disappointed demand for a product flow to other products or other companies. Parlar (1988) was the first to consider a competitive model. He proved the existence of the Nash solution under the condition in which two ordering decision makers know the demand densities for two substitutable products. This model is model of a duopoly in which dissatisfied customers leave one firm and begin to purchase from the other firm. Lippman and McCardle (1997) examined the relationship between equilibrium inventory levels and the allocation rule, and (Netessine and Rudi 2003) established the uniqueness of the equilibrium for the n-product case, thus extending the work of Parlar (1988). The models of Lippman and McCardle (1997), Parlar (1988) and Netessine and Rudi (2003) included a deterministic proportion of disappointed customers' demand transferring to another product. Avsar and Gursoy (2002) treated the same condition on an infinite horizon. Ahn and Olsen (2007) extended the work of Lippman and McCardle (1997) to multiple periods with consideration of a subscription sale. Nettesine, Rudi, and Wang (2006) presented four scenarios of customers' backlogging and transfer behaviour in a dynamic environment and suggested managerial insights reducing the level of competition. They examined a retailer's monetary incentive to persuade customers to back order a product rather than going to a competitor. Olsen and Parker (2008) addressed the relationship between market size and stock outs while allowing inventory and backlog carryover from period to period.

The second stream of research focused on a replacement model in which there are two demand sources: demand for new products and demand for failed product replacement. Khmelnitsky and Gerchak (2002) analysed a continuous review policy by using a deterministic inventory model in which demand is a function of the instantaneous inventory level, shortages are possible and the production capacity is limited. They proposed an optimal production control for this model. Baker and Urban (1988) investigated the continuous, deterministic case of an inventory system in which the demand rate for an item is a function of that inventory level. They proposed an optimal policy to maximise average profit. Cohen, Nahmias, and Pierskalla (1980) assumed that a fixed proportion of the products sold are returned. They proposed an approximation-based stock policy. Kelle and Silver (1989) identified a reorder point for purchasing new containers in which products are sold. They proposed an approximation scheme for solving a classical dynamic lot-sizing problem. Yuan and Cheung (1998) developed for this model a continuous review (s, S) policy with returns based on the sum of the on-hand stock and the number of items in the field. Feinberg and Lewis (2005) proved the existence of an optimal solution for a single-commodity inventory system in which a negative demand value is possible (i.e. returns exceed demand). Decroix et al. (2005) and Decroix (2006) investigated a multi-echelon remanufacturing system. Khawam, Hausman, and Cheng (2007) formulated several models that include complexities and found near-optimal policies for each one. Huang, Kulkarni, and Swaminathan (2007) proposed a base-stock policy that achieves cost saving under warranty. Huang, Kulkarni, and Swaminathan (2008) considered item age and showed a significant cost saving. Zhou, Tao, and Chao (2011) developed a remanufacturing system with multi-type product returns and described the optimal inventory policy.

The third stream of research focused on emergency models in which the supplier has two replenishment modes: regular lead time with lower cost and shorter lead time with higher cost. Decroix (2006) and Neuts (1964) proposed an optimal order-up-to policy with regular and emergency replenishments for the case in which the regular order lead time is one period and the emergency lead time is zero. Fukuda (1964) investigated the case in which there is a longer lead time for emergencies. Whittemore and Saunders (1977) derived the optimal policy for the case in which the regular and replenishment lead times are multiples of the periodic review period. Rosenshine and Obee (1976) investigated a standing order inventory system in which a constant quantity is received every period, and a fixed-size emergency order can be placed when the inventory falls below the reordering point. Blumenfeld, Hall, and Jordan (1985) analysed the trade-off between expediting cost and safety stock cost. Emergency orders are considered to be sufficiently large to avoid stock outs and to be received with a zero lead time. Chiang and Gutierrez (1996) analysed inventory models in which the lead time is shorter than the periodic review period with a periodic review system. Chiang and Gutierrez (1998) subsequently identified the optimal emergency policy for a similar model in which multiple emergency orders can be placed continuously. Tagaras and Vlachos (2001) modelled the problem in which the lead time is shorter than the replenishment cycle with the capacitated emergency mode. The model is limited to only one emergency order per review period. Teunter and Vlachos (2001) allowed more than one emergency order per review period. Alfredsson and Verrijdt (1999) showed the effectiveness of emergency flexibility, including transshipments from other retailers. Chartniyom et al. (2007) proposed a solution approach to avoiding stock outs that incorporates two options, lateral transshipment and emergency order.

