4.3 Riemann Sums and Definite Integrals

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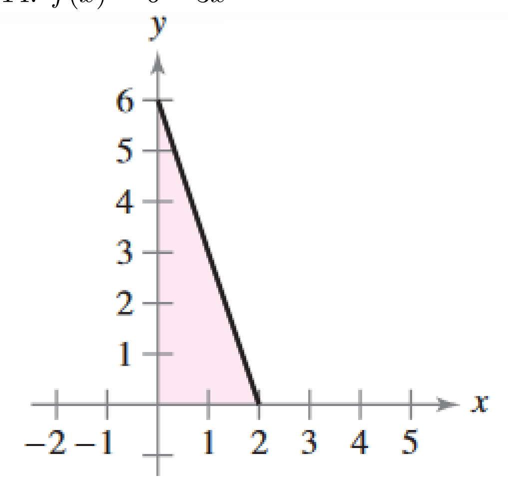
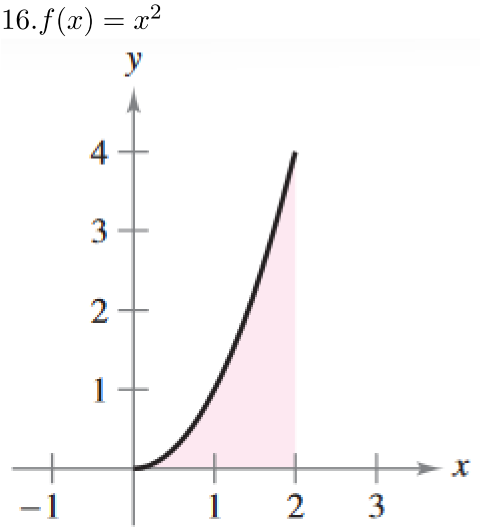
1 Set up a definite integral that yields the area of the region. (Do not evaluate the integral.)

14. $f(x) = 6 - 3x$



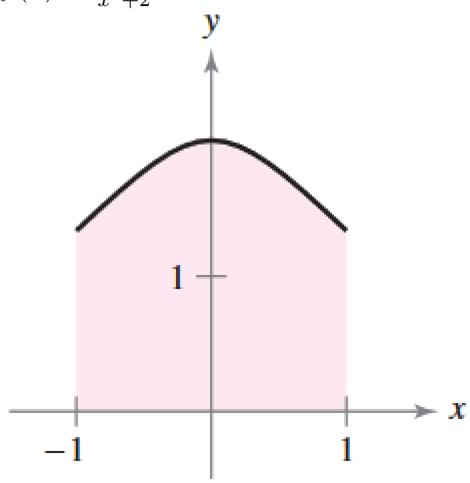
$$\int_0^2 (6 - 3x)dx \tag{1}$$

16.$f(x) = x^2$



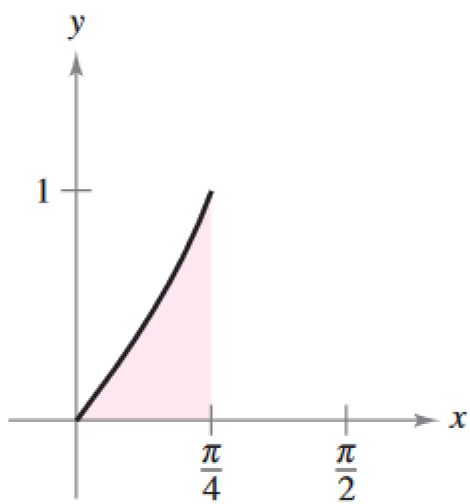
$$\int_0^2 x^2 dx \tag{1}$$

$18. f(x) = \frac{4}{x^2+2}$



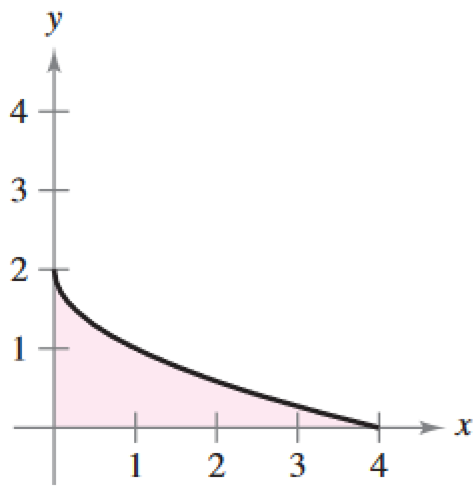
$$\int_{-1}^{1} \frac{4}{x^2+2} dx \tag{1}$$

