5) Sea la Señal Gaussiana $x(t)=e^{-at^2}$

$x(t)=e^{-at^2}$ $a \in R^+$

Sistema A: $y_A(t)=x^2(t)$

Sistema B: Un SLIT con respuesta impulso

$hB(t)=Be^{-bt^2}$

a) salida serie

$x(t) \xrightarrow[hB(t)]{} y(t) \xrightarrow{\text{Cuadrado}} y_A(t)$

1) $x(t) \times hB(t) \longrightarrow y(t)$

2). $y_A(t)=y^2(t)$

Convolución de $x(t) * hB(t)$

$$y(t)=x(t)*hB(t)=\int_{-\infty}^{\infty} x(\tau)\, hB(t-\tau)\, d\tau$$

$$x(\tau)=e^{-a\tau^2} * hB(t-\tau)=Be^{-b(t-\tau)^2}$$

$$y(t)=\int_{-\infty}^{\infty} e^{a\tau^2} - Be^{-b(t-\tau)^2}\, d\tau$$

se factoriza

$$y(t)=\int_{-\infty}^{\infty} e^{-a\tau^2} \cdot Be^{-b(t-\tau)^2}\, d\tau \longrightarrow (t-\tau)^2=(t^2-2at\tau+\tau^2)$$

Sustituyendo: $$y(t)=B\int_{-\infty}^{\infty} e^{-a\tau^2} e^{-b(t^2-2(t\tau+\tau^2)}\, d\tau$$

$$y(t)=Be^{-bt^2}\int_{-\infty}^{\infty} e^{-(a+b)(\tau^2-\frac{2bt}{a+b}\tau)}\, d\tau$$

Completando. diferencia de cuadrados y terminos

$$\tau^2-\frac{2bt}{a+b}\tau=\left(\tau-\frac{bt}{a+b}\right)^2-\left(\frac{bt}{a+b}\right)^2$$

Sustituyendo

$$y(t) = Be^{-bt^2} \int_{-\infty}^{\infty} e^{-(a+b)\left(\left[\tau - \frac{bt}{a+b}\right]^2 - \left[\frac{bt}{a+b}\right]^2\right)} d\tau$$

$$= Be^{-bt^2} e^{\frac{b^2t^2}{a+b}} \int_{-\infty}^{\infty} e^{-(a+b)(\tau-u)^2} d\tau \quad \boxed{u = \frac{bt}{a+b}}$$

El resultado de la integral Gaussiana sera:

$$\int_{-\infty}^{\infty} e^{-K(\tau-u)^2} d\tau = \sqrt{\frac{\pi}{K}} \quad \boxed{K = a+b}$$

$$y(t) = B\sqrt{\frac{\pi}{a+b}}\, e^{-bt^2 + \frac{b^2t^2}{a+b}}$$

Se simplifica la exponente.

$$-bt^2 + \frac{b^2t^2}{a+b} = t^2\left(\frac{-b(a+b)+b^2}{a+b}\right) = -\frac{abt^2}{a+b}$$

$$y(t) = B\sqrt{\frac{\pi}{a+b}}\, e^{-\frac{abt^2}{a+b}}$$

Se aplica $y_A(t) = y^2(t)$

$$y_A(t) = \left(B\sqrt{\frac{\pi}{a+b}}\, e^{-\frac{abt^2}{a+b}}\right)^2$$

$$\boxed{y(t) = B^2 \frac{\pi}{a+b} e^{-2\frac{abt^2}{a+b}}}$$

- Salida del sistema.

$$x(t) \longrightarrow y_A(t) = x^2(t) \xrightarrow{h_B(t)} y(t)$$

→ Aplicar A directamente:

$$y_A(t) = x^2(t) = (e^{-at^2})^2 = \boxed{e^{-2at^2}}$$

Convolución con $hB(t) = Be^{-bt}$

$$Y(t) = Y_A(t) * hB(t)$$

$$= \int_{-\infty}^{\infty} e^{-2a\tau^2} * \cdot Be^{-b(t-\tau)^2} d\tau$$

$$Y(t) = B\int_{-\infty}^{\infty} e^{-2a\tau^2} * e^{-b(b-\tau)^2} d\tau$$

$$\boxed{Y(t) = B\sqrt{\frac{\pi}{2a+b}} * e^{-2\frac{abt^2}{2a+b}}}$$

8) Demuestra las siguientes propiedades sin utilizar tablas de propiedades

i) $\mathcal{L}\{x(t-t_0)\} = e^{-st_0} X(s)$

$$\mathcal{L}\{x(t-t_0)\} = \int_0^{\infty} x(t-t_0) e^{-st} dt$$

$u = t - t_0 \therefore t = u + t_0$ $\quad t \to \infty = u \to \infty$

$t \to \infty = u \to -\infty$

$$\boxed{\mathcal{L}\{x(t-t_0)\} = e^{-st_0} \int_{-\infty}^{\infty} x(u) e^{-su} du = e^{-st_0} X(s)}$$

