3.1 Extrema on an interval

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1 Value of derivate (if it exists) at each indicated extremum

2.

$$f(x) = \cos\frac{\pi}{2}$$

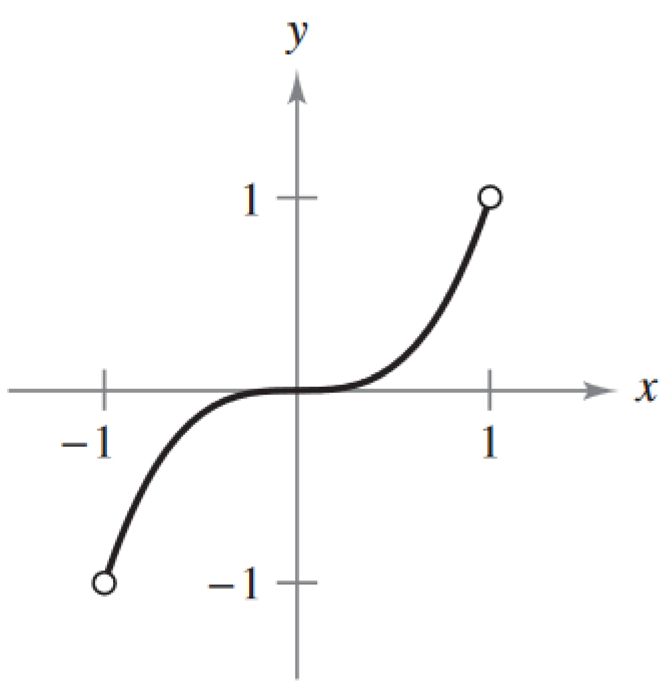
$$f'(x) = \frac{(2x)(x^2+4)-(x^2)(2x)}{(x^2+4)^2} = \frac{8x}{(x^2+4)^2} \therefore f'(0) = 0$$

4.

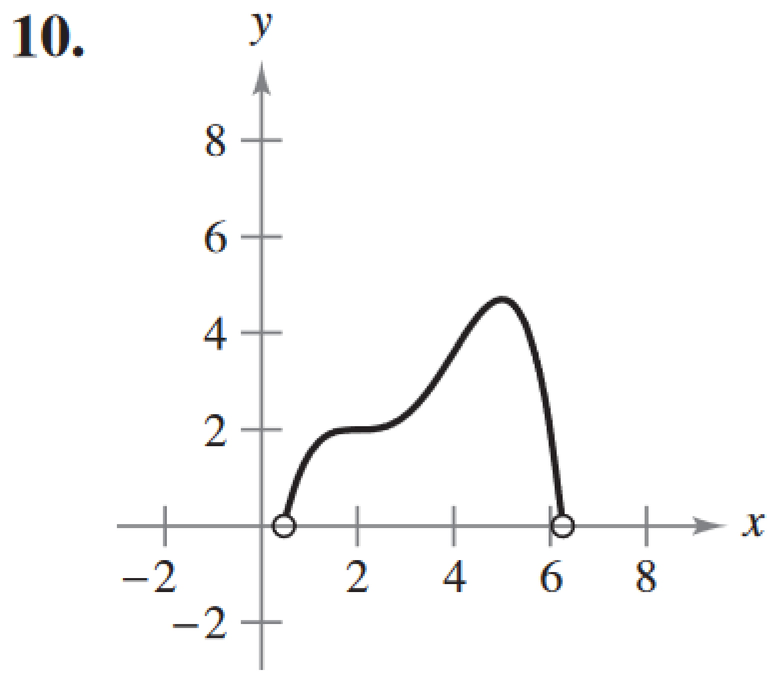
$$f(x) = -3x\sqrt{x+1}$$

$$f'(x) = -3x\left(\frac{1}{2}(x+1)^{-1/2}\right) + \sqrt{x+1}(-3)$$

2 Approximate the critical numbers of the function shown in the graph. Determine whether the function has a relative maximum, a relative minimum, an absolute maximum, an absolute minimum, or none of these at each critical number on the interval shown



8. $x = 0$ is a critical number. There is no minima or maxima at x=0.



10. $x = 2, 5$ are critical numbers. There is an absolute and relative maximum at $x = 5$, and no minima or maxima at $x = 2$

3 Find any critical numbers of the function.

12.

$$g(x) = x^4 - 4x^2$$
$$g'(x) = 4x^3 - 8x = 4x(x^2 - 2)$$

$x = 0, 2$ are critical numbers.

