```
!pip install ipympl
import cv2
import numpy as np
import matplotlib.pyplot as plt
%matplotlib inline
%matplotlib ipympl
import os
from random import random
import time
import scipy
import scipy.sparse.linalg
import utils
from google.colab import output
output.enable_custom_widget_manager()
from google.colab import output
output.disable_custom_widget_manager()
```

→ Part 1 Toy Problem (20 pts)

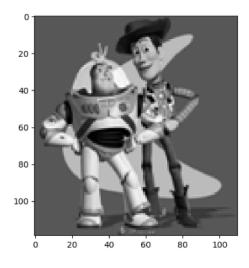
plt.imshow(toy_img, cmap="gray")

```
def toy_reconstruct(img):
   The implementation for gradient domain processing is not complicated, but it is easy to make a mistake, so let's start with a toy example
   1. minimize (v(x+1,y)-v(x,y) - (s(x+1,y)-s(x,y)))^2
   2. minimize (v(x,y+1)-v(x,y) - (s(x,y+1)-s(x,y)))^2
   Note that these could be solved while adding any constant value to v, so we will add one more objective:
   3. minimize (v(1,1)-s(1,1))^2
    :param toy_img: numpy.ndarray
   im_h, im_w = img.shape
                             #Shape: (119,110)
    im2var = np.arange(im_h * im_w).reshape(im_h, im_w)
    #For objective 1, there are (h-1)*w contraints
    #For objective 2, there are (w-1)*h constraints
    #For objective 3, there is only one constraint
    neq = (im_h - 1)*im_w + (im_w - 1)*im_h + 1 #the number of equations
   A = scipy.sparse.lil_matrix((neq, im_h*im_w), dtype='double') # init lil
    b = np.zeros((neq,1), dtype='double')
    e = 0
    #Objective 1, difference between each pixel and its bottom neighbor
    for x in range (im_w):
     for y in range(im_h - 1):
       A[e,im2var[y + 1][x]] = 1
        A[e,im2var[y][x]] = -1
        b[e] = img[y+1][x] - img[y][x]
        e += 1
    #Objective 2, difference between each pixel and its right neighbor
    for x in range (im_w - 1):
      for y in range(im_h):
        A[e,im2var[y][x+1]] = 1
        A[e,im2var[y][x]] = -1
        b[e] = img[y][x+1] - img[y][x]
    #Objective 3, same first pixel in the source and target image
   A[e,im2var[0][0]] = 1
   b[e] = img[0][0]
   v = scipy.sparse.linalg.lsqr(A.tocsr(),b, atol = 1e-7)[0]
   output_img = v.reshape(im_h,im_w)
    return output_img
toy_img = cv2.cvtColor(cv2.imread('toy_problem.png'), cv2.COLOR_BGR2GRAY).astype('double') / 255.0
```

```
plt.show()

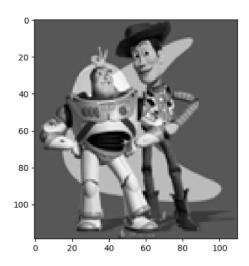
im_out = toy_reconstruct(toy_img)
plt.imshow(im_out, cmap="gray")
plt.show()
print("Max error is: ", np.sqrt(((im_out - toy_img)**2).max()))
```

Figure 1



x=90.9 y=2.4 [0.345]

Figure 1



x=90.9 y=2.4 [0.345]