ii) $\mathcal{L}\{x(at)\} = \frac{1}{|a|} X(s/a)$

$$\mathcal{L}\{x(at)\} = \int_{-\infty}^{\infty} x(at) e^{-st} dt$$

si $a > 0$

$u = at \therefore t = u/a$ $\quad t \to +\infty \quad u \to +\infty$

$t \to -\infty \quad u \to -\infty$

$$\frac{1}{a}\int_{-\infty}^{\infty} x(u) e^{-su/a} = \frac{1}{a} X(s/a)$$

$a < 0$

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$$(-1)\frac{1}{a}\int_{+\infty}^{-\infty} x(u)e^{-s\frac{u}{a}} = -\frac{1}{a}X(s/a)$$

$$\boxed{\mathcal{L}\{x(at)\} = \frac{1}{|a|}X(s/a)}$$

iii) $\mathcal{L}\{\frac{dx(t)}{dt}\} = sX(s) = \int_{-\infty}^{\infty} x'(t)e^{-st}\,dt$

$$u = e^{-st} \qquad v = x(t)$$
$$du = -se^{-st} \qquad dv = x'(t)$$

$$= e^{-st}x(t)\Big|_0^{\infty} \int_{-\infty}^{\infty} se^{-st}x(t)\,dt = 0 - x(0) + s\mathcal{L}\{x(t)\}$$

$$\mathcal{L}\{\frac{dx(t)}{dt}\} = sX(s) - x(0)$$

$$\boxed{\mathcal{L}\{\frac{dx(t)}{dt}\} = sX(s)}$$

iv) $\mathcal{L}\{x(t) * y(t)\} = X(s)\,Y(s)$

$$\mathcal{L}\{x(t) * y(t)\} = \int_{-\infty}^{\infty}[x(t) * y(t)]\,e^{st}$$

$$\mathcal{L}\{x(t) * y(t)\} = \int_{-\infty}^{\infty}(x(t) * y(\tau - t)\,dt$$

$$\mathcal{L}\{x(t) * y(t)\} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x(\tau)\,y(\tau - t)\,d\tau\,e^{-st}\,dt$$

$$= \int_{-\infty}^{\infty} x(\tau) * \int_{-\infty}^{\infty} y(\tau - t)e^{-st}\,dt\,d\tau$$

$$u = \tau - t \quad du = -dt \quad t = \tau + u$$

$$= \int_{-\infty}^{\infty} x(\tau)(+1)\int_{-\infty}^{\infty} y(u)\,e^{-s(\tau+u)}\,dt\,d\tau$$

Entonces:

$$x(t) = -e^{-t} + e^{-2t} + te^{-2t}, \quad t \geq 0$$

$$x(t) = (-e^{-t} + e^{-2t} + te^{-2t})\,u(t)$$

$$= \int_{-\infty}^{\infty} x(\tau)\, e^{-s\tau}\, d\tau \int_{-\infty}^{\infty} y(u)\, e^{-su}\, du$$

$$\boxed{= X(s) \cdot Y(s)}$$

9) Encuentre la transformada de Laplace dibuje la región de ceros y polos y la región de convergencia ROC de:

i) $e^{-2t}u(t) + e^{-3t}u(t)$

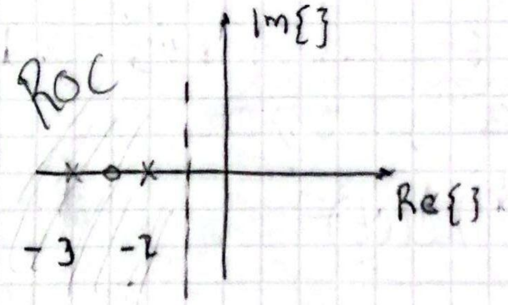
$$\mathcal{L}\{e^{-2t}u(t)\} + \mathcal{L}\{e^{-3t}u(t)\} = \int_0^{\infty} e^{-2t} e^{-st}\, dt + \int_0^{\infty} e^{-3t} e^{-st}\, dt$$

$$= \int_0^{\infty} e^{-(s+2)t}\, dt + \int_0^{\infty} e^{-(s+3)t}\, dt = \frac{e^{-(s+2)t}}{s+2}\Bigg|_0^{\infty} - \frac{e^{-(s+3)t}}{s+3}\Bigg|_0^{\infty}$$