$20. f(x) = \tan x$



$$\int_{0}^{\pi/4} \tan x dx \tag{1}$$

$22. f(y) = (y-2)^2$



$$\int_0^2 (y-2)^2 dy \tag{1}$$

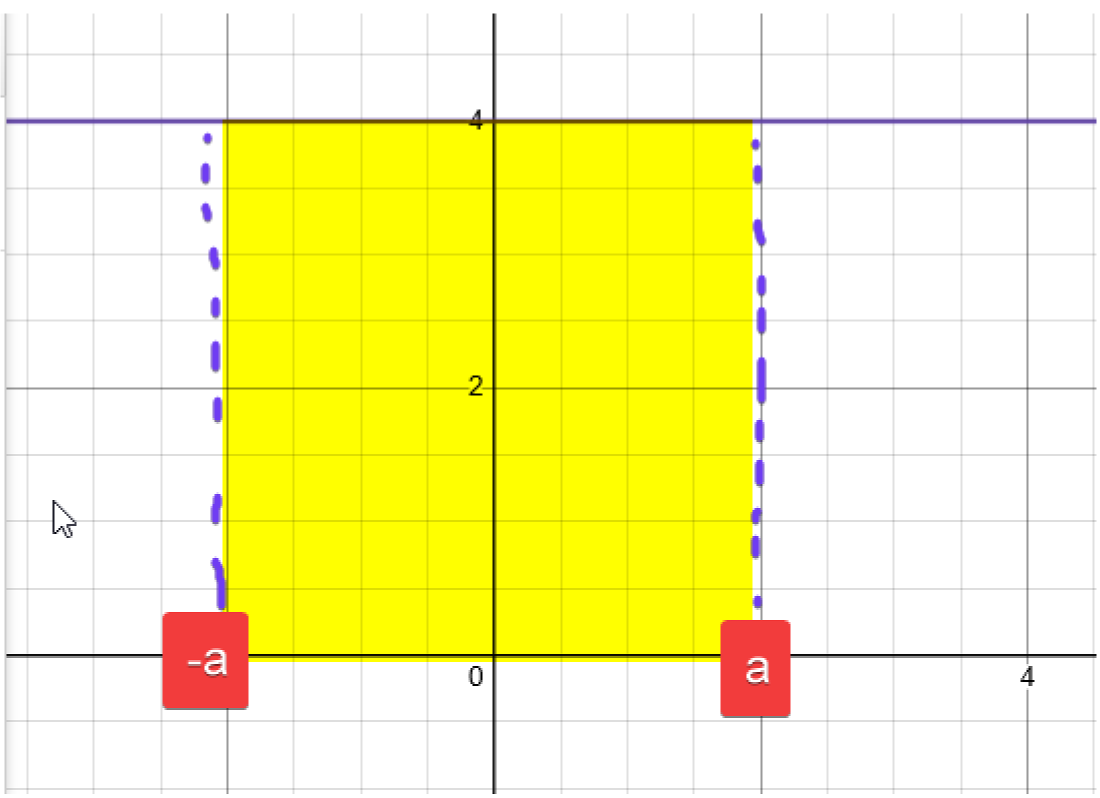
2 Sketch the region whose area is given by the definite integral. Then use a geometric formula to evaluate the integral $(a > 0,\ r > 0)$

24.