Max error is: 7.284309720184678e-05

▼ Preparation

```
# %matplotlib ipympl
background_img = cv2.cvtColor(cv2.imread('mountain.jpg'), cv2.COLOR_BGR2RGB).astype('double') / 255.0
plt.figure()
plt.imshow(background_img)
plt.show()
object_img = cv2.cvtColor(cv2.imread('wolf.jpg'), cv2.COLOR_BGR2RGB).astype('double') / 255.0
plt.imshow(object_img)
plt.show()

use_interface = True  # set to true if you want to use the interface to choose points (might not work in Colab)
if not use_interface:
```

```
XS = (65, 359, 359, 65)
   ys = (24, 24, 457, 457)
   object_mask = utils.get_mask(ys, xs, object_img)
   bottom_center = (500, 2500) \# (x,y)
   object_img, object_mask = utils.crop_object_img(object_img, object_mask)
   bg_ul = utils.upper_left_background_rc(object_mask, bottom_center)
   plt.imshow(utils.get_combined_img(background_img, object_img, object_mask, bg_ul))
if use interface:
   import matplotlib.pyplot as plt
   %matplotlib ipympl
   mask_coords = utils.specify_mask(object_img)
   # Penguin: mask_coords = [[103,80,68,107,184,260,300,338,331,298,261,193,135],[180,228,307,385,437,434,379,322,251,185,104,34,94]]
    \texttt{\# Bear: mask\_coords = [[163,120,107,94,72,78,78,96,155,184,243,288,362,386,368,365,391,399,368,368,368,304], [85,197,311,444,550,644,787,886,986], [85,197,311,444,550,644,787,886,986], [85,197,311,444,550,644,787,886], [85,197,311,444,550,644,787,886], [85,197,311,444,550,644,787,886], [85,197,311,444,550,644,787,886], [85,197,311,444,550,644,787,886], [85,197,311,444,550,644,787,886], [85,197,311,444,550,644,787,886], [85,197,311,444,550,644,787,886], [85,197,311,444,550,644,787,886], [85,197,311,444,550,644,787,886], [85,197,311,444,550,644,787,886], [85,197,311,444,550,644,787,886], [85,197,311,444,550,644,787,886], [85,197,311,444,550,644,787,886], [85,197,311,444,550,644,787,886], [85,197,311,444,550,644,787,886], [85,197,311,444,550,644,787,886], [85,197,311,444,550,644,787,886], [85,197,311,444,550,644,787,886], [85,197,311,444,550,644,787], [85,197,311,444,550,644,787], [85,197,311,444,550,644,787], [85,197,311,444,550,644], [85,197,311,444,550,644], [85,197,311,444,550,644], [85,197,311,444], [85,197,311,444], [85,197,311,444], [85,197,311,444], [85,197,311,444], [85,197,311,444], [85,197,311,444], [85,197,311,444], [85,197,311,444], [85,197,311,444], [85,197,311,444], [85,197,311,444], [85,197,311,444], [85,197,311,444], [85,197,311,444], [85,197,311,444], [85,197,311,444], [85,197,311,444], [85,197,311,444], [85,197,311,444], [85,197,311,444], [85,197,311,444], [85,197,311,444], [85,197,311,444], [85,197,311,444], [85,197,311,444], [85,197,311,444], [85,197,311,444], [85,197,311,444], [85,197,311,444], [85,197,311,444], [85,197,311,444], [85,197,311,444], [85,197,311,444], [85,197,311,444], [85,197,311,444], [85,197,311,444], [85,197,311,444], [85,197,311,444], [85,197,311,444], [85,197,311,444], [85,197,311,444], [85,197,311,444], [85,197,311,444], [85,197,311,444], [85,197,311,444], [85,197,311,444], [85,197,311,444], [85,197,311,444], [85,197,311,444], [85,197,311,444], [85,197,311,444], [85,197,311,444], [85,197,311,444], [85,197,311], [85,197,311], [85,197,311], [85,197,311], [85,197,311], 
   mask\_coords = [[147,138,147,169,217,262,306,343,315,354,419,441,447,467,560,629,708,713,732,803,808,827,854,885,854,827,755,593,408,310,262]
   #Joe mask:
   \# mask_coords = [[52.1,56.6,78,108,120,138,164,179,192,210,245,228,202,198,224,246,259,244,223,199,210,226,224,210,192,166,140,126,124,122,
   #Owl mask:
   # mask_coords = [[1364,1467,1571,1638,1699,1796,1888,1991,2095,2156,2131,2064,2003,1851,1589,1437,1370,1285,1291],[1395,1547,1639,1706,1767
if use_interface:
   xs = mask_coords[0]
   ys = mask_coords[1]
   %matplotlib inline
   import matplotlib.pyplot as plt
   plt.figure()
   object_mask = utils.get_mask(ys, xs, object_img)
if use_interface:
   %matplotlib inline
    import matplotlib.pyplot as plt
   bottom_center = utils.specify_bottom_center(background_img)
   bottom_center = [3640,3220]
   %matplotlib inline
   import matplotlib.pyplot as plt
   object_img, object_mask = utils.crop_object_img(object_img, object_mask)
   bg_ul = utils.upper_left_background_rc(object_mask, bottom_center)
   plt.imshow(utils.get_combined_img(background_img, object_img, object_mask, bg_ul))
def poisson_blend(object_img, object_mask, bg_img, bg_ul):
       Returns a Poisson blended image with masked object_img over the bg_img at position specified by bg_ul.
       Can be implemented to operate on a single channel or multiple channels
       :param object_img: the image containing the foreground object
        :param object_mask: the mask of the foreground object in object_img
```

