LQR Control of Fighter Aircraft A4D

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I. Introduction

Linear Quadratic Regulator (LQR) is nothing but a special case of Optimal control with system dynamics being described by a set of linear differential equations and the cost by a quadratic function. The LQR algorithm reduces the amount of work done by the control systems engineer to optimize the controller. However, the one still needs to specify the cost function parameters, and compare the results with the specified design goals.// For a continuous time system described by :

$$\dot{x} = Ax + Bu \tag{1}$$

A cost function J is defined for the system

$$J = \int_0^\infty (x^T Qx + u^T Ru)dt \tag{2}$$

Here, Q and R are the weight matrices and relate to the dependence of the state variable(x) or control variable (u) on the total cost. The overall goal is to minimize this cost function to get an optimal solution for the system and hence, more the respective weights, less is the expenditure of that variable.
The feedback control law that minimizes the value of the cost is :

$$u = -Kx \tag{3}$$

here, K is given by :

$$K = R^{-1}B^T S \tag{4}$$

and S is obtained from the solution of the following Riccati Equation :

$$A^T S + SA - SBR^{-1}B^T S + Q = 0 \tag{5}$$

Here, the similar control is implemented on the fighter aircraft A4D and the results for the same have been discussed. The general characteristics and properties of the fighter Aircraft A4D are given in Appendix.

II. Longitudinal Control

A. System

Longitudinal control has been implemented on the fighter Aircraft on Matlab and the response of pitch angle has been observed. For the longitudinal control the state space form is given as :

$$\dot{x} = Ax + Bu \tag{6}$$

$$\dot{y} = Cx + Du \tag{7}$$

Here,

$$\dot{x} = \begin{bmatrix} \Delta\dot{u} \\ \Delta\dot{w} \\ \Delta\dot{q} \\ \Delta\dot{\theta} \end{bmatrix} \qquad A = \begin{bmatrix} X_u & X_w & 0 & -g \\ Z_u & Z_w & u & 0 \\ \mathrm{M}_u + M_{\dot{w}} * Z_u & \mathrm{M}_w + M_{\dot{w}} * Z_w & \mathrm{M}_q + M_{\dot{w}} * u & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ 0 \\ M_\delta + M_{\dot{w}} * Z_\delta \\ 0 \end{bmatrix}$$

C and D matrix was chosen in such a manner so that the output was Pitch angle :

$$C = [\ 0\ 0\ 0\ 1\ ]\ and\ D = 0 \tag{8}$$

For the fighter Aircraft A4D the A and B matrices with values :

$$A = \begin{bmatrix} -0.0151 & -0.0050 & 0 & -9.8000 \\ -0.1413 & -0.8784 & 136.1160 & 0 \\ 0.0004 & -0.0695 & -1.4605 & 0 \\ 0 & 0 & 1.0000 & 0 \end{bmatrix} \qquad and \qquad B = \begin{bmatrix} 0 \\ 0 \\ -12.8485 \\ 0 \end{bmatrix}$$

The weights were chosen were more costly for any change in the pitch angle and less for changes in the controller. The following weight matrices were chosen for the purpose :

$$Q = 50 * C * C^T \qquad and \qquad R = 1$$

The feedback gain was obtained using lqr function of MATLAB as :

$$K = \begin{bmatrix} -0.0000 & 0.0051 & -0.8896 & -7.0706 \end{bmatrix}$$

K is obtained from solving the Riccati Equation and the numerical process is automatically computed by MATLAB.

B. Simulink Model & Response

The response for the change in pitch angle was noted w.r.t time in presence of the LQR controller. The model was implemented in Simulink as shown in Fig1. θ_{ref} was given a step input of 0.2 radians at t=1s and then