14.

$$f(x) = \frac{4x}{x^2+1}$$
$$f'(x) = \frac{(4x)(x^2+1) - (4x)(2x)}{(x^2+1)^2} = \frac{4(1-x^2)}{(x^2+1)^2}$$

$x = \pm 1$ are critical numbers.

16.

$$\begin{aligned} f(\theta) &= 2\sec(\theta) + \tan(\theta), \quad 0 < \theta < 2\pi && (1)\\ f'(\theta) &= 2\sec\theta\tan\theta + \sec^2\theta && (2)\\ &= \sec\theta(2\tan\theta + \sec\theta) && (3)\\ &= \sec\theta\left(2\left(\tfrac{\sin\theta}{\cos\theta}\right) + \tfrac{1}{\cos\theta}\right) && (4)\\ &= \sec^2\theta(2\sin\theta + 1) && (5) \end{aligned}$$

$\theta = \frac{7\pi}{6}, \frac{11\pi}{6}$ are critical numbers in $(0, 2\pi)$.

4 Locate the absolute extrema of the function on the closed interval.

18.

$$f(x) = \frac{2x+5}{3}, [0,5] \tag{1}$$

$$f'(x) = \frac{2}{3} \therefore \text{there are no critical numbers} \tag{2}$$

$(0, \frac{5}{3})$ is the left endpoint and minimum, and $(5,5)$ the right endpoint and maximum.

22.

$$f(x) = x^3 - 12x, \;\; [0,4] \tag{1}$$

$$f'(x) = 3x^2 - 12 = 3(x^2 - 4) \tag{2}$$

$(0,0)$ is the left endpoint, $(2,-16)$ is a critical number and the minimum, and $(4,16)$ the right endpoint and the maximum.

24.

$$g(x) = \sqrt[3]{x}, \;\; [-1,1] \tag{1}$$

$$g'(x) = \frac{1}{3x^{2/3}} \tag{2}$$

$(-1,1)$ is the left endpoint and minimum, $(0,0)$ is a critical number, and $(1,1)$ is the right endpoint and maximum.

26.

$$f(x) = \tfrac{2x}{x^2+1}, \;\; [-2,2] \tag{1}$$

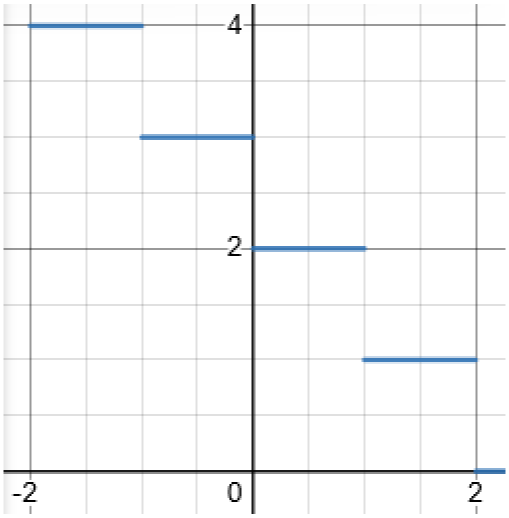
$$f'(x) = \tfrac{(2)(x^2+1)-(2x)(2x)}{(x^2+1)^2} = \frac{2-2x^2}{(x^2+1)^2} \tag{2}$$

$$= \tfrac{2(1-x^2)}{(x^2+1)^2} \tag{3}$$

$(-2,-\frac{4}{5})$ is the left endpoint, $(-1,-1)$ is the minimum and a critical number, $(1,1)$ is the maximum and a critical number, and $(2,\frac{4}{5})$ is the right endpoint.

32.

$$\lfloor 2-x \rfloor, \;\; [-2,2]$$



Analyzing the graph, the maximum h value is 4 at $x = -2$, and the minimum is 0 for $1 < x \leq 2$.

36.

$$y = \tan(\frac{\pi x}{8}), \quad [0, 2] \tag{1}$$

$$y' = \frac{\pi}{8}\sec^2(\frac{\pi x}{8}) \therefore y' \neq 0 \tag{2}$$

$(0, 0)$ is the minimum and left endpoint, and $(2, 1)$ is the maximum and right endpoint.

5 Locate the absolute extrema of the function (if any exist) over each interval.

38. $f(x) = 5 - x$

(a) $[0, 2]$

i. Minimum: (4, 1)
ii. Maximum: (1, 4)

(b) $[0, 2)$

i. Maximum: (1, 4)

(c) $(0, 2]$

i. Minimum: (4, 1)

(d) $(0, 2)$

i. The function has no extrema at this interval.