$$\int_{-a}^{a} 4dx$$

$$\text{Rectangle} A = bh = 2(4)(a) = 8a \tag{1}$$

$$A = \int_{-a}^{a} 4dx = 8a \tag{2}$$

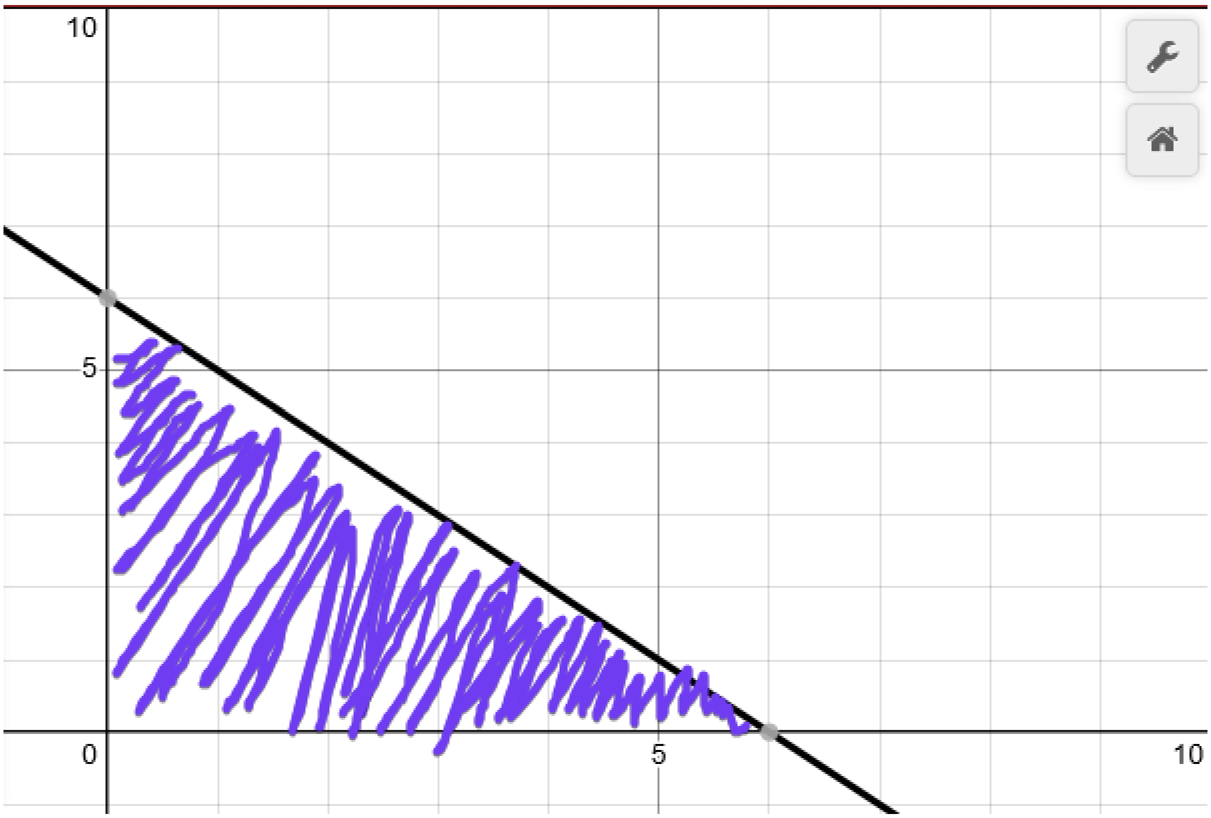


28.