The fourth stream of research led to the development of a perishable inventory model in which a product deteriorates with age. Nahmis and Pierskalla (1973) considered the case in which products have a two-period lifetime and there is backlogging and lost sales. Fries (1975) and Nahmis (1975) allowed for a product with a general lifetime. Weiss (1980) proposed an optimal reorder point and order-up-to level with continuous review and a zero lead time. Kalpakam and Arivarignan (1988) investigated a model in which there is an exponential lifetime and a zero reorder point. Liu (1990) allowed back orders in a similar model. Moorthy, Narasimhulu, and Basha (1992) analysed a model in which items are replaced when a previous item is sold or outdated. Lian and Liu (1999) investigated a discrete time model (s, S) including back ordering policies. Liu and Lian (1999) considered a continuous review model with a constant lifetime and a general renewal demand process. Tekin, Gurler, and Berk (2001) investigated a (Q, r, T) model in which a replenishment order of size Q is placed either when the inventory drops to r or when T units of time have elapsed since the last instance at which the inventory level hit Q, whichever occurs first. Nahmias, Perry, and Stadje (2004) considered the condition in which the demand rate is not affected by the product lifetime. Avinadav and Arponen (2009) assumed that the demand rate increases in proportion to lifetime. Avinadav, Herbon, and Spiegel (2013) modelled demand that decreases in proportion to price and polynomially with lifetime.

Other streams have focused on a substitutability model, a random yield model, a duopoly model, etc. Moreover, an optimal policy has been proposed for whether the set-up cost is included or not, the production system is multi stage or single stage, the production capacity is limited or infinite, the order sizes are discrete or continuous, the planning horizon is given or infinite and a disappointed order is back ordered or is met in next period. However, they have common development, making the models complex and realistic.

3. Fluctuation stock

As mentioned above, fluctuation stock is closely related to safety stock. In traditional inventory systems, if actual demand is larger than expected demand, inventory shortage occurs. The excess of actual demand over expected demand is equal to the shortage. Safety stock, which has been focused on in previous research, is needed to avoid inventory shortage. In the inventory system proposed in this paper, the excess of actual demand over expected demand is not equal to inventory shortage. Even if actual demand is larger than expected demand, inventory shortage may not occur. This means that less safety stock is needed to avoid inventory shortage than in traditional inventory systems. In Section 6.4, this situation is explained using a numerical example. The new system should thus be used in the consideration of an optimal policy.

4. Inventory system

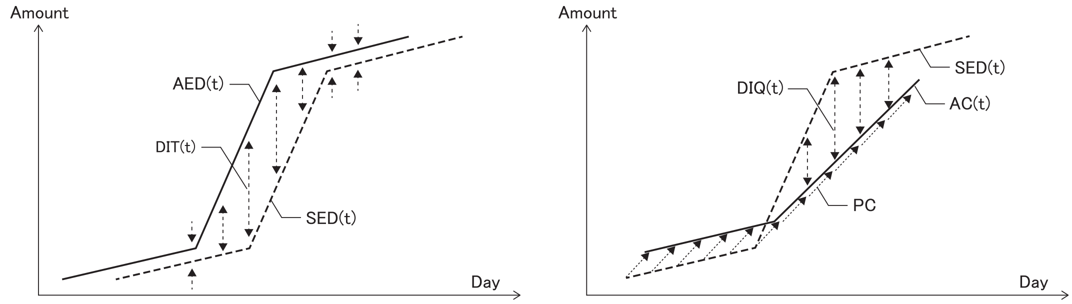
Here, modellisation is defined as the simplification of a complex system, and systemisation is defined as the structuring of factors. This section does not present an inventory model but an inventory system. This means that the background of the problem is not provided. Instead, a method for calculating fluctuation stock, which is the objective of this paper, is provided.