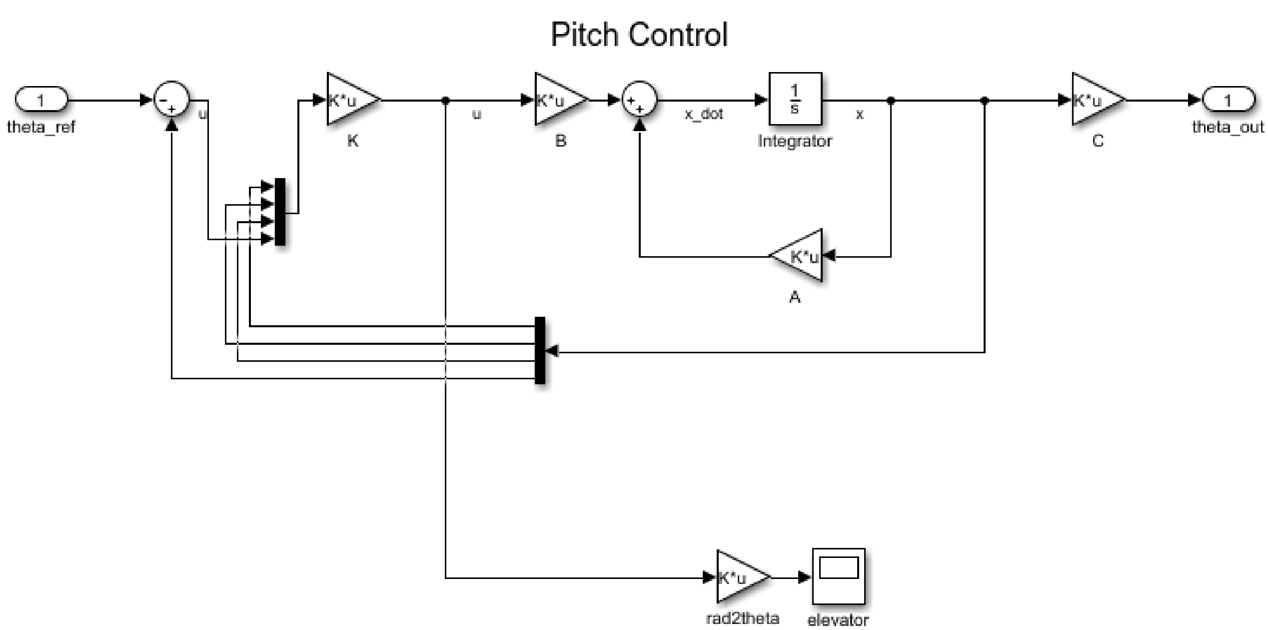

Fig1. Simulink Model of LQR Control of the Pitch of an Aircraft

the response of the pitch was analysed via the output variable θ_{out} as well as the changes in the controller were analysed.