$$\int_0^6 (6-x)dx$$

$$\text{Triangle}A = \frac{1}{2}bh = \frac{1}{2}(6)(6) = 18 \tag{1}$$

$$A = \int_0^8 (6-x)dx = 18 \tag{2}$$

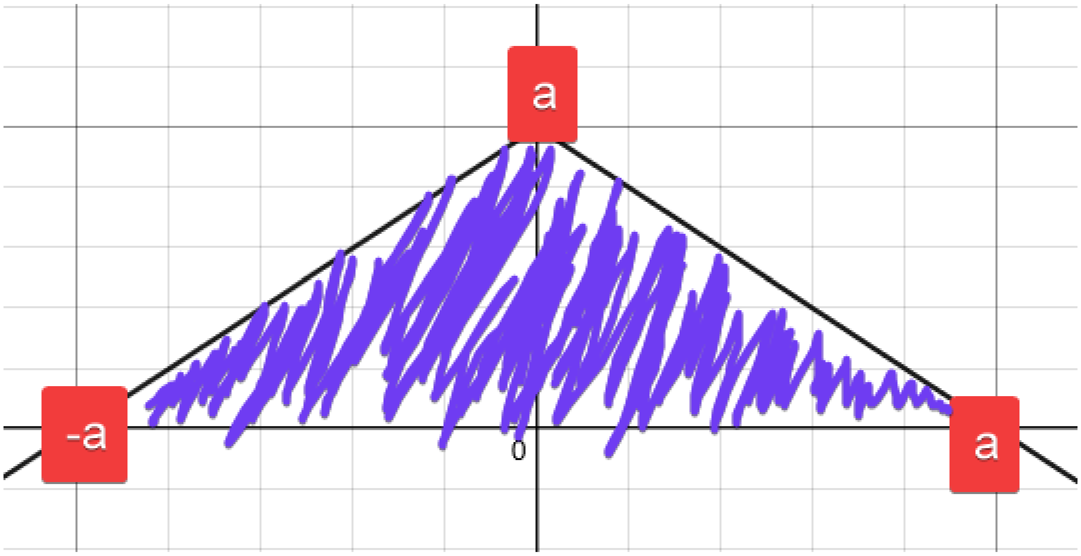


30.

$$\int_{-a}^{a} (a-|x|)dx$$

$$\text{Triangle}A = \frac{1}{2}bh = \frac{1}{2}(2a)a = a^2 \tag{1}$$

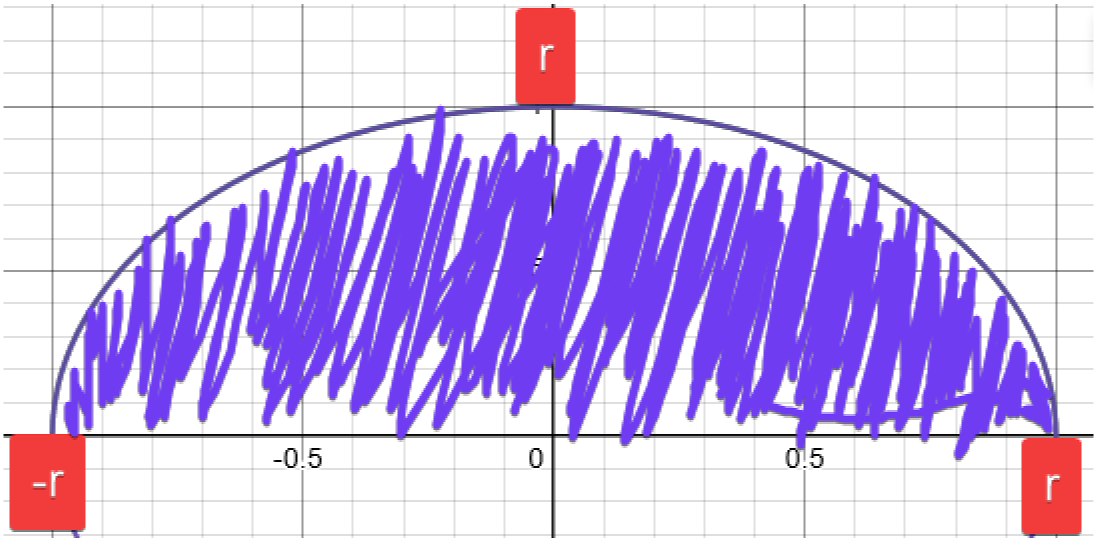
$$A = \int_{-a}^{a} (a-|x|)dx = a^2 \tag{2}$$



32.

$$\int_{-r}^{r}\sqrt{r^2-x^2}dx$$

$$A=\frac{1}{2}\pi r^2 \tag{1}$$
$$A=\int_{-r}^{r}\sqrt[root]{r^2-x^2}dx=\frac{1}{2}\pi r^2 \tag{2}$$



