Notes - 3.7 Optimization Problems

Juan J. Moreno Santos

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1 Warm-up

1. An equation of the line tangent to the graph of $f(x) = x(1-2x)^3$ at the point (1, -1) is
Expanding this quartic results in the following simplification:

$$a^3 + 3a^2b + 3ab^2 + b^3 \tag{1}$$
$$1^3 + 3(1)^{-}2x + 3(1)^1 - (-2x)^2 + (-2x)^3 \tag{2}$$
$$1 - 6x + 12x^2 - 8x^3 \tag{3}$$
$$x - 6x^2 + 12x^3 - 8x^4 \tag{4}$$
$$\tag{5}$$

Therefore,

$$y + 7 = -7(x-1) \tag{6}$$
$$y + 1 = -7x + 7 \tag{7}$$
$$y = -7x + 6 \tag{8}$$
$$\tag{9}$$

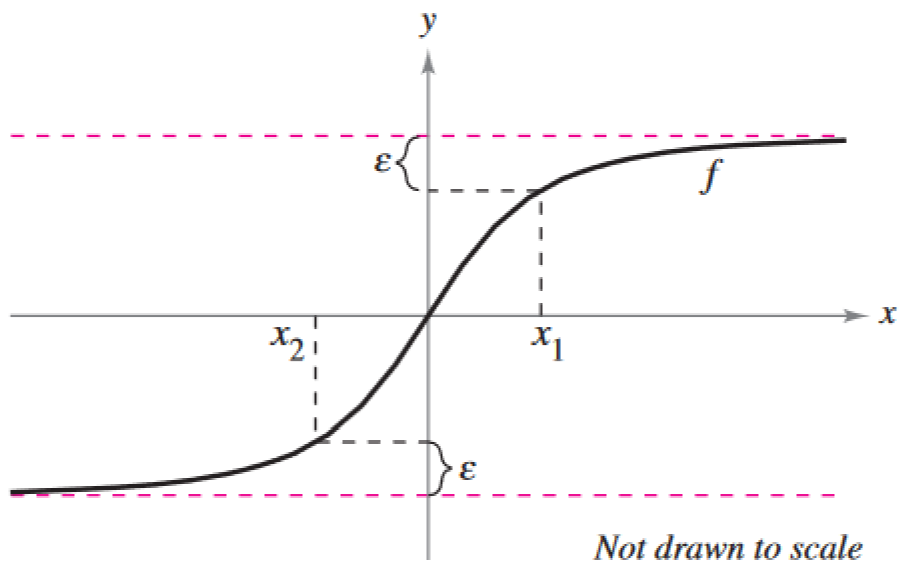
Using the product rule,

$$f'(x) = 1(1-2x)^3 + x3(1-2x)^2(-2) \tag{1}$$
$$= (1-2x^3) - 6x(1-2x)^2 \tag{2}$$
$$= -1 - 6 = -7 = m \tag{3}$$

Refer to equation 6.

2 Reminder about limits

98. The graph of $f(x) = \dfrac{6x}{\sqrt{x^2 + 2}}$ is shown.



(a) Find $L = \lim_{x\to\infty} f(x)$ and $K = \lim_{x\to-\infty} f(x)$.

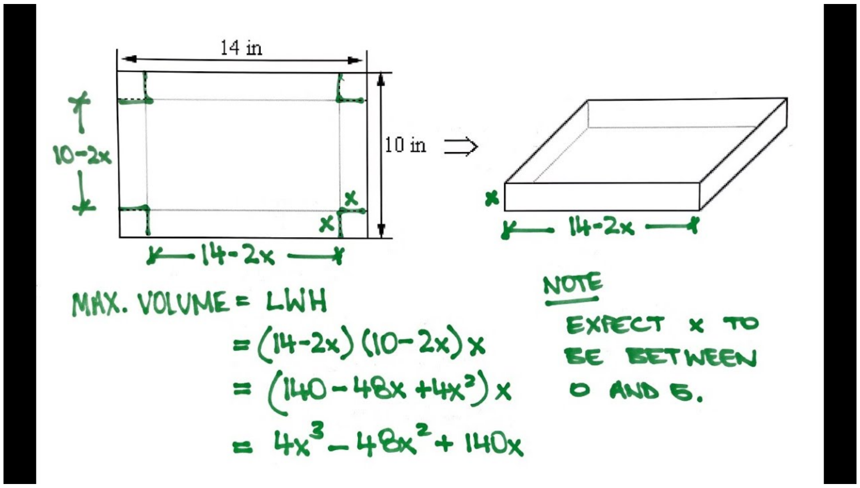
(b) Determine x_1 and x_2 in terms of ε.

(c) Determine M, where $M > 0$, such that $|f(x) - L| < \varepsilon$ for $x > M$.

(d) Determine N, where $N < 0$, such that $|f(x) - K| < \varepsilon$ for $x < N$.

Remember that $|x| = \sqrt{x^2}$.
Dividing coefficients will produce both a negative and a positive asymptote, so the limits approach different values from both sides.