4.1 *Calculation factors*

There are four calculation factors. The first is fluctuating demand. For example, the daily demand for necessities or miscellaneous goods varies slightly, while the demand for moving house tends to be high at the end of the month. The second is production capacity, which is determined by the number of workers, working hours and/or installation capacity. The third factor is order lead time. For example, goods sold in a supermarket are readily available while a new car may not available for one or two months. The fourth factor is replenishment lead time, which is typically determined by machining time, transit time and/or storage time. These four factors are the components comprising an inventory system including fluctuation stock.

A method for calculating fluctuation stock described in this section can be used in any industries. Here, the four factors for two industries are discussed.

First, let us consider an auto assembly plant. The demand for automobiles fluctuates greatly. Automobile demand is strongly affected by economic ups and downs since automobiles are high-priced durable products. The demand for automobiles



(a) Difference in terms of time. (b) Difference in terms of quantity.

Figure 1. Method for calculating difference between demand and supply in terms of time and quantity.

generally increases when measures are implemented to stimulate demand, and the demand generally decreases with its rebound. The production capacity of an automobile assembly plant is generally low compared to peak demand because of ‘production levelling’, which is a key element of the Toyota Production System. Order lead time is about one–two months. A person buying an automobile generally has a long wait for delivery. Nobody expects to go home with a new car on the day the order is placed. An auto assembly plant includes pressing, welding, painting and assembly process. These processes take about 30–40 h in total for one automobile. The replenishment lead time is close to the production lead time plus the time required for transportation by ship. That is, the order lead time is determined by replenishment lead time.

Next, let us consider daily necessities such as toothbrushes and pencils. Daily necessities are typically bought as they are used up. There is little change in frequency or amount of use, so the demand fluctuation is small. Production capacity is generally low. Daily necessities are produced on a large scale to drive down manufacturing costs since daily necessities are typically low priced. The machinery and equipment required for mass production are set to very little excess capacity since demand fluctuation is small. Order lead time is zero since daily necessities are generally available on store shelves. They are typically manufactured in low-labour-cost countries, and transportation by ship can take several months. The replenishment lead time is therefore quite long compared to order lead time.

In short, there are few industries in which demand fluctuation is equal to production capacity and order lead time is equal to replenishment lead time. Inventory is required to absorb the difference.

The difference between supply and demand in terms of time is represented by the difference between the lead time from when an order is received to its due date (‘order lead time’) and the lead time from production to inventory (‘replenishment lead time’). The difference in terms of quantity between supply and demand is represented by the difference between demand per day and the limited production capacity. The replenishment order released each day must be less than production capacity. Fluctuation stock can be represented as the difference between supply and demand in terms of time plus quantity.

4.2 *Calculation method*

Let $I(t)$ be the on-hand inventory available at the beginning of the tth day, and let $ED(t)$ be the expected demand needed for delivery to a customer on the tth day. We assume that demand is expected each month. Thus, fluctuation stock is calculated each month, meaning that the periodic review period is one month. Let $AD(t)$ be the actual demand received at the beginning of the tth day. It is directly satisfied from on-hand inventory. Replenishment order $O(t)$ to reach the target level is released for processing each day.

Let DL be the difference between replenishment lead time and order lead time. It is calculated by subtracting order lead time from replenishment lead time. DL is negative if the replenishment lead time is less than the order lead time. Let $AED(t)$ be the accumulation of $ED(t)$, and let $SED(t)$ be equal to $AED(t - DL)$. The difference in terms of time (see Figure 1) is calculated using

$$DIT(t) = \max\{AED(t) - SED(t), 0\}. \quad (3)$$

Let PC be the daily production capacity, and let $AC(t)$ be the accumulation of PC. The latter is calculated by adding PC to the smaller of $SED(t - 1)$ and $AC(t - 1)$. The difference in terms of quantity (see Figure 1) is calculated using

$$DIQ(t) = \max\{SED(t) - AC(t), 0\}. \quad (4)$$

Table 1. Values of parameters used for calculating fluctuation stock.