Part 2 Poisson Blending (50 pts)

```
:param background_img: the background image
:param bg_ul: position (row, col) in background image corresponding to (0,0) of object_img
#To prevent going out of bounds:
object_img = np.pad(object_img, pad_width=1, mode='constant',constant_values=0)
object_mask = np.pad(object_mask, pad_width=1, mode='constant',constant_values=0)
nnz = (object_mask>0).sum()
im2var = -np.ones(object_img.shape[0:2], dtype='int32')
im2var[object_mask>0] = np.arange(nnz) #Outside of object mask is -1's
                #The number of equations, each pixel has 4 constraint equations
neq = 4*(nnz)
A = scipy.sparse.lil_matrix((neq,nnz), dtype='double') # init lil
b = np.zeros((neq,1), dtype='double')
e = 0
                                            #equation counter
x_pixels,y_pixels = np.nonzero(object_mask) #indexes of each pixel in mask
```

```
for index in range(len(x\_pixels)): #loop through each pixel in the mask
      row = x_pixels[index]
      col = y_pixels[index]
      #Bottom Neighbor
      if (im2var[row + 1][col] == -1): #on the border
        A[e,im2var[row][col]] = 1
        b[e] = object\_img[row+1][col] - object\_img[row][col] + bg\_img[bg\_ul[0] + row + 1,bg\_ul[1] + col]
      else:
        A[e,im2var[row + 1][col]] = 1
        A[e,im2var[row][col]] = -1
        b[e] = object_img[row+1][col] - object_img[row][col]
      e += 1
      #Right Neighbor
      if (im2var[row][col + 1] == -1):
        A[e,im2var[row][col]] = 1
        b[e] = object\_img[row][col + 1] - object\_img[row][col] + bg\_img[bg\_ul[0] + row,bg\_ul[1] + col + 1]
      else:
        A[e,im2var[row][col+1]] = 1
        A[e,im2var[row][col]] = -1
        b[e] = object_img[row][col+1] - object_img[row][col]
      e += 1
      #Left Neighbor
      if (im2var[row][col - 1] == -1):
        A[e,im2var[row][col]] = 1
        b[e] = object_img[row][col - 1] - object_img[row][col] + bg_img[bg_ul[0] + row,bg_ul[1] + col - 1]
      else:
        A[e,im2var[row][col-1]] = 1
        A[e,im2var[row][col]] = -1
        b[e] = object_img[row][col-1] - object_img[row][col]
      e += 1
      #Upper Neighbor
      if (im2var[row - 1][col] == -1): #on the border
        A[e,im2var[row][col]] = 1
        b[e] = object\_img[row - 1][col] - object\_img[row][col] + bg\_img[bg\_ul[0] + row - 1,bg\_ul[1] + col]
      else:
        A[e,im2var[row - 1][col]] = 1
        A[e,im2var[row][col]] = -1
        b[e] = object_img[row - 1][col] - object_img[row][col]
      e += 1
    v = scipy.sparse.linalg.lsqr(A.tocsr(),b)[0]
    fusion = object_img * object_mask
    counter = 0
    for index in range(len(x_pixels)):
      col = y_pixels[index]
      row = x_pixels[index]
      fusion[row][col] = v[counter]
      counter += 1
    plt.figure()
    plt.imshow(fusion)
    for index in range(len(x_pixels)):
      col = y_pixels[index]
      row = x_pixels[index]
      bg\_img[bg\_ul[0] + row,bg\_ul[1] + col] = fusion[row][col]
    return bg_img
im_blend = np.zeros(background_img.shape)