ROC $s > -2$ y $s > -3$ $\therefore$ $s > -3$

$$\frac{1}{s+2} + \frac{1}{s+3} = \frac{2s+5}{(s+2)(s+3)}$$

Polos: $s_1 = -2$, $s_2 = -3$ Cero $= -5/2$



(ii) $e^{2t}u(t) + e^{-3t}u(-t)$

$\mathcal{L}\{e^{2t}u(t)\} + \mathcal{L}\{e^{-3t}u(-t)\}$

$= \int_0^{\infty} e^{2t} e^{-st}\,dt + \int_{-\infty}^{0} e^{-3t} e^{-st}\,dt$

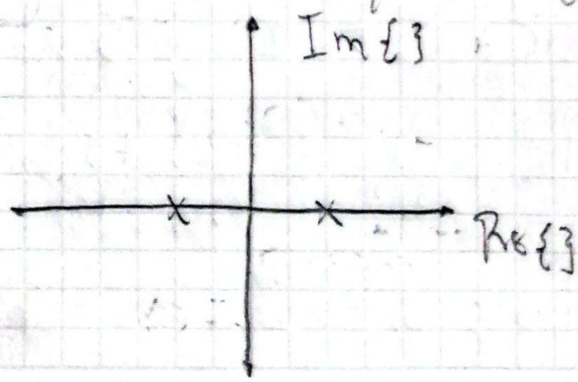
$= \int_0^{\infty} e^{-(s-2)t}\,dt + \int_{-\infty}^{0} e^{-(3+s)t}\,dt$

$= -\dfrac{e^{-(s-2)t}}{(s-2)}\Big|_0^{\infty} - \dfrac{e^{(-3-s)}}{(3+s)}\Big|_{-\infty}^{0}$

RoC $s > 2$ y $s < -3$

$= \dfrac{1}{s-2} - \dfrac{1}{3+s} = \dfrac{5}{(s-2)(3+s)}$

2 y -3 no hay ceros si fuera posible su transformada.



iii) $\mathcal{L}\{e^{-a|t|}\} = \int_{-\infty}^{0} e^{-a(-t)} e^{-st}\,dt + \int_{0}^{\infty} e^{-at}\,dt$

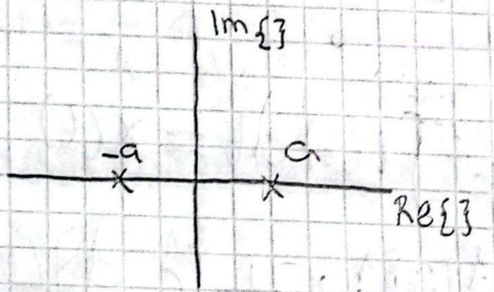
$$= \int_{-\infty}^{0} e^{(a-s)t}\,dt + \int_{0}^{\infty} e^{-(a+s)t}\,dt$$

$$= \frac{e^{(a-s)t}}{a-s}\Big|_{-\infty}^{0} - \frac{e^{-(a+s)t}}{a+s}\Big|_{0}^{\infty}$$

ROC: $s < a$ y $s > -a$ $\therefore$ ROC es $t(-a, a)$

$$= \frac{1}{a-s} + \frac{1}{a+s} = \frac{2a}{a^2-s^2}$$

por lo tanto $-a$ y a ningun cero



iv) $e^{-2t}[u(t) - u(t-5)]$ $\quad t \in [0,5]$

$$\mathcal{L}\{e^{-2t}u(t) - u(t-5)\} = \int_{0}^{5} e^{-2t} e^{-st}\,dt$$

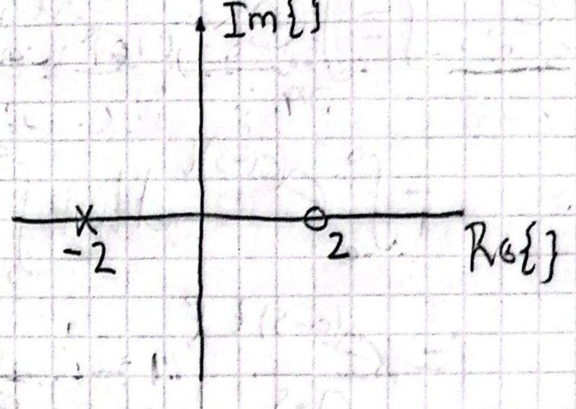
$$= \int_{0}^{5} e^{(2+s)t}\,dt = \frac{e^{(2+s)t}}{2+s}\Big|_{0}^{5}$$