t	$ED(t)$	$AED(t)$	$SED(t)$	$DIT(t)$	$AC(t)$	$DIQ(t)$
1	20	1220	1000	220	840	160
2	20	1240	1100	140	920	180
3	20	1260	1200	60	1000	200
4	20	1280	1220	60	1080	140
5	20	1300	1240	60	1160	80
6	20	1320	1260	60	1240	20
7	20	1340	1280	60	1320	0
8	20	1360	1300	60	1360	0
9	20	1380	1320	60	1380	0
10	20	1400	1340	60	1400	0
11	100	1500	1360	140	1420	0
12	100	1600	1380	220	1440	0
13	100	1700	1400	300	1460	0
14	100	1800	1500	300	1480	20
15	100	1900	1600	300	1560	40
16	100	2000	1700	300	1640	60
17	100	2100	1800	300	1720	80
18	100	2200	1900	300	1800	100
19	100	2300	2000	300	1880	120
20	100	2400	2100	300	1960	140

If DL is negative, $DIT(t)$ is always zero, but $DIQ(t)$ is not always zero. That is, the proposed calculation method shows that there is a need to hold inventory if the production capacity is low and the demand fluctuation is large, even if the replenishment lead time is less than the order lead time.

Fluctuation stock is calculated using

$$FS = \max_{t=1,2,\cdots,T} \{DIT(t) + DIQ(t)\}, \tag{5}$$

where there are T days in the periodic review period.

Let $AO(t)$ be the accumulation of $O(t)$, and let $TI(t)$ be the tentative inventory. The latter is calculated by subtracting the accumulation of actual demand, $AAD(t)$, from the accumulation of replenishment orders, $AO(t-1)$. The latter is the smaller of subtracting $TI(t)$ from FS and subtracting it from PC.

$$O(t) = \min[FS - \{AO(t-1) - AAD(t)\}, PC]. \tag{6}$$

5. Numerical example

This is the numerical example presented here, we assume (i) there are 20 working days in a month, (ii) expected demand is 20 during the first 10 days and 100 during the last 10 days, (iii) production capacity is 80 per day and (iv) replenishment lead time is three days longer than order lead time.

Under the initial condition that the tentative inventory level is equal to the target inventory level, the proposed calculation method is applied. The values of the parameters used for calculating fluctuation stock are listed in Table 1. Here, as an example, we explain the process for calculating DIT and DIQ on the fourth day. DL is three days, and $SED(4)$:1220 is equal to $AED(1)$:1220. $DIT(4)$:60 is calculated by subtracting $SED(4)$:1220 from $AED(4)$:1280. Next, $AC(4)$ is calculated. In this example, $AC(3)$:1000 is smaller than $SED(3)$:1220. $AC(4)$:1080 is calculated by adding $AC(3)$:1000 to PC:80. $DIQ(4)$:140 is calculated by subtracting $AC(4)$:1080 from $SED(4)$:1220. The sum of $DIT(t)$ and $DIQ(t)$ is calculated on day t. Fluctuation stock is equal to the maximum of the sum for each day t. In this way, fluctuation stock is calculated on the basis of expected demand.

The replenishment order is calculated on the basis of actual demand. The values of the parameters used for calculating replenishment order are listed in Table 2. In this example, order lead time is two days, replenishment lead time is five days and actual demand is equal to expected demand. The replenishment order for the third day is calculated as follows. $TI(3)$:340 is calculated by subtracting $AAD(3)$:1260 from $AO(2)$:1600. $O(3)$:80 is the smaller of the two values and is the amount

Table 2. Values of parameters used for calculating replenishment order.

