# Angular Momentum

Shaping the Universe

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University of Alabama

# Emmy Noether's Theorem

- Born in Bavaria (Germany) March 1882
- Despite proficiency in English, French, decided to pursue mathematics at Erlangen
  - Allowed only to audit classes
  - "Overthrew all academic order"
  - Passed final exams 1903
- Teaches without pay at Erlangen, 1908-1915
  - Begins work on abstract algebra
- Hired by David Hilbert (Hilbert space!) to work at University of Gottingen
  - Other faculty protest
  - Teaches as Hilbert's "assistant" without pay
- In 1915, she proves Noether's Theorem
  - Recognized after being presented by a colleague in 1918
  - Most important mathematical proof since Pythagorean theorem?
  - "Every differential symmetry of the action of conservative forces has a corresponding conservation law"
    - Translation symmetry→linear momentum
    - Rotational symmetry→ angular momentum
    - Temporal symmetry→ mass-energy
- In 1919 after WWI Emmy Noether is given tenure at Gottingen
  - Extremely productive during this time of acceptance
- In 1933 began teaching at Bryn Mawr College after being forced to leave Germany by the Nazi party

# Conservation Laws

The Universe only cares about a few things:

Amount of metal? No, nuclear reactions!

Amount of mass? No, nuclear reactions again!

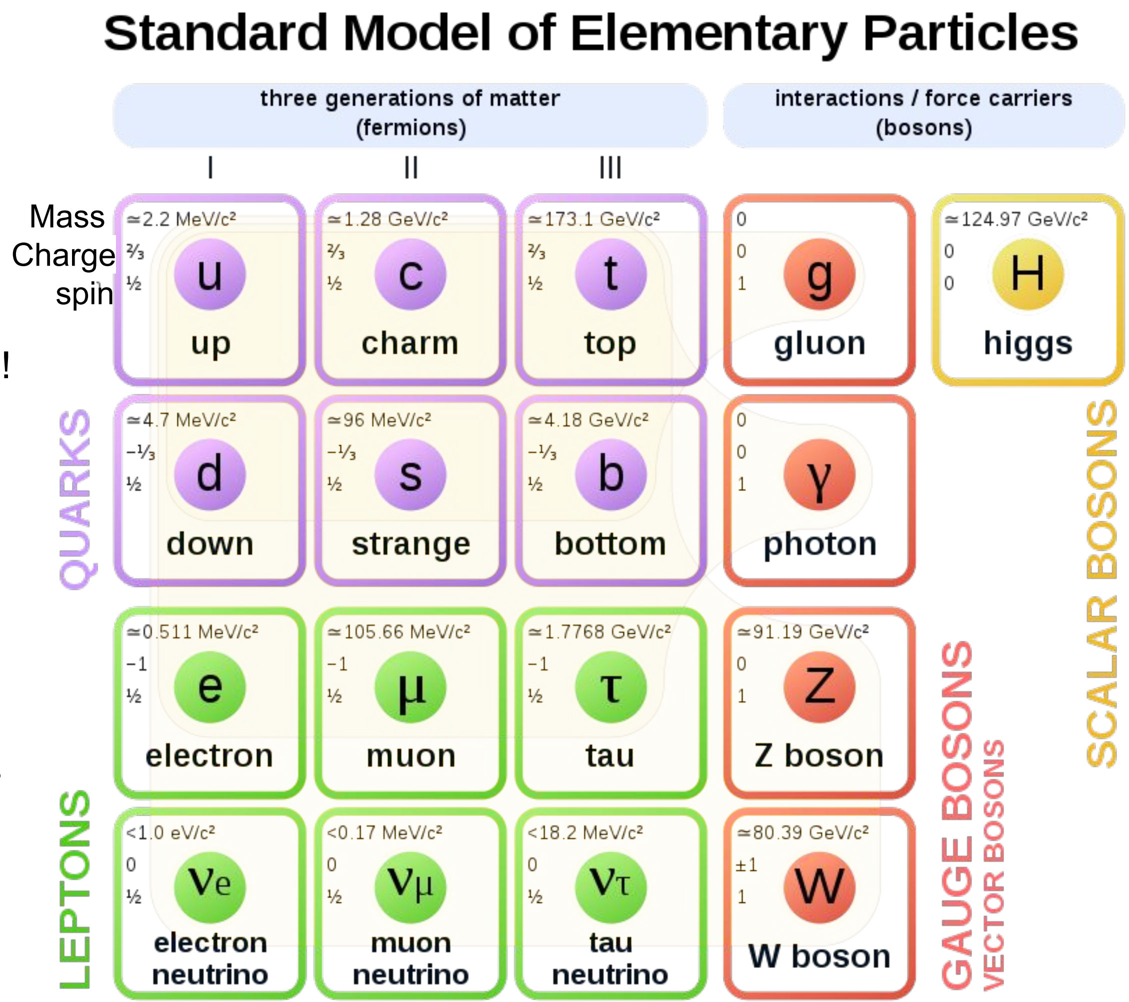
Energy? See above

But, mass-energy IS conserved! $E=mc^2$

Sometimes things we assume are separate are actually the same.

Velocity? Almost→ Momentum=velocity*mass
$p=mv$

Some weird ones: muon and lepton number, baryon (quark) number, electric charge



# Angular Momentum, Right Hand Rule

L=p*r=m*v*r=m*(2$\pi$r*f)*r $\Rightarrow$ L=mr$^2$w (angular momentum of small bit of matter)

How is angular momentum conserved?