$$= \frac{1 - e^{-(5s+10)}}{s+2} \quad \text{ROC } s > -2$$

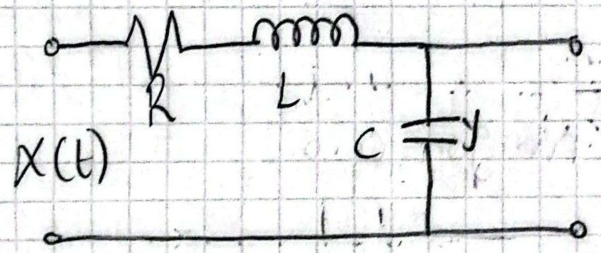
$$1 - e^{-(5s+10)} = 0$$

$0 = -5s + 10$

$s = 2$



(10) La función de transferencia en lazo abierto par RLC serie y circuito RLC paralelo



$$i(t) = \frac{C\,dv_c}{dt} = \frac{C\,dv(t)}{dt}$$

$$i(t)R_L + \frac{L\,di(t)}{dt} + \frac{1}{C}\int i(t)\,dt = x(t)$$

$$RC\frac{dv(t)}{dt} + L\frac{d}{dt}\left(\frac{C\,dy(t)}{dt}\right) + \frac{1}{C}\int C\,\frac{dy(t)}{dt} = x(t)$$

$$x(t) = LC\frac{d^2y(t)}{dt} + RC\,\frac{dy(t)}{dt} + y(t)$$

$$X(s) = LC\left(s^2Y(s)\right) - sY(0) - Y'(0)\Big) + RC\left(sY(s) - Y(0)\right) + Y(s)$$

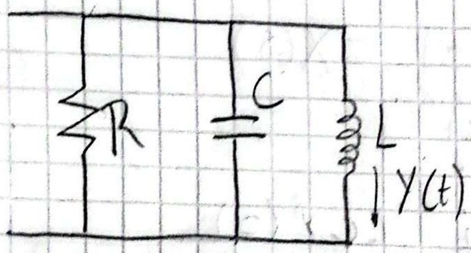
$$X(s) = LCs^2Y(s) - Lss\,Y(0) - LC\,Y(0) + RCs\,Y(s) - RC\,Y(0) + Y(s)$$

$$X(s) = Ys(LCs^2 + RCs + 1) - Y(0)(LCs + RC) - LC\,Y'(0)$$

$$Y(s) = \frac{X(s) + Y(0)(LCs + RC) + LCY'(0)}{LCs^2 + RCs + 1}$$

$$H(S) = \frac{Y(S)}{X(S)} = \frac{X(S) + Y(0)(LCS + RC) + LC\,Y'(0)}{X(S)\,(LCS^2 + RCS + 1)}$$

RLC Paralelo



$$V_L = L\frac{d_i(t)}{dt} = L\frac{dy(t)}{dt}$$

$$X(t) = i_R(t) + i_C(t) + y(t)$$

$$X(t) = \frac{V_L}{R} + C\frac{dV_L}{dt} + y(t)$$

$$X(t) = \frac{L}{R}\frac{dy(t)}{dt} + LC\frac{d^2}{dt^2}y(t) + y(t)$$

$$RX(S) = L(Sy(S) - y(0)) + RCL(y(S)S^2 - y(0)S - y'(0)) + y(S)R$$

$$RX(S) = y(S)(RLCS^2 + LS + R) - y(0)(RLCS + 1) - y'(0)RLC$$

$$y(S) = \frac{RX(S) + y(0)(RLCS + 1) + y'(0)RLC}{RLCS^2 + LS + R}$$

$$\boxed{H(S) = \frac{y(S)}{X(S)} = \frac{RX(S) + y(0)(RLCS + L) + y'(0)RLC}{X(S)(RLCS^2 + LS + R)}}$$