t	$AD(t)$	$AAD(t)$	$O(t)$	$QO(t)$	$TI(t)$	$I(t)$
1	20	1220	80	1520	300	
2	20	1240	80	1600	360	
3	20	1260	80	1680	420	60
4	20	1280	40	1720	440	120
5	20	1300	20	1740	440	180
6	20	1320	20	1760	440	240
7	20	1340	20	1780	440	300
8	20	1360	20	1800	440	360
9	20	1380	20	1820	440	380
10	20	1400	20	1840	440	380
11	100	1500	80	1920	420	380
12	100	1600	80	2000	400	380
13	100	1700	80	2080	380	300
14	100	1800	80	2160	360	220
15	100	1900	80	2240	340	140
16	100	2000	80	2320	320	120
17	100	2100	80	2400	300	100
18	100	2200				80
19	100	2300				60
20	100	2400				40

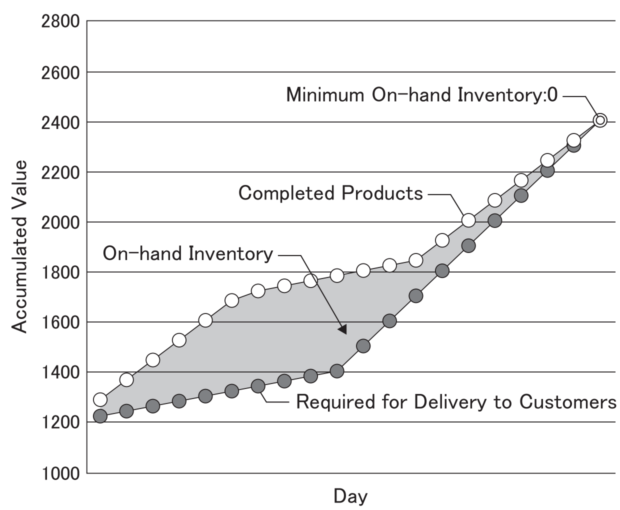
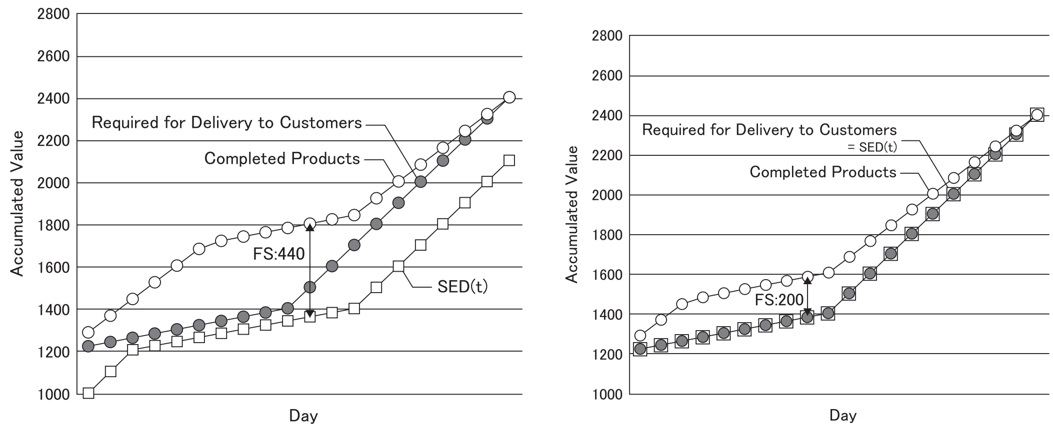


Figure 2. Results of calculation: DL = 3, PC = 80, demand = 20 during first 10 days and 100 during last 10 days.

that should be replenished and that can be replenished. The amount that should be replenished is calculated by subtracting $TI(3)$:340 from FS:440. The amount that can be replenished is equal to PC:80. $O(3)$:80 is added to the inventory on the eighth day since replenishment lead time is five days, and $AD(6)$:20 is subtracted from the inventory on the eighth day since order lead time is two days. $I(8)$:360 is calculated by adding $O(3)$:80 to $I(7)$:300 and subtracting $AD(6)$:20 from 380. The replenishment order on the fourth day is calculated as follows. $TI(4)$:400 is calculated by subtracting $AAD(4)$:1280 from $AO(3)$:1680. $O(4)$:40 is the smaller of the two values and is the amount that should be replenished and that can be replenished. The amount that should be replenished is calculated by subtracting $TI(4)$:400 from FS:440. The amount that can be replenished is equal to PC:80. $O(4)$:40 is added to the inventory on the ninth day since replenishment lead time is five days, and $AD(7)$:20 is subtracted from the inventory on the ninth day since order lead time is two days. $I(9)$:380 is calculated by adding $O(4)$:40 to $I(8)$:360 and subtracting $AD(7)$:20 from 400.