3 Evaluate the integral using the following values.

$$\int_2^4 x^3dx=60,\ \int_2^4 xdx=6,\ \int_2^4 dx=2$$

36.

$$\int_2^4 25dx \tag{1}$$
$$=25\int_2^4 dx \tag{2}$$
$$=25(2)=50 \tag{3}$$

40.

$$\int_2^4 (10 + 4x - 3x^3)dx \tag{1}$$
$$= 10\int_2^4 dx + 4\int_2^4 dx - 3\int_2^4 dx \tag{2}$$
$$= 10(2) + 4(6) - 3(60) \tag{3}$$
$$= -136 \tag{4}$$

4 Given $\int_0^3 f(x)dx = 4$ and $\int_3^6 f(x)dx = -1$, evaluate

42.

(a)

$$\int_0^6 f(x)dx \tag{1}$$
$$= \int_0^3 f(x)dx + \int_3^6 f(x)dx \tag{2}$$
$$= 4 + (-1) = 3 \tag{3}$$

(b)

$$\int_3^6 f(x)dx \tag{1}$$
$$= -\int_3^6 f(x)dx \tag{2}$$
$$= -(-1) = 1 \tag{3}$$

(c)

$$\int_3^3 f(x)dx \tag{1}$$
$$= 0 \tag{2}$$

(d)

$$\int_3^6 -5f(x)dx \tag{1}$$
$$= -5\int_3^6 f(x)dx \tag{2}$$
$$= -5(-1) = 5 \tag{3}$$

5 Given $\int_{-1}^1 f(x)dx = 0$ and $\int_0^1 f(x)dx = 5$, evaluate

42.

(a)

$$\int_{-1}^0 f(x)dx \tag{1}$$
$$= \int_{-1}^1 f(x)dx - \int_0^1 f(x)dx \tag{2}$$
$$= 0 - 5 = -5 \tag{3}$$

(b)

$$\int_0^1 f(x)dx - \int_{-1}^0 f(x)dx \quad (1)$$
$$= 5 - (-5) = 10 \quad (2)$$

(c)

$$\int_{-1}^1 3f(x)dx \quad (1)$$
$$= 3\int_{-1}^1 f(x)dx \quad (2)$$
$$= 3(0) = 0 \quad (3)$$

(d)

$$\int_0^1 3f(x)dx \quad (1)$$
$$= 3\int_0^1 f(x)dx \quad (2)$$
$$= 3(5) = 15 \quad (3)$$

46. Use the table of values to estimate $\int_0^6 f(x)dx$. Use three equal subintervals and the (a) left endpoints, (b) right endpoints, and (c) midpoints. If f is an increasing function, how does each esti- mate compare with the actual value? Explain your reasoning.

x	0	1	2	3	4	5	6
$f(x)$	−6	0	8	18	30	50	80

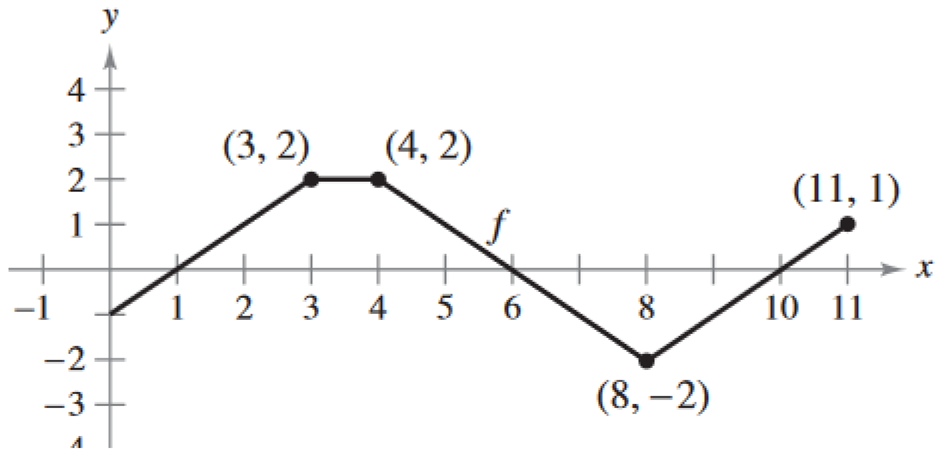
(a) Left endpoint estimate: $(-6 + 8 + 30)(2) = 64$