40. $f(x) = \sqrt{4 - x^2}$

(a) $[-2, 2]$

i. Minima: (2, 0) and (-2, 0)
ii. Maximum: (0, 2)

(b) $[-2, 0)$

i. Minimum: (-2, 0)

(c) $(-2, 2)$

i. Maximum: (0, 2)

(d) $[1, 2)$

i. Maximum: (1,$\sqrt{3}$)

6 Locate the absollute of the function on the given interval.

42.

$$f(x) = \begin{cases} 2x - x^2, & 1 \leq x < 3 \\ 2 - 3x, & 3 \leq x \leq 5 \end{cases}, \quad [0, 3] \tag{1}$$

(1, 1) is the left endpoint and the maximum.

(5, -13) is the right endpoint and the minimum.

44.

$$f(x) = \frac{2}{2-x}, \quad [0, 2)$$

(0, 1) is the left endpoint and the miminum

The function doesn't have a right endpoint or maximum.

46.

$$f(x) = \sqrt{x} + \cos\frac{x}{2}, \quad [0, 2\pi]$$

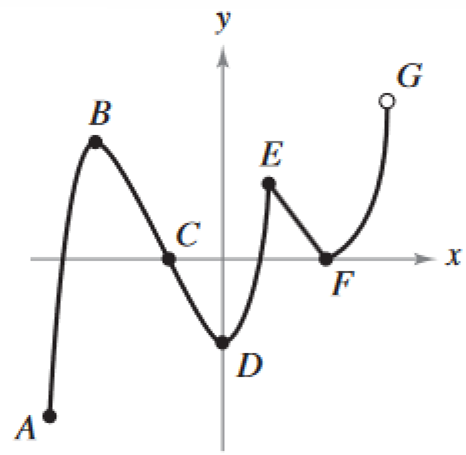
$$f'(x) = \frac{1}{2\sqrt{x}} - \frac{1}{2}\sin\frac{x}{2}$$

(0,1) is the left endpoint and minimum.

(1.729, 1.964) is the maximum.