- If you move inward on merry-go-round (reduce r), w must increase (merry-go-round speeds up) to conserve L!
- If the merry-go-round is rotating and you jump on (increase m), it must slow down to conserve L!

Right hand rule:

Curl right fingers in direction of rotation $\Rightarrow$ right thumb points in direction of angular momentum!

# Angular Momentum of Objects

L=mr$^2$w=iw
(angular momentum of small bit)

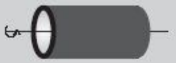
$$I_P = \iiint_Q \rho(x, y, z) \|\mathbf{r}\|^2 \, dV$$

I=(geometric factor)*MR$^2$,
L=(geometric factor)*MR$^2$W
(total angular momentum)

Angular momentum has 4 different values that can all change together to keep L constant!

1) Mass of object (M)
2) Size (R)
3) Rotation speed (W)
4) Shape (geometric factor)

*Angular momentum is a basic conserved quantity like mass-energy, But unlike other basic conserved quantities it cares about the shape of objects
*L is responsible for setting the shape of objects in the universe!

| Z | Body | Axis | Figure | I |
|---|---|---|---|---|
| (1) | Thin circular ring, radius $R$ | Perpendicular to plane, at centre |  | $MR^2$ |
| (2) | Thin circular ring, radius $R$ | Diameter |  | $MR^2/2$ |
| (3) | Thin rod, length L | Perpendicular to rod, at mid point |  | $ML^2/12$ |
| (4) | Circular disc, radius $R$ | Perpendicular to disc at centre |  | $MR^2/2$ |
| (5) | Circular disc, radius $R$ | Diameter |  | $MR^2/4$ |
| (6) | Hollow cylinder, radius $R$ | Axis of cylinder |  | $MR^2$ |
| (7) | Solid cylinder, radius $R$ | Axis of cylinder |  | $MR^2/2$ |
| (8) | Solid sphere, radius $R$ | Diameter |  | $2MR^2/5$ |

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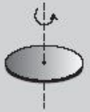
| (4) | Circular disc, radius $R$ | Perpendicular to disc at centre |  | $MR^2/2$ |
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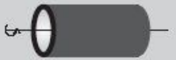
| (6) | Hollow cylinder, radius $R$ | Axis of cylinder |  | $MR^2$ |
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# Stellar Birth in Molecular Clouds

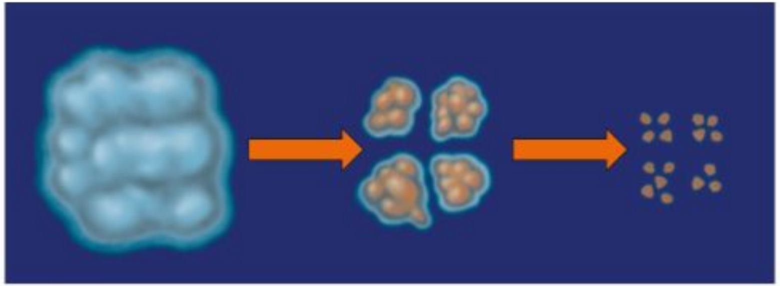
Confusingly, to create hot stars you first need very cold gas! For gas clouds to collapse, individual atoms/molecules need to lose energy so that gravity can move them. This energy is usually given off as light.

Problem; the hotter the gas, the harder it is to release the heat as light!

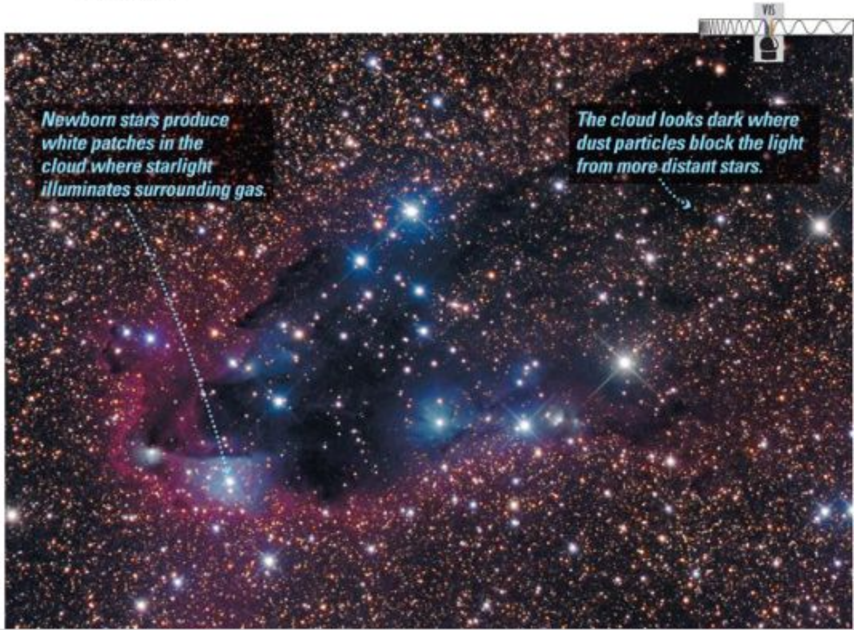
Molecular bonds in the gas allow light to be released more easily. Once these form the cooling is rapid, causing the cloud to collapse under its own gravity. As it does, the cloud fragments.

Some regions of the cloud get dense enough that the molecules cannot radiate heat properly; these places become “proto-stars” (and eventually stars).