(a) Replenishment lead time is three days longer than order lead time.

(b) Replenishment lead time is equal to order lead time.

Figure 3. Comparison of replenishment lead times.

Figure 2 shows the results of these calculations. Because the fluctuation stock is calculated on the assumption that expected demand repeats, the situation on the first day is equal to the situation at the end of the last day. The initial inventory is thus zero since the inventory at the end of the last day is zero. Since the accumulation of quantity produced exceeds the accumulation of quantity required for delivery throughout the month, a deficiency does not occur. Since there is a day in which the inventory quantity is zero, excess inventories are avoided. Therefore, the fluctuation stock and replenishment order are calculated without deficiency or excess with the proposed method. Furthermore, a method for calculating the amount of each type of inventory included in safety stock can be established.

6. Quantitative evaluation

Three numerical examples based on the proposed method are presented here to illustrate the quantitative effect of three factors on inventory. As mentioned in the Introduction, the calculated results given here are obvious if the results are evaluated qualitatively, but this is the first presentation of a quantitative evaluation.

6.1 *Replenishment lead time*

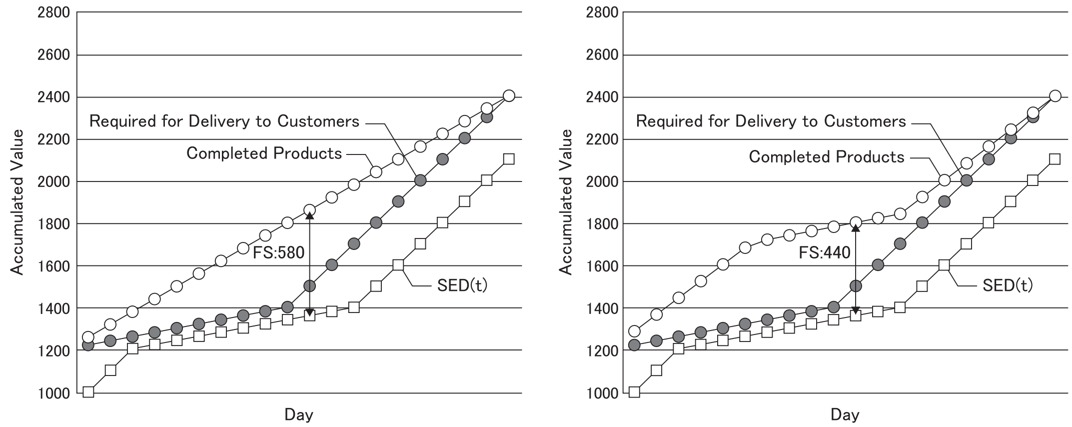
Two lead time cases are used as examples. Case 1: replenishment lead time is three days longer than order lead time. Case 2: replenishment lead time is equal to order lead time. In both cases, expected demand is 20 during the first 10 days and 100 during the last 10 days of the month, and production capacity is 80 per day. Figure 3 shows the results of calculation using the proposed method for both cases.

The fluctuation stock in Case 1 is 440 and that in Case 2 is 200. Shortening replenishment lead time by three days leads to a reduction in inventory (220) under these conditions. Note that this reduction in inventory can be quantified only by forecasting demand and determining production capacity since expected demand, $ED(t)$, and production capacity, PC, are used in the proposed method.

Reducing replenishment lead time can be achieved by holding intermediate products, by automating work, etc. which require a management decision. Since inventory reduction due to reducing replenishment lead time can be quantified using the proposed method, the inventory reduction effect can be compared with the cost of the actions that can be taken. Thus, the proposed method can be used by management to make replenishment-related decisions.