13. Encuentra la expresión de salida en el tiempo para una configuración en lazo cerrado del sistema en función RLC:

i) Impulso:

$$X(t) = \delta(t) \qquad \mathcal{L}\{\delta(t)\} = 1 = X(s)$$

$$X(0) = 1 \qquad X'(0) = 1$$

$$Y(s) = \frac{1}{LCs^2 + RCs + 1} + \frac{(LCs + RC)\,Y(0)}{LCs^2 + RCs + 1} + \frac{LC\,Y'(0)}{LCs^2 + RCs + 1}$$

$$Y(s) = \frac{1}{LC} * \frac{1}{s^2 + \frac{R}{L}s + \frac{1}{LC}} + \frac{s\,Y(0)}{s^2 + \frac{R}{L}s + \frac{1}{LC}} + \frac{\frac{R}{L}Y(0)}{s^2 + \frac{R}{L}s + \frac{1}{LC}} + \frac{Y'(0)}{s^2 + \frac{R}{L}s + \frac{1}{LC}}$$

$$Y(s) = \frac{1}{LC} * \frac{1}{\left(s + \frac{R}{2L}\right)^2 + \frac{4L - CR^2}{4L^2C}} + \frac{Y(0)\left(s - \frac{R}{L} + \frac{R}{2L}\right)}{\left(s + \frac{R}{2L}\right)^2 + \frac{4L - CR^2}{4L^2C}} + \dots$$

$$+ \frac{R\,Y(0)}{\left(s + \frac{R}{2L}\right)^2 + \frac{4L - CR^2}{4LC}} + \frac{Y'(0)}{\left(s + \frac{R}{2L}\right)^2 + \frac{4L - CR^2}{4L^2C}}$$

$$Y(t) = \left(\frac{\frac{1}{LC} + R\,Y(0) + Y(0) - \frac{R}{2L}}{\sqrt{\frac{4L - CR^2}{4L^2C}}}\right) e^{-\frac{R}{2L}t} \sin\left(\sqrt{\frac{4L - CR^2}{4L^2C}}\,t\right)$$

$$+ e^{-\frac{R}{2L}t} \cos\left(\sqrt{\frac{4L - CR^2}{4L^2C}}\,t\right)$$

$$Y(s) = \frac{R + Y(0)RLCs + Y(0)L + Y'(0)RLC}{RLCs^2 + Ls + R}$$

$$Y(s) = \frac{(R + Y(0))L + Y'(0)RLC - Y(0)\,{}^{1}/_{RLC}2RC)}{RLC}$$

$$* \frac{1}{(s+\frac{1}{2RC})^2 + \frac{4R^2C-1}{4R^2LC^2}} + Y(0) \frac{(s + 1/2\,RC)}{(s+\frac{1}{2RC})^2 + \frac{4R^2C-1}{4R^2LC^2}}$$

$$Y(t) = \frac{2R + RL\,Y(0) - L\,Y(0)}{L\sqrt{4R^2C-L}} e^{-1/2\,RCt} * \sin\left(\sqrt{\frac{4R^2C-L}{4R^2LC^2}}\,t\right)$$

$$+ Y(0)e^{-\frac{1}{2}RCt} * \sin\left(\sqrt{\frac{4R^2C-L}{4R^2LC^2}}\,t\right) + \ldots$$

$$\ldots + Y(0)e^{-\frac{1}{2}RCt} \cos\left(\sqrt{\frac{4R^2C-L}{4R^2LC^2}}\,t\right)$$

ii) Escalón Unitario

$$\mathcal{L}\{u(t)\} = 1/s$$

$$Y(s) = \frac{1/s + Y/0\,(LCs + RC) + LCY'(0)}{LCs^2 + RCs + 1}$$

$$Y(s) = \frac{1 + Y(0)LCs^2 + RCs + LCs\,Y'(0)}{s(LCs^2 + RCs + 1)}$$

$$Y(s) = \frac{1}{s(LCs^2+RCs+1)} + \frac{s\,Y(0)LC}{(LCs^2+RCs+1)} + \frac{V_0RC + LC + V(0)}{LCs^2 + RCs + 1}$$

$$\frac{1}{s(LCs^2+RCs+1)} = \frac{a}{s} + \frac{bs+d}{LCs^2+RCs+1}$$

$$a(LCs^2 + RCs + 1) + bs^2 + ds + 1$$

$$a = 1 \quad b = -LC \quad d = RC$$

$$Y(s)=\frac{1}{s}-\frac{LCs+RC}{LCs^2+RCs+1}+\frac{Y(0)LCs}{LCs^2+RCs+1}+\frac{RC+LCY'(0)}{LCs^2+RCs+1}$$