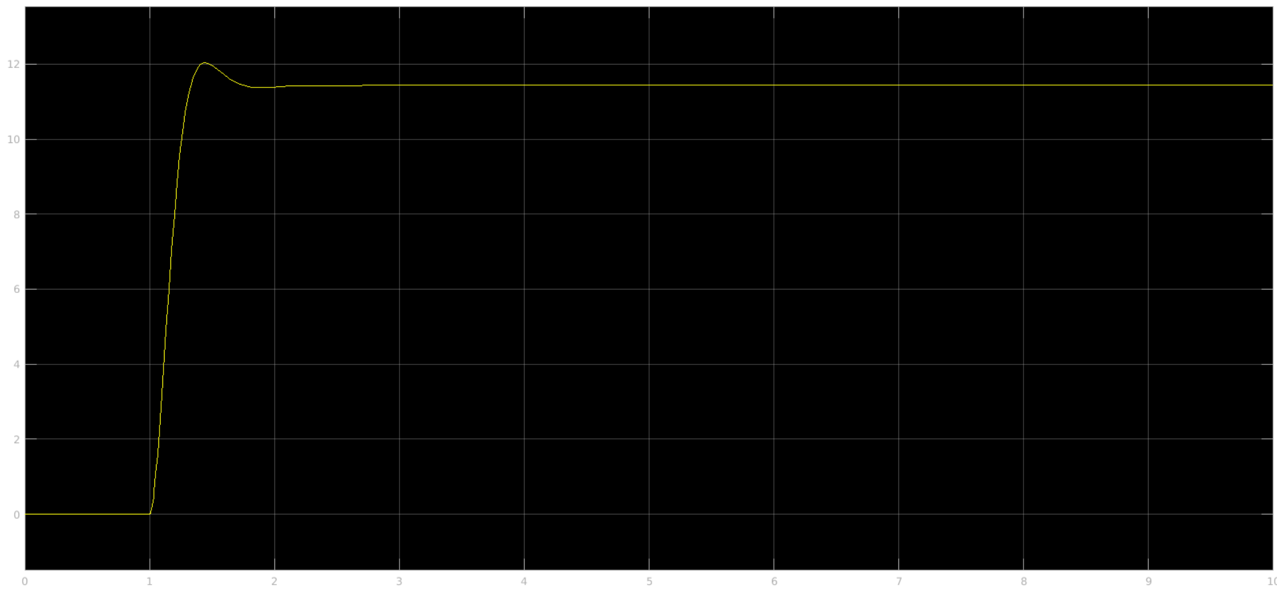

Fig. 1 Response of the Pitch Angle in presence of lqr Controller

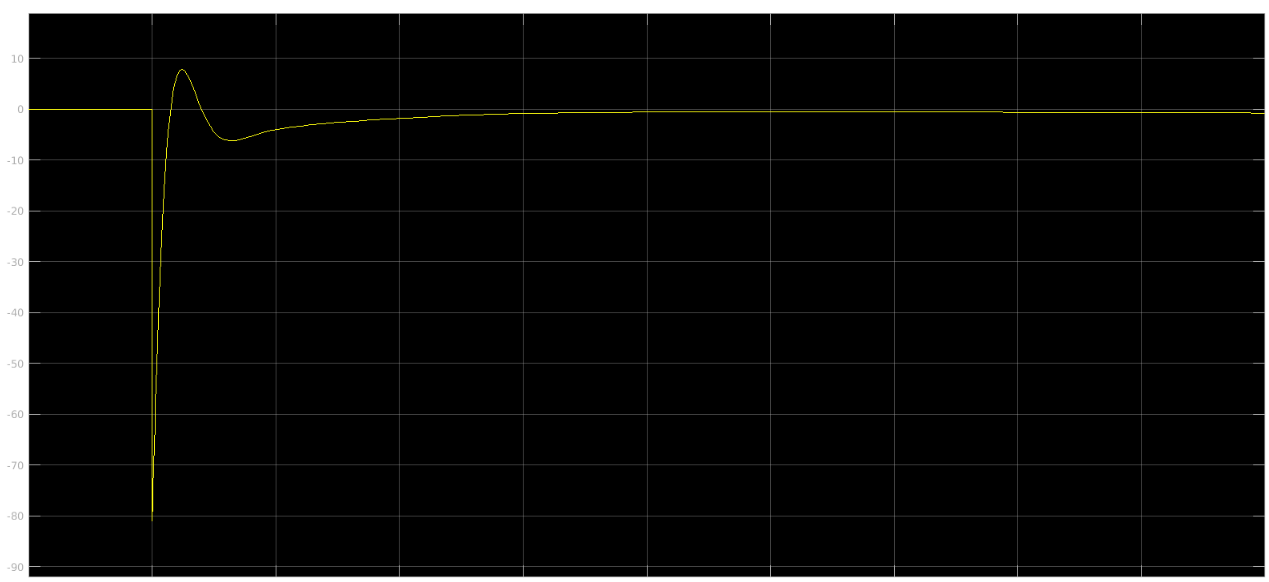

Fig. 2 Response of the Elevator in presence of lqr Controller

III. Lateral Control

A. System

Lateral control has been implemented on the fighter Aircraft on Matlab and the response of pitch angle has been observed. For the lateral control the state space form is given as :

$$\dot{x} = Ax + Bu \tag{9}$$

$$\dot{y} = Cx + Du \tag{10}$$

Here,

$$\dot{x} = \begin{bmatrix} \Delta\dot{\beta} \\ \Delta\dot{p} \\ \Delta\dot{r} \\ \Delta\dot{\phi} \end{bmatrix} \qquad A = \begin{bmatrix} \frac{Y_\beta}{u} & \frac{Y_p}{u} & -(1 - \frac{Y_r}{u}) & \frac{g*cos(0.0872)}{u} \\ L_{\beta 1} + \frac{I_{xz}}{I_x N_{\beta 1}} & L_{p1} + \frac{I_{xz}}{I_x N_{p1}} & L_{r1} + \frac{I_{xz}}{I_x N_{r1}} & 0 \\ N_{\beta 1} + \frac{I_{xz}}{I_z L_{\beta 1}} & N_{p1} + \frac{I_{xz}}{I_z L_{p1}} & N_{r1} + \frac{I_{xz}}{I_z L_{r1}} & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & Y_{\delta_r} \\ L_{\delta_{a1}} + \frac{I_{xz}}{I_x N_{\delta_{a1}}} & L_{\delta_{r1}} + \frac{I_{xz}}{I_x N_{\delta_{r1}}} \\ N_{\delta_{a1}} + \frac{I_{xz}}{I_z L_{\delta_{a1}}} & N_{\delta_{r1}} + \frac{I_{xz}}{I_z * L_{\delta_{r1}}} \\ 0 & 0 \end{bmatrix}$$

C and D matrix was chosen in such a manner so that the output was Pitch angle :

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad and \qquad D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

For the fighter Aircraft A4D the A and B matrices with values :

$$A = \begin{bmatrix} -0.2474 & 0 & -1.0000 & 0.0717 \\ -22.9697 & -1.6826 & 0.5441 & 0 \\ 13.4863 & -0.0356 & -2.2332 & 0 \\ 0 & 1.0000 & 0 & 0 \end{bmatrix} \qquad and \qquad B = \begin{bmatrix} 0 & 5.8405 \\ 17.4422 & -21.8526 \\ 4.2587 & 0.8843 \\ 0 & 0 \end{bmatrix}$$

The weights were chosen were more costly for any change in the side-slip angle and bank and less for changes in the controller. The following weight matrices were chosen for the purpose :

$$Q = 10 * C * C^T \qquad and \qquad R = 1$$

The feedback gain was obtained using lqr function of MATLAB as :

$$K = \begin{bmatrix} 0.5606 & 0.3743 & -0.0729 & 2.4916 \\ 2.5560 & -0.2752 & -0.0399 & -1.9553 \end{bmatrix}$$

K is obtained from solving the Riccati Equation and the numerical process is automatically computed by MATLAB.

B. Simulink Model & Response

The Simulink model for the lateral control of A4D aircraft was designed in such a way that the control input 'u' was decoupled and then step input of 1.0472 radians(or 60°) was given individually to the roll angle. Similarly the output was decoupled and the response of sidelslip angle and bank angle was observed with respect to the step change in roll angle.

