Descomposición de Cholesky.

$$A = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} \qquad A = B . B^T$$

1. halle su Raiz

$$\begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix} \begin{pmatrix} b_{11} & b_{21} \\ b_{12} & b_{22} \end{pmatrix} = \begin{pmatrix} b_{11}^2 + b_{12}^2 & b_{11} b_{21} + b_{11} b_{12} \\ b_{21} b_{11} + b_{22} . b_{12} & b_{21}^2 + b_{22}^2 \end{pmatrix} = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$$

$$\begin{array}{r} b_{21} b_{11} + b_{22} . b_{12} = 1 \\ - \quad b_{21} b_{11} + b_{11} \, b_{12} = 1 \\ \hline (b_{22} - b_{11}) b_{12} = 0 \end{array} \rightarrow b_{22} = b_{11} = x$$

$$\begin{array}{r} b_{11}^2 + b_{12}^2 = 2 \\ - \quad b_{22}^2 + b_{21}^2 = 2 \\ \hline b_{12}^2 - b_{21}^2 = 0 \end{array} \longrightarrow b_{12} = b_{21} = y$$

$$x^2 + y^2 = 2$$
$$xy + xy = 1$$
$$2xy = 1$$
$$y = \frac{1}{2x}$$

$$x^2 + \frac{1}{4x^2} = 2$$
$$x = 1{,}375$$
$$y = 0{,}363$$

$$B = \begin{pmatrix} 1{,}375 & 0{,}363 \\ 0{,}363 & 1{,}375 \end{pmatrix}$$

$B = Q R$

Q es una matriz ortogonal normalizada

$Q . Q^T = I \qquad Q^T = Q^{-1}$

R es una matriz triangular s.d. positiva.

$$\left.\begin{array}{l} B = Q R \\ B^* = (Q R)^* = R^* Q^* \end{array}\right\} B = B^* \Rightarrow QR = R^* Q^*$$

Demostrar que $A = R . R^*$

$$A = \underbrace{Q R}_{B} \underbrace{R^* Q^*}_{B^*}$$

Como Q es una matriz ortonormal, puedo afirmar que $Q . C$, siendo C cualquier matriz $n x n$ transformaré la matriz C alineándola a los ejes ortogonales de Q. Luego, al multiplicar $Q.C.Q^T$, realizo a transformación inversa, dejando a C inmutada.

$$Q . C . Q^T = Q.C.Q^{-1} = C$$