$$X(s) = \frac{1}{(s+1)(s+2)^2}$$

$$R\{s\} \geq -1$$

① Se descompone en fracciones parciales

$$\frac{1}{(s+1)(s+2)^2} = \frac{A}{s+1} + \frac{B}{s+2} + \frac{C}{(s+2)^2}$$

Se multiplica ambos lados por $(s+1)(s+2)^2$:

$$\frac{(s+1)(s+2)^2}{(s+1)(s+2)^2} = A\frac{\cancel{(s+1)}(s+2)^2}{\cancel{(s+1)}} + B\frac{(s+1)(s+2)^{\cancel{2}}}{\cancel{(s+2)}} + C\frac{(s+1)\cancel{(s+2)^2}}{\cancel{(s+2)^2}}$$

$$1 = A(s+2)^2 + B(s+1)(s+2) + C(s+1)$$

se expande cada termino

$$A(s+2)^2 = A(s^2+4s+4)$$

$$B(s+1)(s+2) = B(s^2+3s+2)$$

$$C(s+1) = C(s+1)$$

Se Suman terminos:

$$1 = A(s^2+4s+4s) + B(s^2+3s+2) + C(s+1)$$

Se agrupan potencias. de s

$$1 = (A+B)s^2 + (4A+3B+C)s + (4A+2B+C)$$

- Se igualan coeficientes.

$$\begin{cases} A+B=0 & (1) \\ 4A+3B+C=0 & (2) \\ 4A+2B+C=1 & (3) \end{cases}$$

Se Resuelve el sistema

(1) $A+B=0 \Rightarrow B=-A$

(2) $4A+3(-A)+C=0 \Rightarrow A+C=0;\ C=-A$

(3) $4A+2(-A)+(-A)=1 \Rightarrow 4A-2A-A=1;\ A=-1$

Entonces:

$$A=-1 \quad ; \quad B=1 \quad ; \quad C=1$$

$$\therefore \frac{1}{(s+1)(s+2)^2} = \frac{-1}{(s+1)} + \frac{1}{(s+2)} + \frac{1}{(s+2)^2}$$

- Ahora se usan las propiedades de transformada inversa de Laplace

- $\mathcal{L}^{-1}\left\{\frac{1}{s+a}\right\} = e^{-at}\,u(t)$

- $\mathcal{L}^{-1}\left\{\frac{1}{(s+a)^2}\right\} = te^{-at}\,u(t)$

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Entonces:

$$x(t) = -e^{-t} + e^{-2t} + te^{-2t}, \quad t \geq 0$$

$$x(t) = \left(-e^{-t} + e^{-2t} + te^{-2t}\right) u(t)$$