Scribe

- $\mathcal{E}\{x\} = \int_{-\infty}^{\infty} N(x|\mu,\sigma^2)\, x\, dx = \mu$ [1]

$\mathcal{E}\{x^2\} = \int_{-\infty}^{\infty} N(x|\mu,\sigma^2)\, x^2\, dx = \mu^2 + \sigma^2$ [2]

$Var\{x\} = \sigma^2$ [3]

$$= \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{|x-\mu|^2}{2\sigma^2}\right) x\, dx$$

$$= \frac{1}{\sqrt{2\pi\sigma^2}} \int_{-\infty}^{\infty} \exp\left(-\frac{|x-\mu|^2}{2\sigma^2}\right) x\, dx$$

Para hallar facilmente el Valor esperado y la Varianza Se Va a Calcular Con la <u>función generadora de momentos</u>

$$m_x(t) = \exp\left[\mu t + \frac{\sigma^2 t^2}{2}\right] \longrightarrow \mathcal{E}\{x\} = \int_{-\infty}^{\infty} x\,(fx)\, dx$$

$$m_x(t) = \frac{1}{\sqrt{2\pi}\,\sigma} \int_{-\infty}^{\infty} \exp\left[tx - \frac{|x-\mu|^2}{2\sigma^2}\right] dx$$

$$= \frac{1}{\sqrt{2\pi}\,\sigma} \int_{-\infty}^{\infty} \exp\left[-\frac{(x-\mu-\sigma^2 t)^2}{2\sigma^2} + \mu t + \frac{\sigma^2 t^2}{2}\right] dx$$

$$= \frac{1}{\sqrt{2\pi}\,\sigma} \exp\left[\mu t + \frac{\sigma^2 t^2}{2}\right] \int_{-\infty}^{\infty} \exp\left[-\frac{(x-\{\mu+\sigma^2 t\})^2}{2\sigma^2}\right] dx$$

$$= \exp\left[\mu t + \frac{\sigma^2 t^2}{2}\right]$$

Por Consiguiente:

$$\mathcal{E}\{x\} = [\mu + \sigma^2 t] \exp\left[\mu t + \frac{\sigma^2 t^2}{2}\right]\Big|_{t=0} = \boxed{\mu} \quad [1]$$

$$\mathcal{E}\{x^2\} = (\sigma^2 + [\mu + \sigma^2 t]^2) \exp\left[\mu t + \frac{\sigma^2 t^2}{2}\right]\Big|_{t=0} = \boxed{\sigma^2 + \mu^2} \quad [2]$$

Luego $Var(x) = \sigma^2$