NOTE: STARS FORM IN CLUSTERS

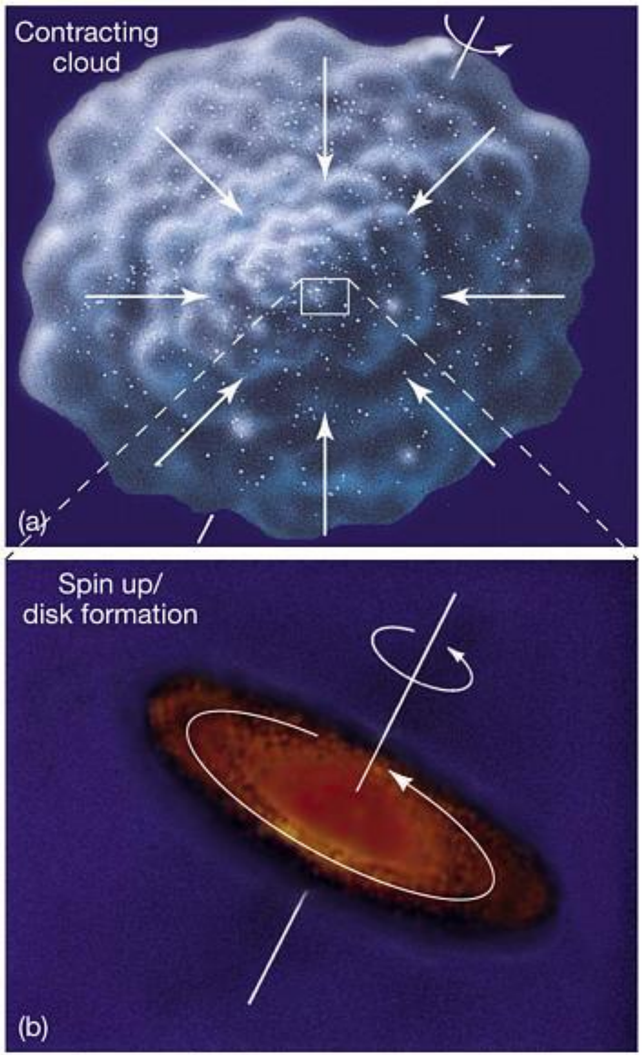






# Angular Momentum and Star Formation

1. Begin with huge cloud of gas, dust (spherical-ish)
   a. Chaotic environment, particles with different masses (m), different distances (r), different speeds, even *different directions*!
   b. Carefully add up all ang mom for each bit, get $L_T$
2. Gravity pulls gas/dust particles toward center, but angular momentum of most particles is too large to “fall in”→ tiny collisions cancel out angular momentum of different particles→particles fall in
3. Right hand rule: whatever direction $L_T$ was in before collapse began, $L_T$ remains in that direction (conservation of L!)
4. Any particles with angular momentum in different directions than $L_T$ fall into center; the particles that are left form a disk of more dense gas



# Angular Momentum and Star Formation

5. Change from sphere to disk changes geometric factor from ⅖ to ½
   a. To keep $L_T$=(geometric factor)*$MR^2W$ constant, change M, R, or W
      i. Mass remains the same
      ii. Size can reduce
      iii. Frequency of rotation usually speeds up

