Modelo de regresión

Sea el modelo de regresión

$t_n = \phi(X_n) W^T + \eta_n$, con $\{t_n \in \mathbb{R}, X_n \in \mathbb{R}^P\}_{n=1}^N$, $w \in \mathbb{R}^Q$,

$\phi : \mathbb{R}^P \to \mathbb{R}^Q$, $Q \geq P$ y $\eta_n \sim N(\eta_n | 0, \sigma_n^2)$.

Presente el problema de optimización (inferencia) y la solución del mismo para los modelos

$$\boxed{t_n = \phi(X_n) W^T + \eta_n} \qquad \eta_n \sim N(\eta_n | 0, \sigma^2)$$

Por hipótesis $E[\eta_n] = 0$ y $Var(\eta_n) = \sigma^2$

entonces

$$t_n \sim N(E[\eta_n]; Var(\eta_n)).$$

$$E[t_n] = E[\phi(x_n) w^T + \eta_n]$$
$$= E[\phi(X_n) w^T] + \underbrace{E[\eta_n]}_{0} \longrightarrow \text{por hip}$$
$$= \phi(X_n) w^T$$

$$Var(t_n) = Var(\phi(X_n) w^T + \eta_n)$$
$$= \underbrace{Var(\cancel{\phi(x_n) w^T})}_{\to 0} + Var(\eta_n)$$
$$= \sigma^2.$$

luego se concluye que $t_n \sim N(\phi(X_n) w^T; \sigma^2)$

$$f(t) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{(t_n - u)^2}{2\sigma^2}}$$

$$f(t_n) = \frac{1}{\sqrt{2\pi}\,\sigma} \cdot e^{-\frac{(t_n - \phi(X_n) w^T)^2}{2\sigma^2}}$$

① $$L(\phi(X_n), w^T / \underset{\sim}{X}) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{(t_n - \phi(X_n) w^T)^2}{2\sigma^2}} \cdots \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{(t_n - \phi(X_n) w^T)^2}{2\sigma^2}}$$

$$L(\phi(X_n), w^T / \underset{\sim}{X}) = \left(\frac{1}{\sqrt{2\pi}\,\sigma}\right)^n e^{-\frac{\sum_{n=1}^{N}(t_n - \phi(X_n) w^T)^2}{2\sigma^2}}$$

$$\ell(\phi(X_n), w^T / \underset{\sim}{X}) = n \ln\left(\frac{1}{\sqrt{2\pi}\,\sigma}\right) - \frac{\sum_{n=1}^{N}(t_n - \phi(X_n) w^T)^2}{2\sigma^2}$$

$$\frac{\partial \ell(\phi(X_n), w^T / \underset{\sim}{X})}{\partial w^T} = 0 - \frac{\sum_{n=1}^{N} \cancel{2}(t_n - \phi(X_n) w^T)(-\phi(X_n))}{\cancel{2}\sigma^2}$$

$$\Rightarrow \sum_{n=1}^{N} (t_n - \phi(X_n) w^T)(\phi(X_n)) = 0$$

$$\Rightarrow \sum_{n=1}^{N} (\phi(X_n) t_n - w^T \phi(X_n)^2) = 0$$

$$\Rightarrow \sum_{n=1}^{N} \phi(X_n) t_n - \sum_{n=1}^{N} \phi(X_n)^2 w^T = 0$$

$$w^T_{MV} = \frac{\sum_{n=1}^{N} \phi(X_n) t_n}{\sum_{n=1}^{N} \phi(X_n)^2}$$