Chapter 4 Review

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1 Free Response

1.1 4.1 Antiderivatives and Indefinite Integration - p.257

71. A ball is thrown vertically upward from a height of 6 feet with an initial velocity of 60 feet per second. How high will the ball go? Use $a(t) = -32$ feet per second per second as the acceleration due to gravity.

$$h(t) = \frac{1}{2}at^2 + v_0 t + h_0 \tag{1}$$
$$\int a(t) = \int -32 \tag{2}$$
$$v(t) = -32t + C \tag{3}$$
$$60 = -32(0) + C \tag{4}$$
$$v(t) = -32t + 60 \tag{5}$$
$$h(t) = \frac{1}{2}(-32)t^2 + 60t + C \tag{6}$$
$$6 = \frac{1}{2}(-32)(0)^2 + 60(0) + C \tag{7}$$
$$\therefore h(t) = \frac{1}{2}(-32)t^2 + (60)t + 6 \tag{8}$$

Remember that $x = t = \frac{-b}{2a}$ can be used. With the t-value, we can calculate the max height. The maximum height is 62.25.

77. A baseball is thrown upward from a height of 2 meters with an initial velocity of 10 meters per second. Determine its maximum height. Use $a(t) = -9.8$ meters per second per second as the acceleration due to gravity. (Neglect air resistance.)

$$y = \frac{1}{2}(-9.8)t^2 + 10t + 2 \tag{1}$$
$$x\frac{-b}{2a} = \frac{-10}{2(-4.9)} = 1.02 \tag{2}$$

Let $t = 1.02$. The maximum height is 7.1m.

1.2 4.2 Area - p.268

37. Find the limit of $s(n) as n \to \infty$.

$$s(n) = \frac{81}{n^4}\left(\frac{n^2(n+1)^2}{4}\right) \tag{1}$$

$$\lim_{n\to\infty} \frac{81}{n^4}\left(\frac{n^2(n+1)^2}{4}\right) = \lim_{n\to\infty} \frac{81n^4 + 162n^3 + 81n^2}{4n^4}\left(\frac{\frac{1}{n^4}}{\frac{1}{n^4}}\right) \tag{2}$$

$$= \lim_{n\to\infty} \frac{81 + \frac{162}{n}}{+} \frac{81}{n^2} \tag{3}$$

$$= \frac{81}{4} \tag{4}$$

1.3 4.3 Riemann Sums and Definite Integrals - p.279

41. Given $\int_0^5 f(x)dx = 10$ and $\int_5^7 f(x)dx = 3$, evaluate

(a)

$$\int_0^7 f(x)dx = 13 \tag{1}$$

(b)

$$\int_5^0 f(x)dx = -10 \tag{1}$$

(c)

$$\int_5^5 f(x)dx = 0 \tag{1}$$

(d)

$$\int_0^5 3f(x)dx = 30 \tag{1}$$

42. Given $\int_0^3 f(x)dx = 4$ and $\int_3^6 f(x) = -1$ evaluate

(a)

$$\int_0^6 f(x)dx = 3 \tag{1}$$

(b)

$$\int_6^3 f(x)dx = -1 \tag{1}$$

(c)

$$\int_3^3 f(x)dx = 0 \tag{1}$$

(d)

$$\int_3^6 -5f(x)dx = 5 \tag{1}$$

1.4 4.4 The Fundamental Theorem of Calculus - p.293

46. Find the value(s) of c guaranteed by the Mean Value Theorem for Integrals for the function over the given interval.

$$f(x) = \frac{9}{x^3}, \quad [1, 3] \tag{1}$$

$$\tag{2}$$

Remember that the average height of a function on an interval is given by

$$\frac{1}{b-a}\int_a^b f(x)dx = f(c) \tag{3}$$

$$\tag{4}$$

Therefore,

$$\begin{aligned}
&\frac{1}{2}\int_1^3 9x^{-3}dx && (5)\\
&= \frac{9}{2}\cdot \frac{x^{-2}}{\Big|_1}^3 && (6)\\
&= -\frac{9}{4}(3)^{-2} - \left(-\frac{9}{4}(1)^{-2}\right) && (7)\\
&= \frac{1}{4} + \frac{9}{4} = \frac{8}{4} = 2 && (8)
\end{aligned}$$

Finding c:

$$\begin{aligned}
2 &= \frac{9}{c^3} && (9)\\
2c^3 &= 9 && (10)\\
c^3 &= 4.5 && (11)\\
c &= \sqrt[3]{4.5} && (12)
\end{aligned}$$

1.5 4.5 Integration by Substitution - p.306

41. Solve the differential equation
1. Start by separating the integral

$$\begin{aligned}
\frac{dy}{dx} &= \frac{x+1}{(x^2+2x-3)^2} && (1)\\
dy &= \frac{x+1}{(x^2+2x-3)^2}dx && (2)\\
& && (3)
\end{aligned}$$

2. Integrate

$$\begin{aligned}
\int dy &= \int \frac{x+1}{(x^2+2x-3)^2}dx, \quad u = x^2+2x-3, \quad du = 2x+2dx \therefore \frac{1}{2}du = x+1dx && (1)\\
y &= \frac{1}{2}\int\left(\frac{1}{u^2}\right)du && (2)\\
&= \int u^{-2}du && (3)\\
&= -\frac{1}{2}u^{-1} + C && (4)\\
&= -\frac{1}{2(x^2+2x-3)} + C && (5)
\end{aligned}$$

1.6 4.6 Numerical Integration - p. 316

8. Use the Trapezoidal Rule to approximate the value of the definite integral for the given value of n Round your answer to four decimal places and compare the results with the exact value of the definite integral.

$$\int_1^4 (4 - x^2)dx, \quad n = 6 \tag{1}$$

$$= \frac{4-1}{2(6)}(f(1) + 2f(\frac{3}{2}) + 2f(2) + 2\frac{5}{2} + 2f(3) + 2f(\frac{7}{2}) + f(4)) \tag{2}$$

$$= -9 \tag{3}$$