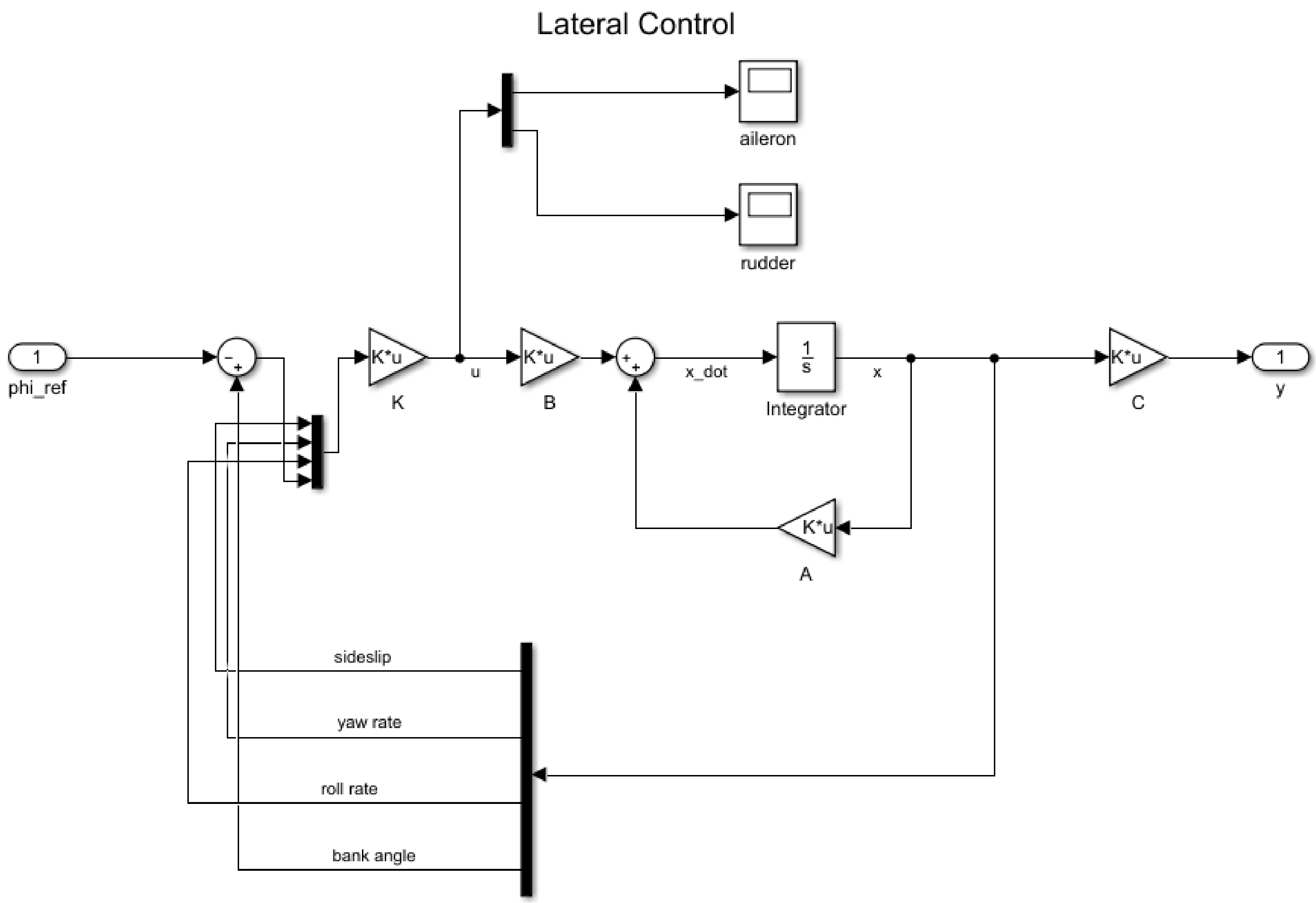

Fig. 3 Simulink Model of lateral control of A4d Aircraft with LQR controller

1. Step Response

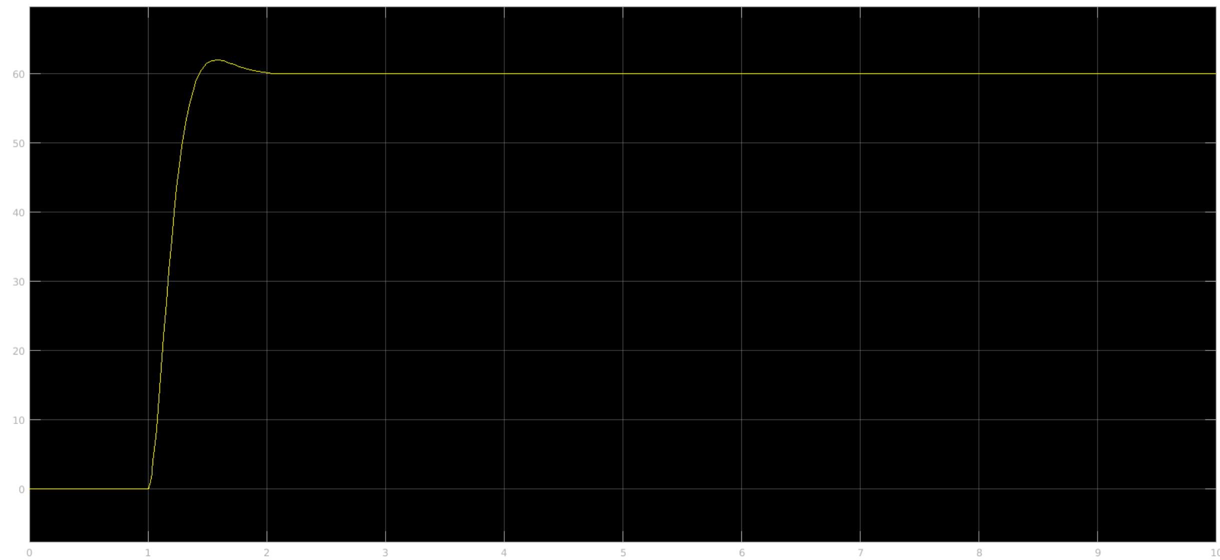

Fig. 4 Response of bank angle to the step input given to ϕ

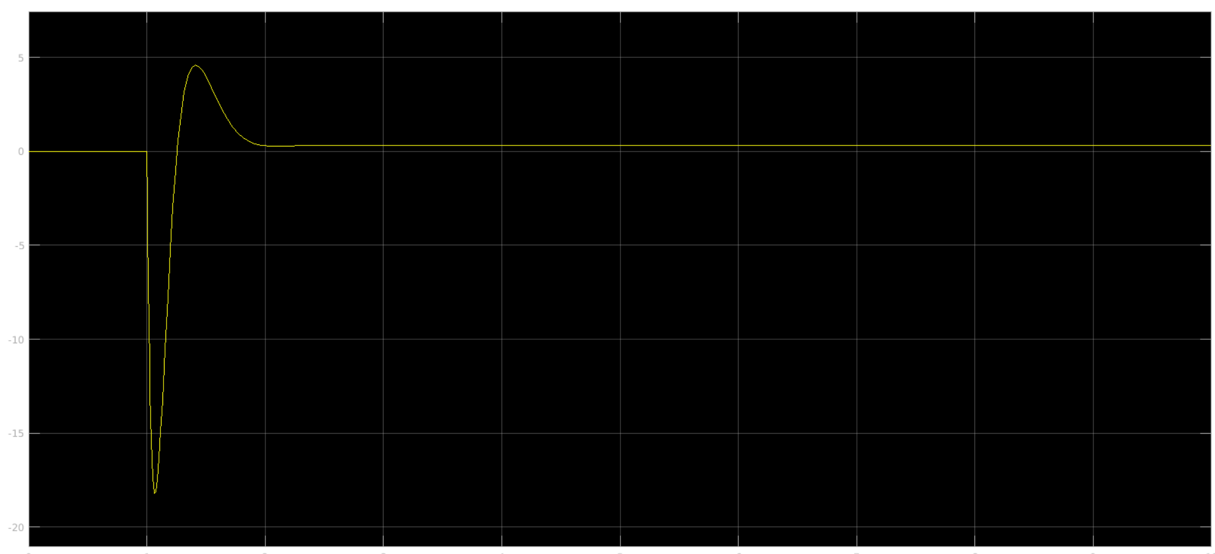

Fig. 5 Response of sideslip angle to the step input given to ϕ

Fig. 6 Change in aileron w.r.t the step input given to ϕ