7 Capstone

54. Decide whether each labeled point is an absolute maximum or minimum, a relative maximum or minimum, or neihter.



A: Absolute minimum

B: Relative maximum

C: Neither

D: Relative minimum

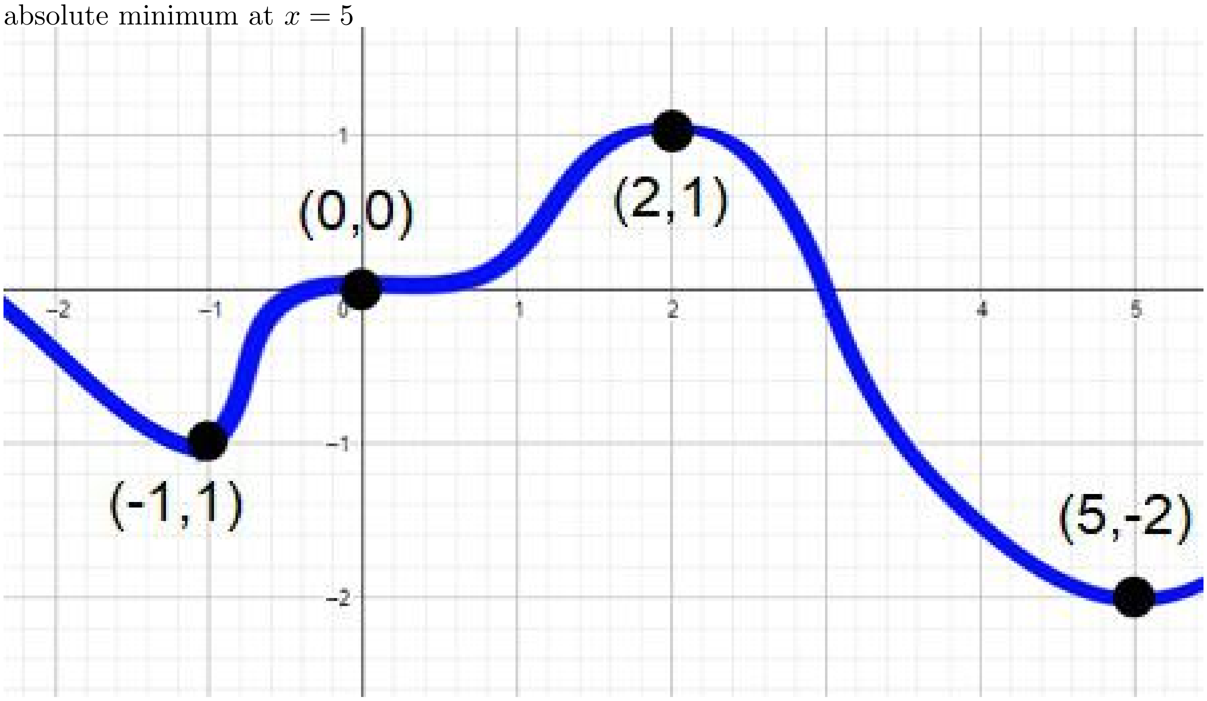
E: Relative maximum

F: Relative minimum

G: Neither

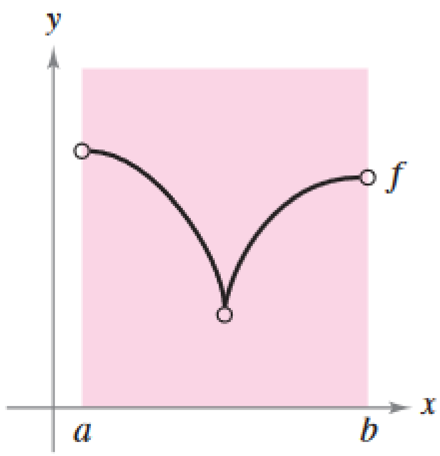
8 Graph a function on the interval [-2, 5] having the given characteristics

56. Relative minimum at $x = -1$, critical number (but no extremum) at $x = 0$, absolute maximum at $x = 2$, absolute minimum at $x = 5$

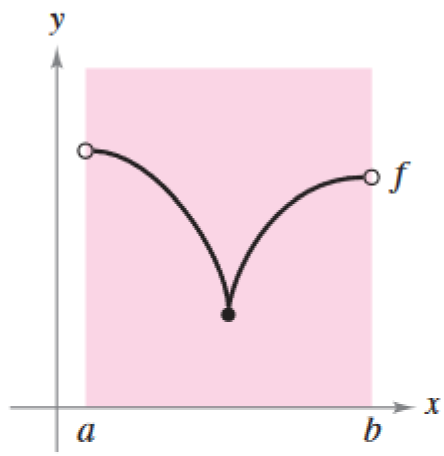


9 Determine from the graph whether f has a minimum in the open interval (a, b)

58. (a)



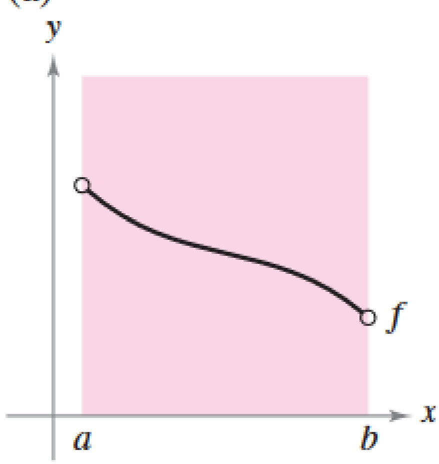
(b)



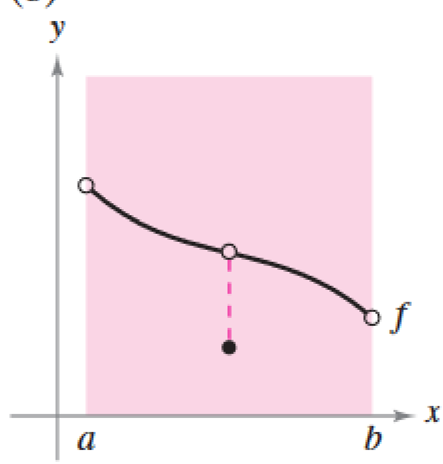
(a) No

(b) Yes

60. (a)



(b)



(a) No

(b) Yes

10 Determine whether the statement is true or false. If it is false, explain why or give an example that shows it is false.

65. The maximum of a function that is continuous on a closed interval can occur at two different values in the interval.

True

66. If a function is continuous on a closed interval, then it must have a minimum on the interval.

True

67. If $x = c$ is a critical number of the function f, then it is also a critical number of the function $g(x) = f(x) + k$, where k is a constant.

True

68. If $x = c$ is a critical number of the function f, then it is also a critical number of the function $g(x) = f(x - k)$, where k is a constant.

False. $x = 0$ is a critical number of $f(x) = x^2$. If $g(x) = f(x - k) = (x - k)^2$ then $x = k$ is a critical number of g.