(b) Right endpoint estimate: $(8 + 30 + 80)(2) = 236$

(c) Midpoint estimate: $(0 + 18 + 50)(2) = 136$

6 Think About It

48. The graph of f consists of line segments, as shown in the figure. Evaluate each definite integral by using geometric formulas.



(a)

$$\int_0^1 -f(x)dx = -\int_0^1 f(x)dx = \frac{1}{2} \tag{1}$$

(b)

$$\int_3^4 3f(x)dx = 3(2) = 6 \tag{1}$$

(c)

$$\int_0^7 f(x)dx = -\frac{1}{2} + \frac{1}{2}(2)(2) + 2 + \frac{1}{2}(2)(2) - \frac{1}{2} = 5 \tag{1}$$

(d)

$$\int_5^{11} f(x)dx = -\frac{1}{2} + 2 + 2 + 2 - 4 + \frac{1}{2} = 2 \tag{1}$$

(e)

$$\int_4^{10} f(x)dx = 2 - 4 = -2 \tag{1}$$

7 Capstone

52. Find possible values of a and b that make the statement true. If possible, use a graph to support your answer. (There may be more than one correct answer.)

(a)

$$\int_{-2}^{1} f(x)dx + \int_1^5 f(x)dx = \int_{-2}^{5} f(x)dx \tag{1}$$

$$a = -2,\ b = 5 \tag{2}$$

(b)

$$\int_{-3}^{3} f(x)dx + \int_3^6 f(x)dx - \int_a^b f(x)dx = \int_{-1}^{6} f(x)dx \tag{1}$$

$$= \int_{-3}^{6} f(x)dx + \int_b^a f(x)dx \tag{2}$$

$$a = -3,\ b = -1 \tag{3}$$

(c)

$$\int_a^b \sin x dx < 0 = \int_\pi^{2\pi} \sin x dx < 0 \quad (1)$$
$$a = \pi,\ b = 2\pi \quad (2)$$

(d)

$$\int_a^b \cos x dx = 0 = \int_0^\pi \cos x dx \quad (1)$$
$$a = 0,\ b = \pi \quad (2)$$

8 Determine which value best approximates the definite integral. Make your selection on the basis of a sketch.

58.

$$\int_0^{1/2} 4 \cos \pi x dx$$

(a) 4

(b) $\frac{4}{3}$

(c) 16

(d) 2π

(e) -6

The answer is (b)$A \approx \frac{4}{3}u^2$.

60.

$$\int_0^9 (1 + \sqrt{x}) dx$$

(a) -3

(b) 9

(c) 27

(d) 3

The answer is (c)$A \approx 27$.

9 Determine whether the statement is true or false. If it is false, explain why or give an example that shows it is false.

65.$\int_a^b (f(x) + g(x))dx = \int_a^b f(x)dx + \int_a^b g(x)dx$
True.

66.$\int_a^b f(x)g(x)dx = \left(\int_a^b f(x)dx\right)\left(\int_a^b g(x)dx\right)$
False because $\int_0^1 x\sqrt{x}dx \neq \left(\int_0^1 xdx\right)\left(\int_0^1 \sqrt{x}dx\right)$.

67. If the norm of a partition approaches zero, then the number of subintervals approaches infinity.
True.

68. If f is increasing on $[a, b]$, then the minimum value of $f(x)$ on $[a, b]$ is $f(a)$.
True.

69. The value of $\int_a^b f(x)dx$ must be positive.
False because $\int_0^2 (-x)dx = -2$.

70. The value of $\int_2^2 \sin(x^2)dx$ is 0.
True.