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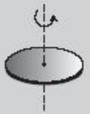
| (4) | Circular disc, radius $R$ | Perpendicular to disc at centre |  | $MR^2/2$ |
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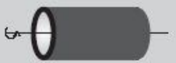
| (6) | Hollow cylinder, radius $R$ | Axis of cylinder |  | $MR^2$ |
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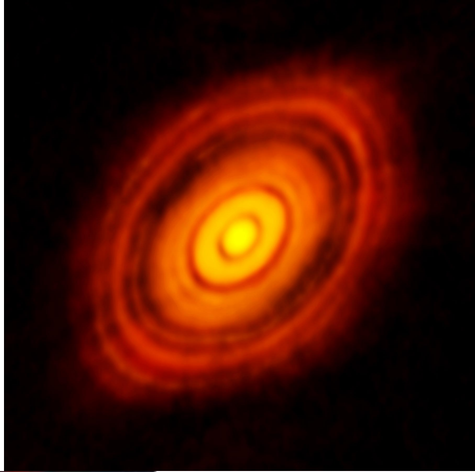
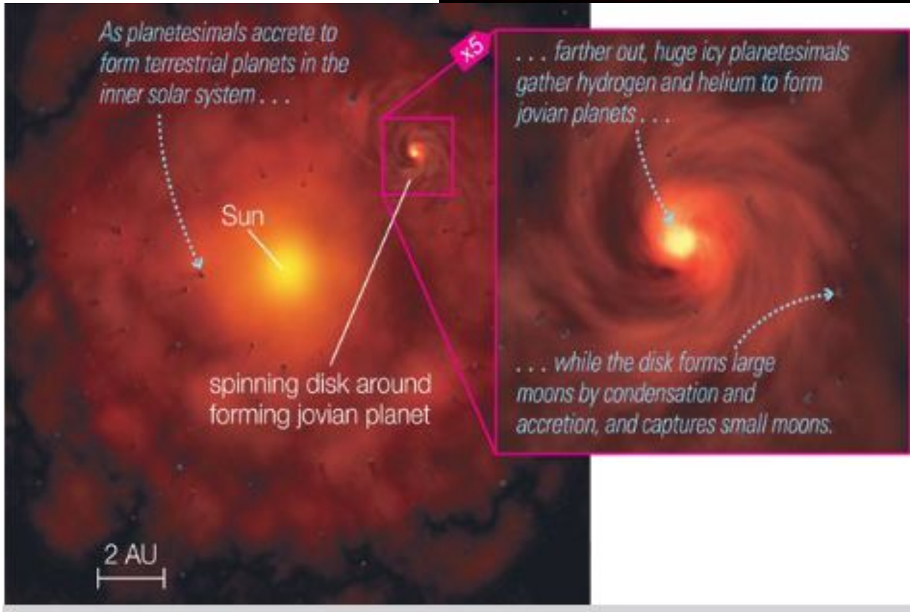
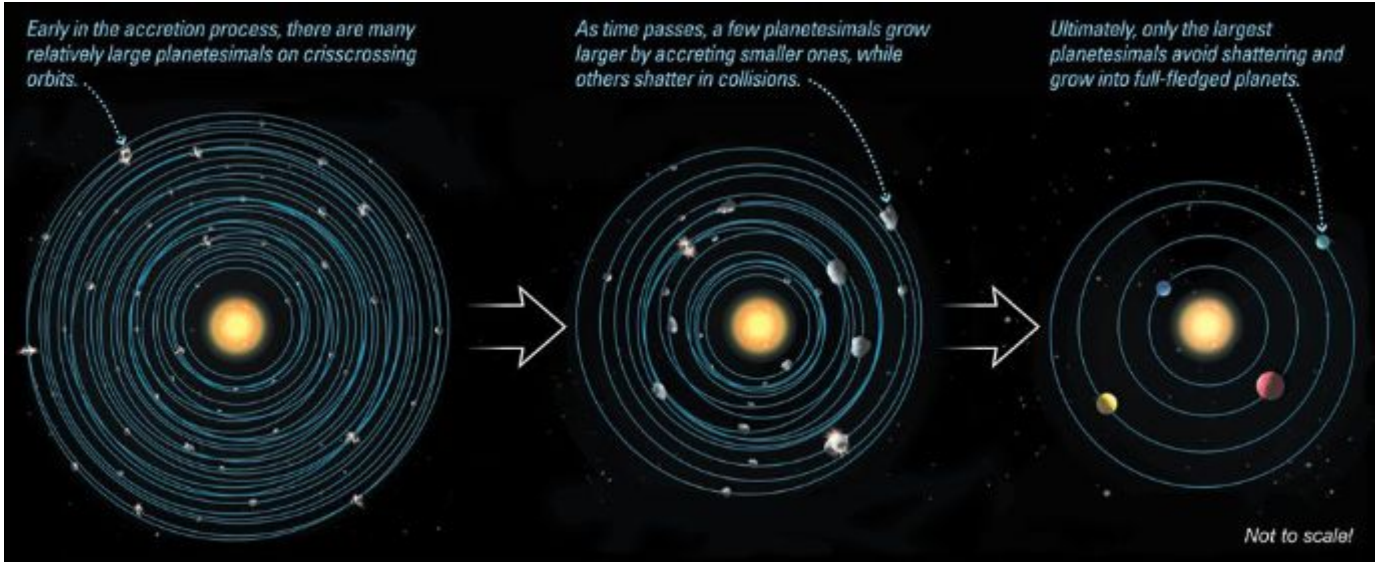
6. Call this “accretion disk”
    a. Disk is much more dense than initial cloud→ more collisions
    b. Compressing cloud into disk heats it
7. Ordinary matter has the ability to radiate light!
    a. As disk heats, the light generated carries away energy and angular momentum
    b. This allows the matter to be compressed even further→more light generated, more energy/L radiated→ more compression….
8. Matter collects in middle of disk, heats and compresses until hydrogen burning!



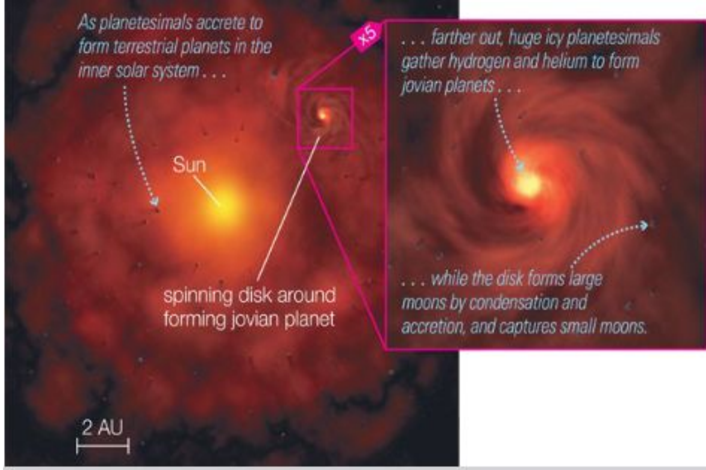


# Angular Momentum and Star Formation

9. The new star heats and evaporates most of the accretion disk, leaving only the largest objects
10. Left with a spherical star and a disk of planets
    a. Angular momentum conservation gives us the initial disk shape because it is perpendicular to the original angular momentum of cloud
    b. Angular momentum gives us spherical shape for stars because at same $MR^2W$, spheres have less $L_T$ than disks and matter with less L is able to fall into central star ($L_{sphere}=(2/5)MR^2W$, $L_{disk}=(1/2)MR^2W$)
        i. Planets are essentially chunks of material that had too much L to fall inward, too dense to radiate light