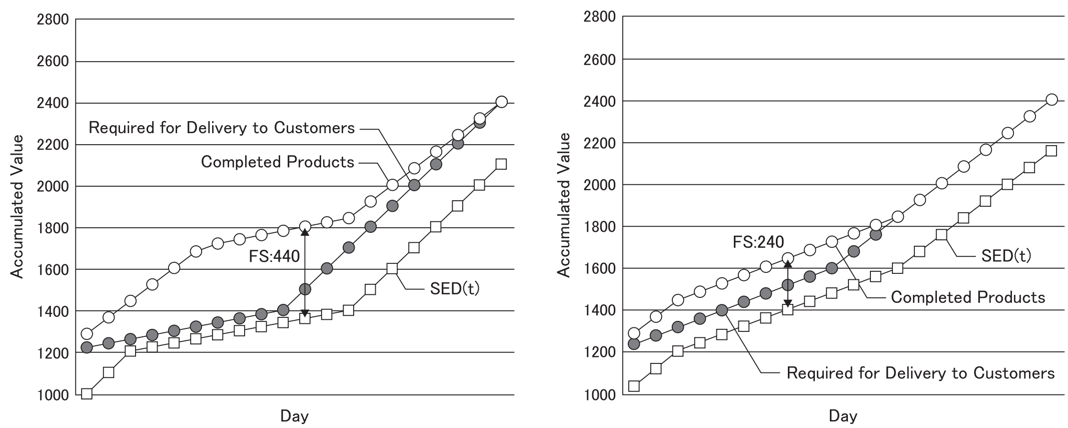
6.2 *Production capacity*

Two production capacity cases are used as examples. Case 1: production capacity is 60 per day, which is equal to the average of demand. Case 2: production capacity is 80, which is greater than the average of demand. In both cases, expected demand is 20 during the first 10 days and 100 during the last 10 days of the month, and replenishment lead time is three days longer than order lead time. Figure 4 shows the results of calculation using the proposed method for both cases.



(a) Production capacity is 60. (b) Production capacity is 80.

Figure 4. Comparison of production capacity.



(a) Demand is 20 during first 10 days and 100 during last 10 days.

(b) Demand is 40 during first 10 days and 80 during last 10 days.

Figure 5. Comparison of demand fluctuation.

The fluctuation stock in Case 1 is 580 and that in Case 2 is 440. Increasing production capacity by 20 per day leads to a reduction of inventory (140) under these conditions. Note that the reduction of inventory due to increasing production capacity can also be quantified by expecting demand and determining replenishment lead time since expected demand and replenishment lead time are used in the proposed method.

Increasing production capacity can be achieved by increasing the number of workers, adding additional equipment, etc. which also need a management decision. Since inventory reduction due to increasing production capacity can be quantified using the proposed method, the inventory reduction effect can be compared with the cost of the actions that can be taken. Thus, the proposed method can be used by management to make production-related decisions.

6.3 *Expected demand*

Two demand pattern cases are used as examples. Case 1: expected demand is 20 during the first 10 days and 100 during the last 10 days of the month. Case 2: expected demand is 40 during the first 10 days and 80 during the last 10 days of the month. In both cases, replenishment lead time is three days longer than order lead time, and production capacity is 80 per day. Figure 5 shows the results of calculation using the proposed method for both cases.

The fluctuation stock in Case 1 is 440 and that in Case 2 is 240. The levelling of demand thus leads to a reduction in inventory (200) under these conditions. Note that the reduction in inventory due to the levelling of demand can also be

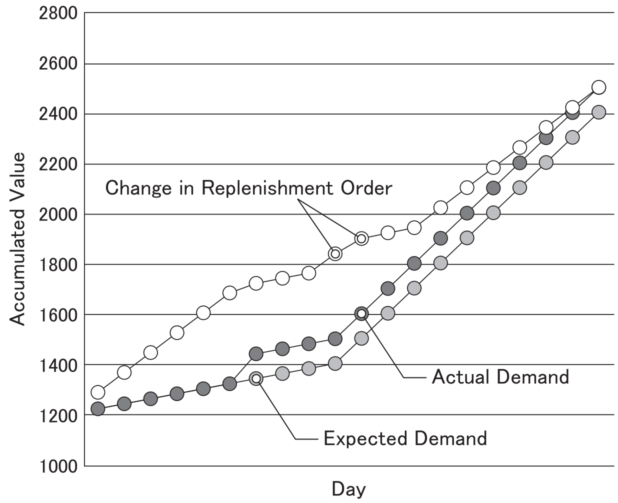


Figure 6. Results of actual demand exceeding expected demand on seventh day.

quantified only by determining replenishment lead time and production capacity since replenishment lead time and production capacity are used in the proposed method.