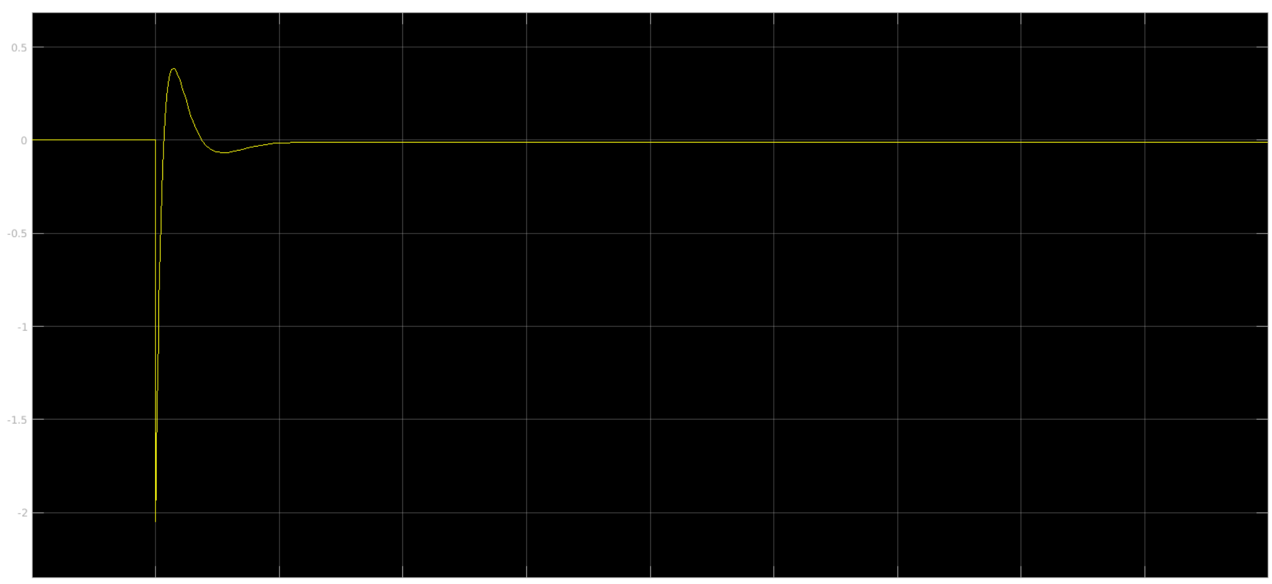

Fig. 7 Change in rudder w.r.t the step input given to ϕ

2. Impulse Response

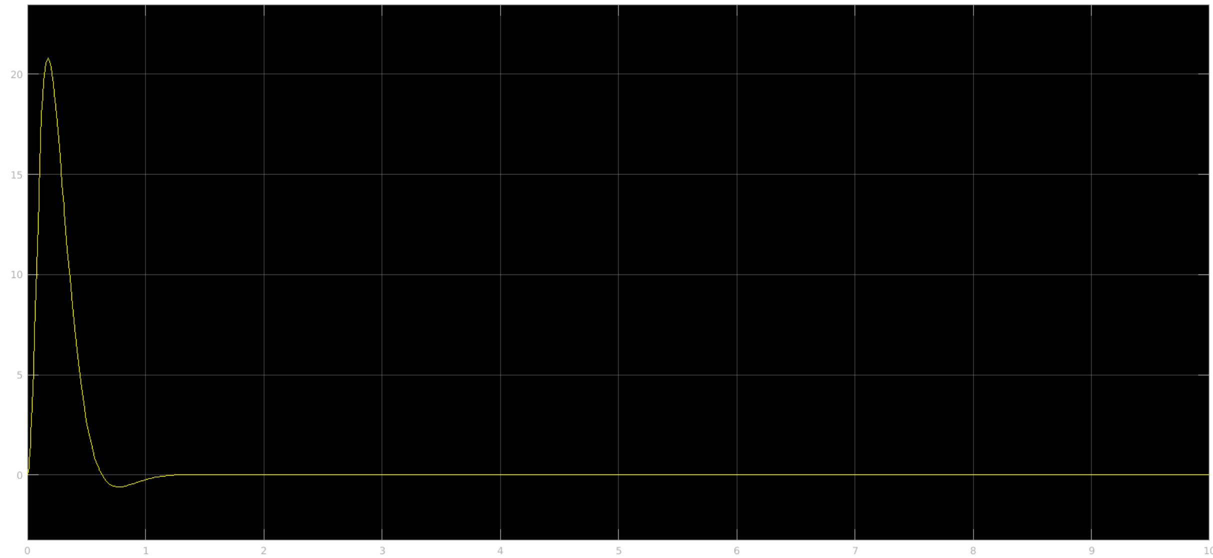

Fig. 8 Response of bank angle to the impulse input given to ϕ

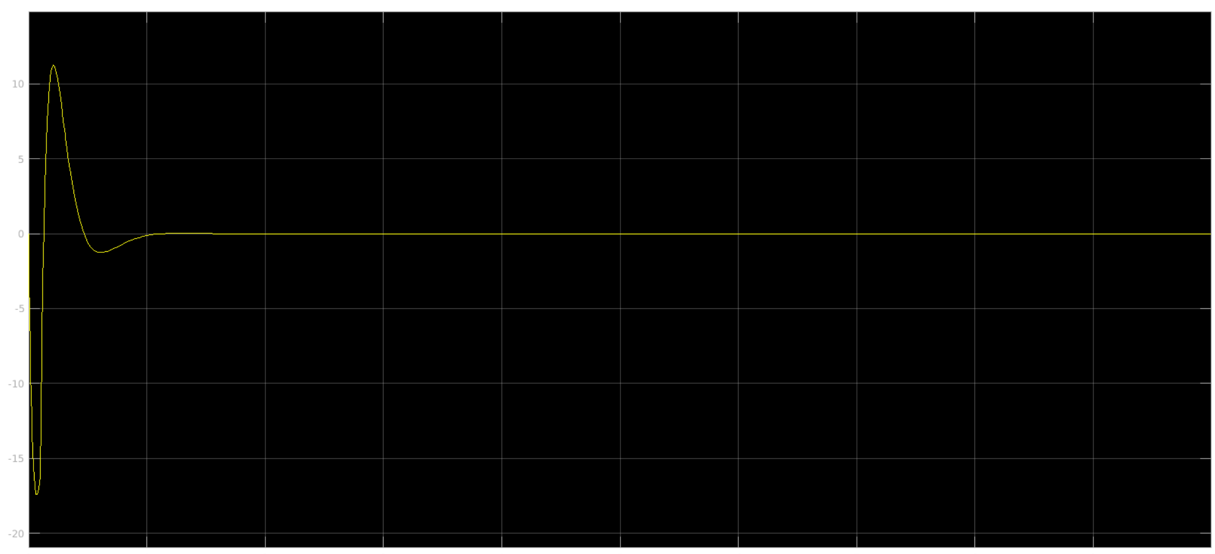

Fig. 9 Response of sideslip angle to the impulse input given to ϕ

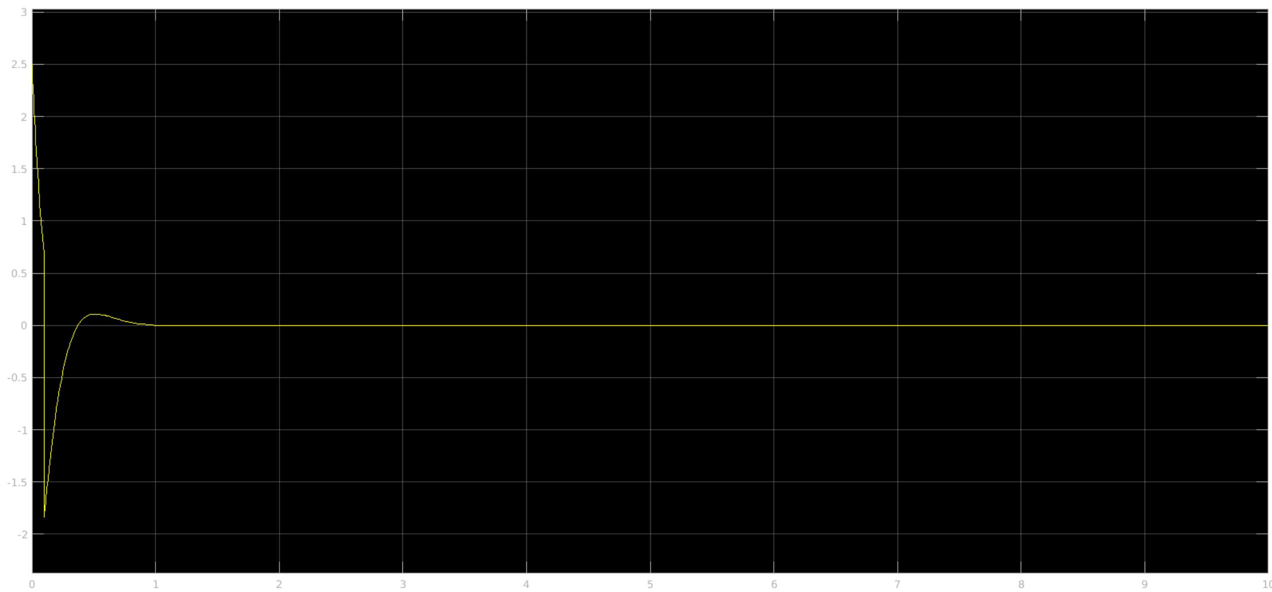

Fig. 10 Change in aileron w.r.t the impulse input given to ϕ