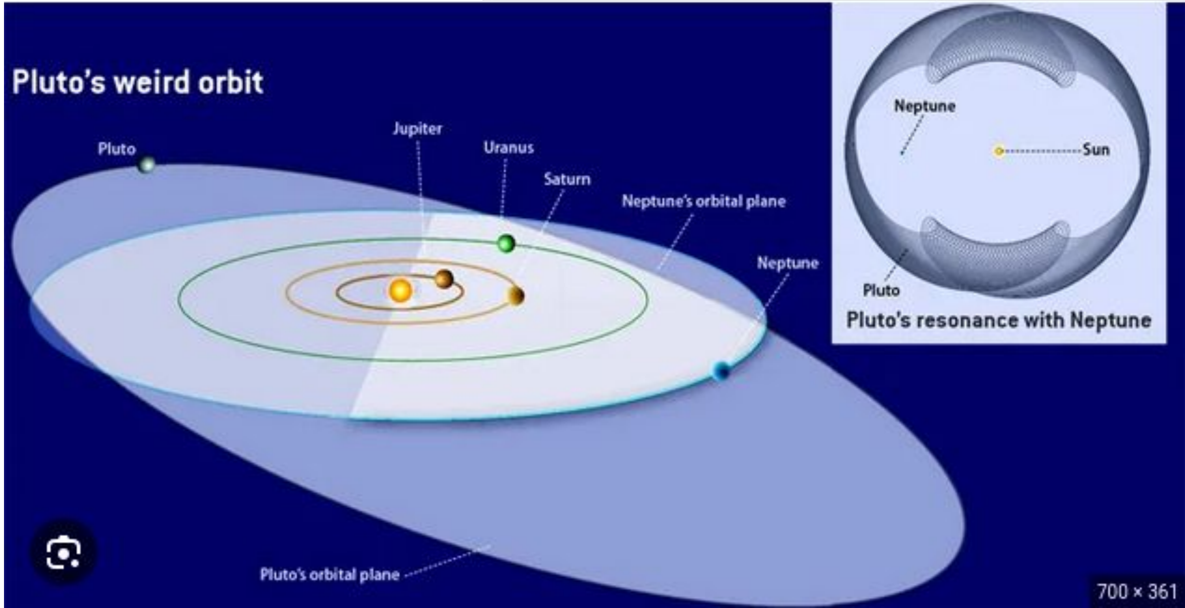
# Angular Momentum and Star Formation




- Until 1990s, was unclear if planets were common
  - Distant stars much larger, brighter than distant planets
  - Now think that all stars form with a planetary disk!
  - Planets are common the the universe thanks to angular momentum!

- Extremely massive planets (Jupiter, Saturn, Uranus, Neptune) can form their own accretion disk within the stars!
  - This is why Jupiter has 70+ moons; they are remnants of its accretion disk

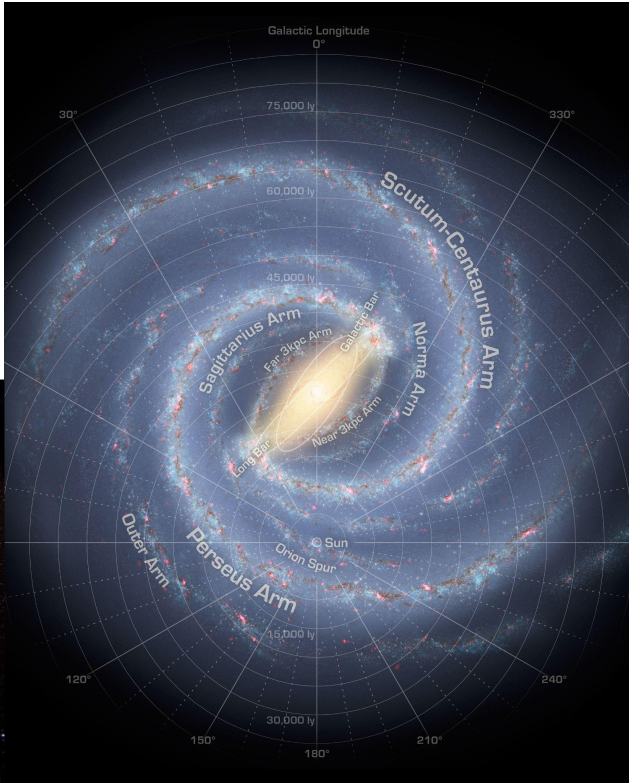
- Pluto's orbit is not in the same disk as other planets! From Kuiper Belt, thicker part of the disk
  - Thousands of similar off-disk objects near Pluto, many are much larger