Levelling of demand and can be achieved by price negotiations with customers, shifting of the decoupling point, etc. which also need a management decision. Since inventory reduction due to the levelling of demand can be quantified using the proposed method, the inventory reduction effect can be compared with the cost of the actions that can be taken. Thus, the proposed method can be used by management to make decisions related to demand levelling.

6.4 *Fluctuation stock and safety stock*

In this section, fluctuation stock is calculated for the conditions described in Section 5, while replenishment order is calculated for different conditions. In Section 5, actual demand is assumed to be equal to expected demand. Here, actual demand is not equal to expected demand. Expected demand is 20 for the seventh day, and actual demand is 120 (see Figure 6). Fluctuation stock is calculated on the basis of expected demand, while replenishment order is calculated on the basis of actual demand. The difference is assumed to be three days, thus the replenishment order after the tenth day differs from the case in Section 5. In the case here, the minimum on-hand inventory is zero on the twentieth day. That is, an inventory shortage does not occur although actual demand exceeds expected demand. In traditional inventory systems, inventory shortage is equal to the excess of actual demand over expected demand. Safety stock is needed to avoid such shortages, and, as described in Section 2, previous research has proposed an optimal policy for safety stock. In the inventory system proposed here, shortages do not occur even if actual demand exceeds expected demand. This means that less safety stock is needed. The optimal policy for safety stock should thus be set using the proposed system.

We refer to the upper limit at which an inventory shortage does not occur as the ‘acceptable amount’. The proposed inventory system differs from traditional ones in terms of the existence of an acceptable amount. Future research is expected to lead to the formulation of a method for calculating the acceptable amount and the amount of safety stock.

7. Conclusion

We have proposed an entirely new approach to finding a method for calculating inventory. Inventories are classified in accordance with their holding purpose, and a calculation method for each category is established. We applied this approach to fluctuation stock. We defined fluctuation stock as the inventory that absorbs the differences in terms of time and quantity between supply and demand and established a method for calculating this inventory. Use of this calculation method enables fluctuation stock and replenishment orders to be calculated without inventory deficiency or excess.

We have clarified an inventory system that includes fluctuation stock as one of its components. However, many types of inventory systems need to be clarified in order to establish a calculation method for each type of inventory included in safety stock. We have clarified only one type of inventory system, so the effect is limited. As more types of inventory systems are clarified, the effect will increase. Here, we established the existence of an inventory system and described an approach to clarifying it. This is the first step towards clarifying many types of inventory systems. The approach to clarifying an inventory system is to clarify the difference between supply and demand, define an inventory that absorbs the difference, identify the components needed to calculate the inventory and finally derive a calculation method. A specific method has not yet been established for each step. We thus need to clarify other types of inventory systems and establish a specific method for each step.

After most types of inventory systems have been clarified, an inventory system that includes inventory for absorbing the uncertainties remains to be clarified. We refer to this inventory as 'uncertain inventory'. We expect that the approach proposed in this paper is not applicable to uncertain inventory. An alternative approach is thus needed to establish a calculation method for uncertain inventory. The acceptable amount is a key component of an alternative approach. In the proposed inventory system, even if actual demand exceeds expected demand, inventory shortages still may not occur. That is, uncertain inventory is affected by the acceptable amount. The acceptable amount is included in other inventory systems and will become increasingly clear as other types of inventory systems are clarified. It will act as a stepping stone to establishing a calculation method for uncertain inventory.

Disclosure statement

No potential conflict of interest was reported by the authors.

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