3 The box problem



4 Find two positive numbers that satisfy the given requirements

5. The product is 147 and the sum of the first number plus three times the second number is a minimum.

$$xy = 147 \tag{1}$$
$$y = \frac{147}{x} \tag{2}$$

$$x + 3y = minimum \tag{3}$$
$$(x+3)(\frac{147}{x}) = minimum \tag{4}$$
$$minimum' = 1 - \frac{3 \cdot 147}{x^2} = 0 \tag{5}$$
$$1 = \frac{3 \cdot 147}{x^2} \tag{6}$$
$$x^2 = 3.147 \therefore \quad x = 21, \quad y = 7 \tag{7}$$
$$\tag{8}$$

5 Find the length and width of a rectangle that has the given perimeter and a maximum area.

10. Perimeter: P units

$$Area_{max} = A_{max} = xy \tag{1}$$
$$Perimeter = P = 2x + 2y \tag{2}$$
$$\frac{P-2x}{2} = y \tag{3}$$
$$Amax = x\left(\frac{p-2x}{2}\right) \tag{4}$$
$$= \frac{1}{2}Px - x^2 \tag{5}$$
$$A' = \frac{P}{2} - 2x = 0 \text{ when } x = \frac{P}{4}, \quad y = \frac{P}{4} \tag{6}$$

6 11/02/2023 Warm-up

1.

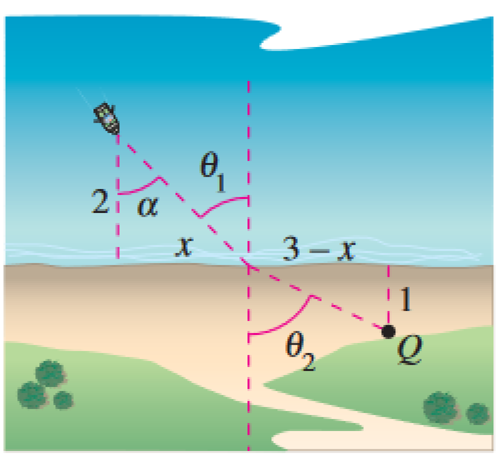
$$
\begin{aligned}
f(x) =& \sqrt{2x} && (1)\\
f'(1) =& \frac{1}{\sqrt{2x}} && (2)\\
=& \frac{1}{\sqrt{2 \cdot 1}}(\frac{\sqrt{2}}{\sqrt{2}}) && (3)\\
=& \frac{\sqrt{2}}{2} && (4)
\end{aligned}
$$

2.

$$
\begin{aligned}
f(x) &= \frac{x^2+1}{x}, \quad g(x) = 4x+7 && (1)\\
h(x) &= f(x) \cdot g(x) && (2)\\
h'(x) &= f'(x)g(x) + f(x)g'(x) && (3)\\
h'(x) &= (1 - x^{-}2)(4x+7) + (x + \frac{1}{x})(4) && (4)\\
h'(1) &= 8, \quad f(1) = 0, \quad g'(1) = 4 && (5)
\end{aligned}
$$

7 3.7 P-set example problems

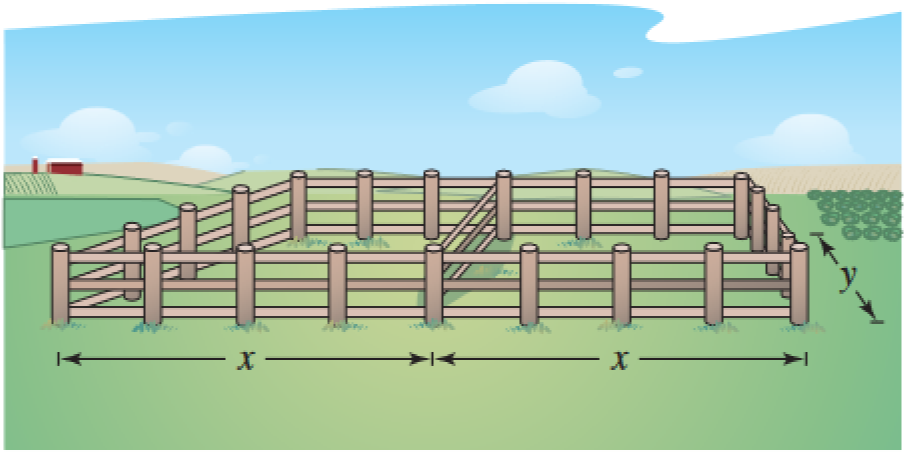
49. ***Minimum Time*** A man is in a boat 2 miles from the nearest point on the coast. He is to go to a point Q, located 3 miles down the coast and 1 mile inland (see figure). He can row at 2 miles per hour and walk at 4 miles per hour. Toward what point on the coast should he row in order to reach point Q in the least time?



$$
Distance = rate_1 time_1 + rate_2 time_2 = D = r_1 t + r_2 t \quad (1)
$$

$$
(2)
$$

22. ***Maximum Area*** A rancher has 400 feet of fencing with which to enclose two adjacent rectangular corrals (see figure). What dimensions should be used so that the enclosed area will be a maximum?



$$400ft = 4x + 3y \tag{1}$$
$$x = \frac{400 - 3y}{4} \tag{2}$$
$$Area = A = 2x \cdot y \tag{3}$$
$$= 2\left(\frac{400 - 3y}{4}\right)y \tag{4}$$
$$= 200y - \frac{3}{2}y^2 \tag{5}$$
$$A' = 200 - 3y = 0y = \frac{200}{3} \tag{6}$$

Solving for x:

$$400 = 3\left(\frac{200}{3}\right) + 4x \tag{7}$$
$$200 = 4x \tag{8}$$
$$50 = x \tag{9}$$

Since the fence is $2x$, the dimensions of the enclosed area will be 100ft by $\frac{200}{3}$ft.