# Galaxies

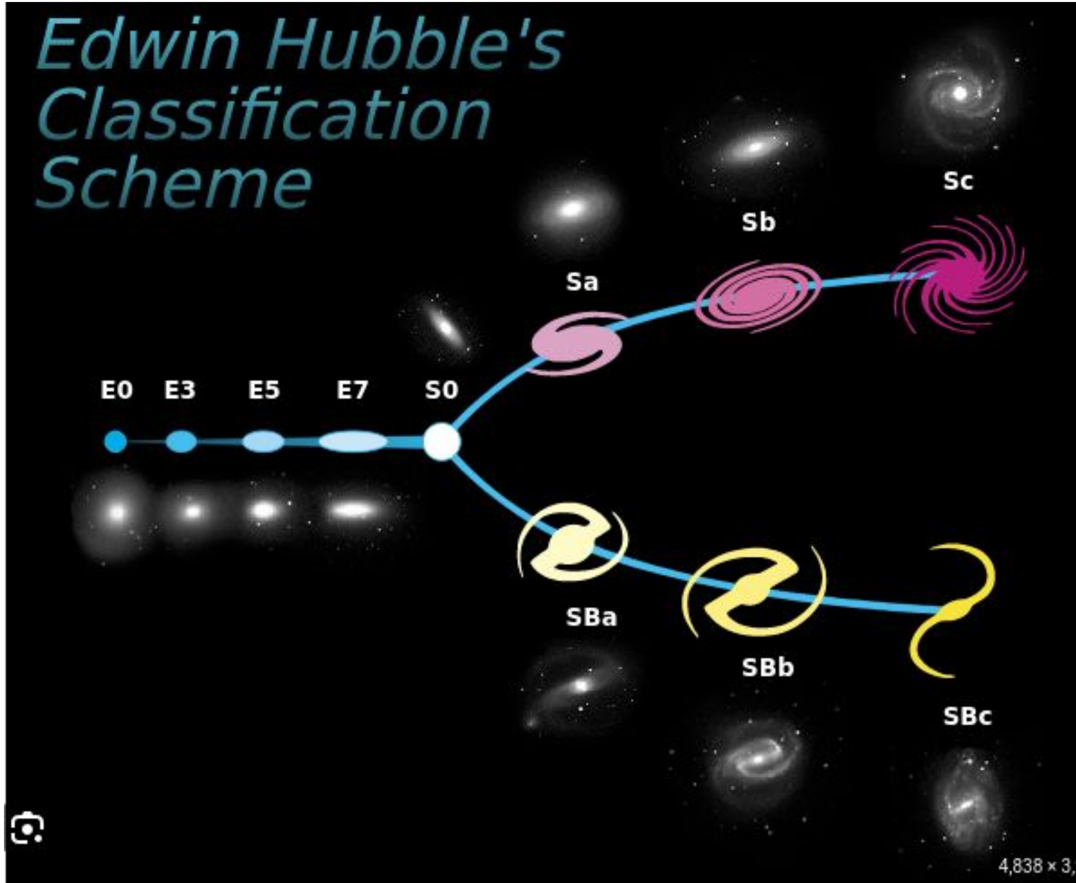
| Galaxy | Solar Systems/stars |
|---|---|
| Billions of stars, central black hole | Several planets orbit 1-2 stars |
| Most formed at roughly the same time | Galaxies are forming new stars today |
| After formation, gas/dust play important role | After formation, gas/dust is pushed away by star |
| Come in 2 varieties: spiral and elliptical | Come in 2 varieties: new and blue and old and red |

Our galaxy is the Milky Way!



# Galaxies

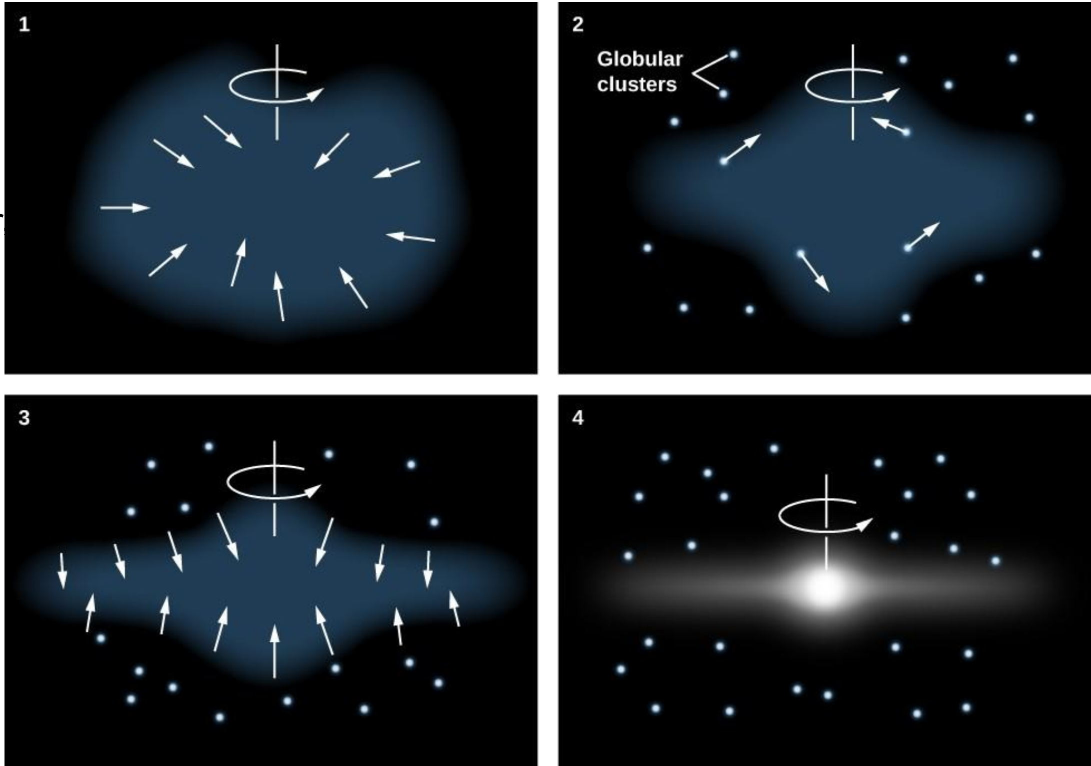
- In 1930s, Edwin Hubble discovered the Universe was much larger than the Milky Way
  - Observed stars in “nebulae” and calculated distance to them→ many stars much too far away to be within Milky Way
  - Later find that the more distant an object is, the faster it moves away from us
  - “Nebulae” must be entirely separate, “island universes”. Galaxies!
- Galaxies come in variety of different shapes. Why? Probably something to do with $L_T$!



