Notes - 5.6 Inverse Trigonometric Functions: Differentiation

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1 arcsin refresher

$$\sin \theta = \frac{opp}{hyp} \tag{1}$$

(4)

$$\arcsin\left(\frac{opp}{hyp}\right) = \sin^{-1}\left(\frac{opp}{hyp}\right) = \theta \tag{2}$$

$$\tan^{-1}\left(\frac{opp}{adj}\right) = \theta \tag{3}$$

2 Inverse trig functions

$$\cos(\arctan(2x^2)) = \frac{1}{\sqrt{1+4x^4}}, \ opp = 2x^2, \ adj = 1, \ hyp = \sqrt{1^2 + (2x^2)^2}$$
 (1)

$$\cos \theta = \frac{1}{\sqrt{1+4x^4}} \tag{2}$$

$$\frac{d}{dx}(\arcsin x) = \frac{1}{\sqrt{1-x^2}}\tag{3}$$

$$\sin y = \sin(\arcsin x) \tag{4}$$

$$\frac{d}{dx}(\sin y = x) \tag{5}$$

$$\cos y dy = dx \tag{6}$$

$$\frac{dy}{dx} = \frac{1}{\cos y} \tag{7}$$

$$\frac{d}{dx}(\cos^{-1}x) = , opp = \sqrt{1^2 - x^2}, adj = x, hyp = 1$$
 (1)

$$\cos y = \cos(\cos^{-1} x) \tag{2}$$

$$\cos y = x \tag{3}$$

$$-\sin y dy = dx \tag{4}$$

$$\frac{dy}{dx} = -\frac{1}{\sin y} \tag{5}$$

$$\frac{d}{dx}(\tan^{-1}x) = \frac{1}{1+x^2}, \ opp = x, \ adj = 1, \ hyp = \sqrt{1^2 + x^2}$$
 (1)

$$y = \tan(\tan^{-1} x) \tag{2}$$

$$tan y = x (3)$$

$$\sec^2 y dx = dx \tag{4}$$

$$\frac{dy}{dx} = \frac{1}{\sec^2 y} = \frac{1}{(\sec y)^2} = \frac{1}{(\sqrt{1+x^2})^2}$$
 (5)

$$\frac{d}{dx}(\sec^{-1}x) = , opp = \sqrt{x^2 - 1^2}, adj = 1, hyp = x$$
 (1)

$$\sec y = \frac{x}{1} = x \tag{2}$$

$$\sec y \tan y dy = dx \tag{3}$$

$$\frac{dy}{dx} = \frac{1}{\sec y \tan y} \tag{4}$$