$$Y(s)=\frac{1}{s}+(Y(0)-1)\frac{s+R/2L}{\left(s+\frac{R}{2L}\right)^2+\frac{4L-CR^2}{4L^2C}}+\left(RC+LCY'(0)\right.$$

$$\left.-\frac{RLC}{2L}+RLCY(0)\right)+\left(\frac{1}{LCs^2+RCs+1}\right)$$

Con calculos anteriores tenemos:

$$Y(t)=1+(Y(0)-1)e^{-\frac{Rt}{2L}}\cos\left(\sqrt{\frac{4L-CR^2}{4L^2C}}t\right)$$

$$+\left(\frac{RC+LCY'(0)-RC/2+RC/2\,Y(0)}{LC\sqrt{4L-CR^2/4L^2C}}t\right)\left(e^{-\frac{R}{2L}t}\sin\sqrt{\frac{4L-CR^2}{4L^2C}}\right)$$

$$Y(t)=1+(Y(0)-1)e^{-\frac{R}{2L}t}\cos\left(\frac{\sqrt{4L-CR^2}}{4L^2C}t\right)+\left(\frac{\sqrt{C}\,(R+2LY'(0)+RY(0))\sqrt{4L-R^2C}}{4L-R^2C}\right)$$

$$\left(e^{-\frac{R}{2L}t}\sin\left(\sqrt{\frac{4L-CR^2}{4L^2C}}t\right)\right)$$

Ahora en el circuito en paralelo.

$$Y(s)=\frac{R/s+Y(0)RLCs+Y(0)L+Y'(0)LRC}{RLCs^2+Ls+R}$$

$$Y(s)=\frac{Ra}{s^2(RLCs^2+Ls+R)}+\frac{Y(0)RCs}{RLCs^2+Ls+R}+\frac{LY(0)+Y(0)RLC}{RLCs^2+Ls+R}$$

Ahora se aplica fracciones parciales.

$$\frac{Ra}{s^2(RLCs^2+Ls+R)}=\frac{bs+d}{s^2}+\frac{es+f}{RLCs^2+Ls+R}$$

$$Ra = bRLCS^3 + bS^2L + bRS + dRLCS^2 + dLS + dR + eS^3 + FS^3$$

$$Rd = dr \qquad br + dl = 0$$

$$\boxed{d = a} \qquad \boxed{b = -\frac{aR}{R}}$$

$$bl + dRLC + f = 0 \qquad e + bRLC = 0$$

$$f = \frac{aL^2}{R} - aRLC \qquad e = aL^2C$$

Ahora queda:

$$Y(s) = \frac{-\frac{dls}{R} + a}{S^2} + aL^2CS + \frac{aL^2}{R} - aRLC + \frac{Y(0)RCS}{RLCS^2 + LS + R}$$

$$+ \frac{LY(0) + Y(0)RLC}{RLCS^2 + LS + R}$$

$$Y(S) = -\frac{aL}{RS} + \frac{a}{S^2}\left(\frac{Y(0)RC + aL^2C}{RLC}\right)\frac{S + R/2L}{\left(\frac{S+R}{2L}\right)^2 + \frac{4L - CR^2}{4L^2C}} + \ldots$$

$$\ldots + \frac{\left(LY(0) + Y(0)RLC + aL^2/R - aRLC + 2/RL\,Y(0)(RC) + \frac{R}{2L}aL^2C\right)}{RLC\sqrt{\frac{4L - CR^2}{4LC}}}$$

$$Y(t) = -\frac{aL}{R} + at + \left(\frac{Y(0)}{L} + \frac{aL}{R}\right)e^{-\frac{R}{2L}t}\cos\left(\sqrt{\frac{4L - CR^2}{4L^2C}}\,t\right)$$

$$+ \frac{2L\sqrt{C}\left(\frac{Y(0)}{RC} + Y(0) + \frac{aL}{R^2} - a + \frac{RY(0)}{2L^2} + \frac{a}{2}\right)}{\sqrt{4L - CR^2}}e^{-\frac{Rt}{2L}} * \ldots$$

$$\operatorname{Sen}\left(\sqrt{\frac{4L - CR^2}{4L^2C}}\,t\right)$$