# Angular Momentum and Galaxy Formation

Like many things in astronomy, galaxies begin as a cloud of gas and dust

- Just like with solar system formation, cloud collapses inward and forms a disk of more dense gas
- During collapse of whole cloud, smaller denser areas inside cloud fragment, collapse first
    - Form tightly bound globular clusters of older stars outside main body of galaxy.
    - The disk can continue to form stars for billions of years after collapse




One theory why we have two main galaxy shapes:

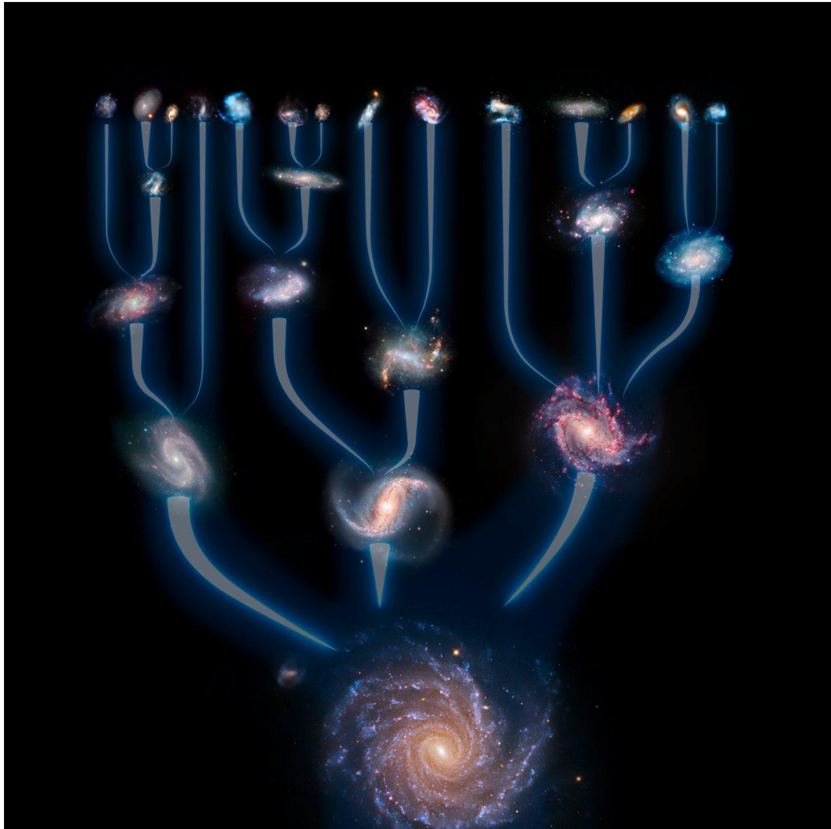
Spirals come from clouds that began with more angular momentum. This allows the disk to form before most gas/dust turned to stars

Ellipticals come from clouds that began with ~0 angular momentum. This allows for faster collapse inward and faster star formation

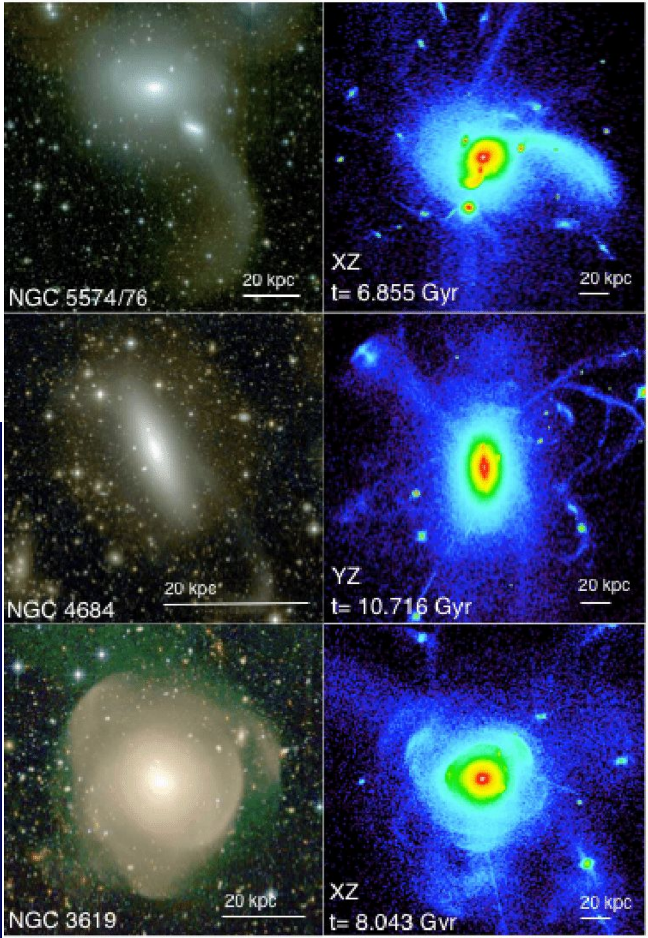
# Angular Momentum and Galaxy Formation

Like many things in astronomy, galaxies begin as a cloud of gas and dust

- Just like with solar system formation, cloud collapses inward and forms a disk of more dense gas. But unlike stars, galaxies MERGE!
- Mergers associated with disruption of galactic disk, enhancing star formation and black hole activity
- Mergers may also bring new gas and dust into elliptical galaxies