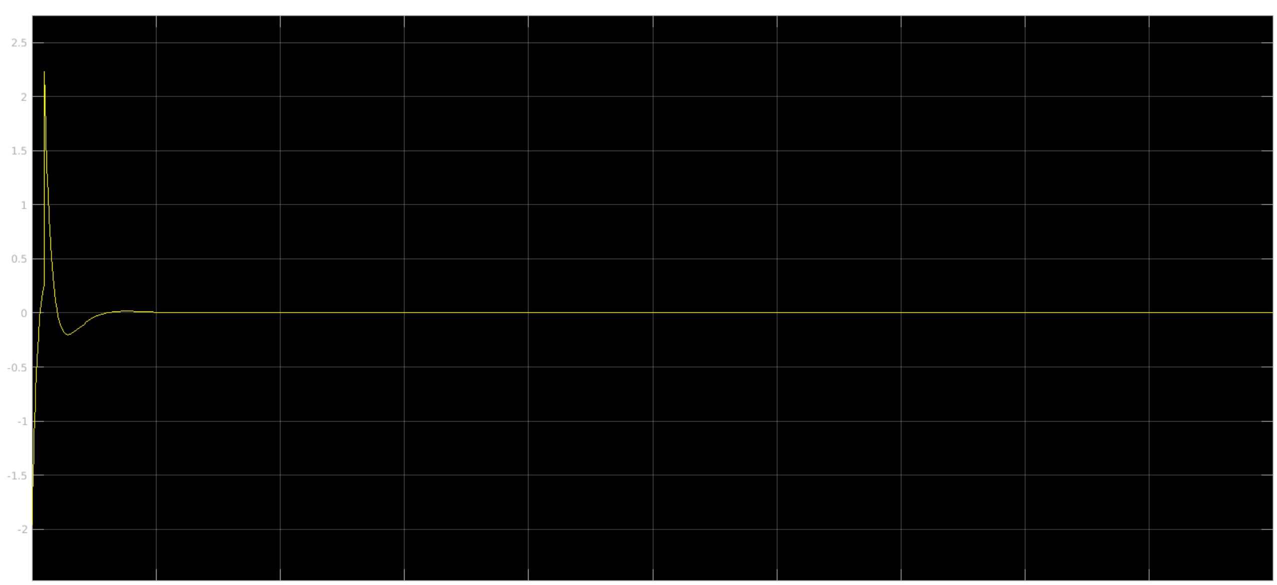

Fig. 11 Change in rudder w.r.t the impulse input given to ϕ

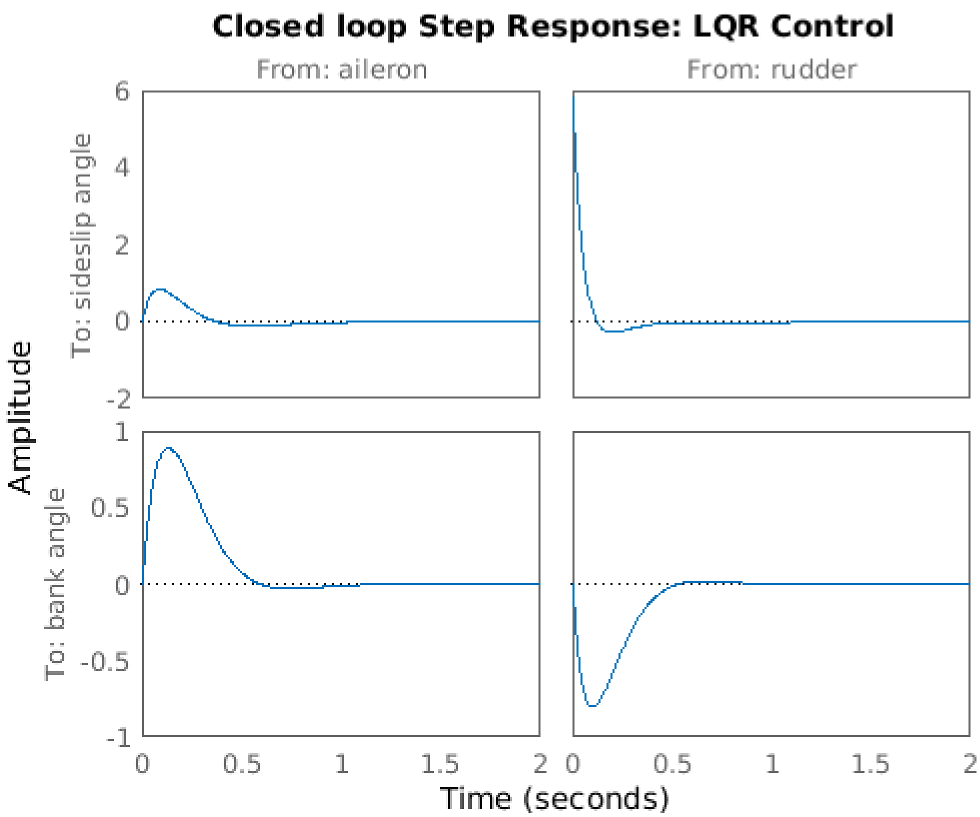

Fig. 12 Response obtained from MATLAB for impulse input

IV. Conclusion

The results for the pitch control of the aircraft are quite straightforward. The reference input is given as step input of 0.2 radians at t=1sec and the aircraft is made to pitch at that angle from trim condition. The Aircraft stabilizes at around t=2 sec. The response of the elevator with respect to the above input is also plotted.

For the lateral control of aircraft, the response is plotted for two different cases, viz,

1) Step input of 1.0472 radians (60°) at t=1sec
2) Impulse input with sample time of 0.1sec and 1 sample per frame

In case of step input, bank angle stabilizes at 60° in t=2 sec and the behaviour is completely normal. Response of the side-slip angle, aileron and rudder has been also attached.

For the impulse input, the responses are compared with the MATLAB plot and it's observed that while SIMULINK plots the responses for the impulse input from the roll angle, MATLAB individually applies the impulse to bank angle as well as sideslip angle and then plots the results from each individual controller input.