# Angular Momentum and Galaxy Formation

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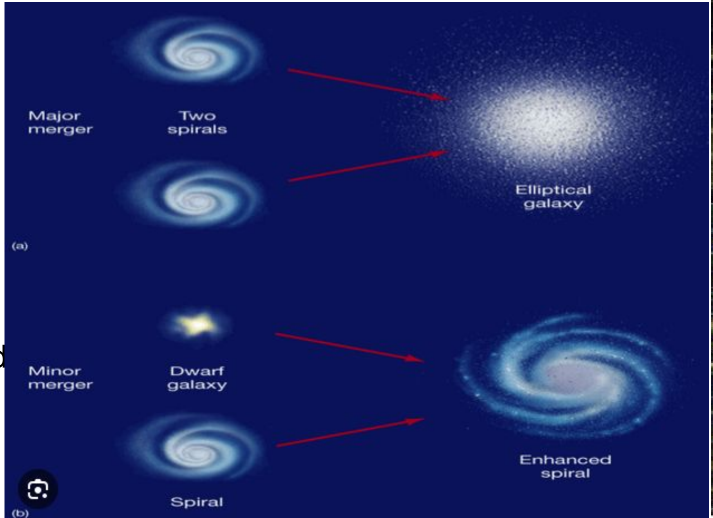
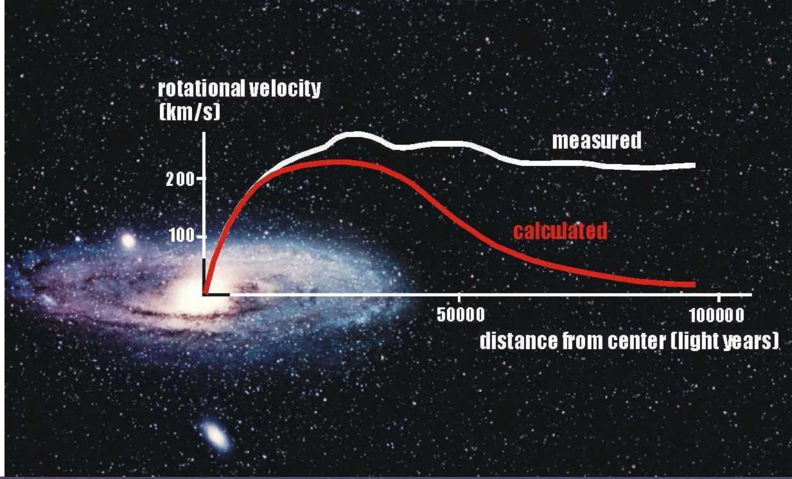
Another theory why we have two main galaxy shapes:

Major mergers disrupt the disk and destroy spiral structure

Minor mergers can impart new gas and enhanced angular momentum to elliptical galaxies

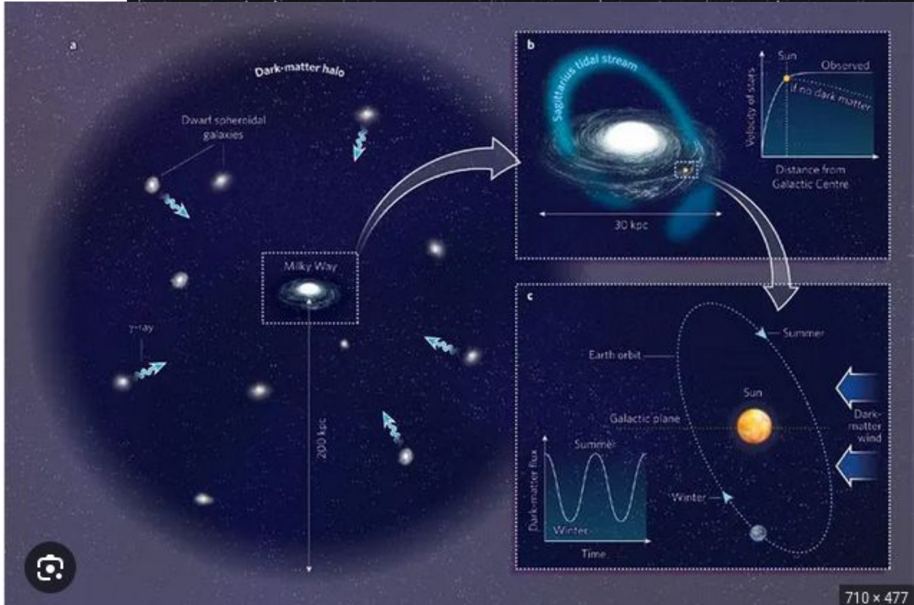


# Angular Momentum and DM Halos




Like many things in astronomy, galaxies begin as a cloud of gas and dust

- Just like with solar system formation, cloud collapses inward and forms a disk of more dense gas.
- This gas cloud sits inside an even larger cloud of dark matter
    - “Dark” matter cannot interact with light→
      No radiating away angular momentum!
      No collapse!
    - Detected through gravitational interactions
    - Simulations and observations show DM halo can have different shapes, and its own angular momentum




Another theory why we have two main galaxy shapes:

If the central galaxy experiences mergers that keep the regular matter and dark matter angular momentum aligned we tend to get spirals. If they become decoupled, tend to get ellipticals

# Conclusions

Angular momentum is an important quantity in both everyday life and in the cosmos

It is useful to study because it is a quantity experienced by humans and stars alike

Humans are responsive to shapes; visual classification of galaxies by an expert is still the most reliable method. Shape is determined by angular momentum

There are theoretical and observed connections between angular momentum and: star formation rate, mass, and age of galaxies