im\_blendr = poisson\_blend(object\_img[:,:,0], \ object\_mask, \ background\_img[:,:,0].copy(), \ bg\_ul)
im\_blendg = poisson\_blend(object\_img[:,:,1], \ object\_mask, \ background\_img[:,:,1].copy(), \ bg\_ul)
im_blendb = poisson_blend(object_img[:,:,2], object_mask, background_img[:,:,2].copy(), bg_ul)
im_blend = cv2.merge((im_blendr, im_blendg, im_blendb))
plt.figure()
plt.imshow(im_blend)
```

```
def mixed_blend(object_img, object_mask, bg_img, bg_ul):
          Returns a mixed gradient blended image with masked object_img over the bg_img at position specified by bg_ul.
          Can be implemented to operate on a single channel or multiple channels
          :param object_img: the image containing the foreground object % \left( 1\right) =\left( 1\right) \left( 1\right
          :param object_mask: the mask of the foreground object in object_img
          :param background_img: the background image
          :param bg_ul: position (row, col) in background image corresponding to (0,0) of object_img
          #TO DO
          object_img = np.pad(object_img, pad_width=1, mode='constant',constant_values=0)
          object_mask = np.pad(object_mask, pad_width=1, mode='constant',constant_values=0)
          nnz = (object_mask>0).sum()
          im2var = -np.ones(object_img.shape[0:2], dtype='int32')
          im2var[object\_mask>0] = np.arange(nnz) \ \#Outside \ of \ object \ mask \ is \ -1's
          neq = 4*(nnz)
                                                       #The number of equations, each pixel has 4 constraint equations
          A = scipy.sparse.lil_matrix((neq,nnz), dtype='double') # init lil
          b = np.zeros((neq,1), dtype='double')
          e = 0
                                                                                                                               #equation counter
          x_pixels,y_pixels = np.nonzero(object_mask) #indexes of each pixel in mask
          for index in range(len(x_pixels)): #loop through each pixel in the mask
               row = x_pixels[index]
               col = y_pixels[index]
               #Bottom Neighbor
               s_gradient = object_img[row,col] - object_img[row+1,col]
                t_gradient = bg_img[row,col] - bg_img[row+1,col]
                gradient = max(abs(s_gradient),abs(t_gradient))
                if (im2var[row + 1][col] == -1): #on the border
                    A[e,im2var[row][col]] = 1
                    b[e] = gradient + bg_img[bg_ul[0] + row + 1,bg_ul[1] + col]
                else:
                    A[e,im2var[row + 1][col]] = 1
                    A[e,im2var[row][col]] = -1
                    b[e] = gradient
                e += 1
                #Right Neighbor
               s_gradient = object_img[row,col] - object_img[row,col + 1]
                t_gradient = bg_img[row,col] - bg_img[row,col + 1]
                gradient = max(abs(s_gradient),abs(t_gradient))
               if (im2var[row][col + 1] == -1):
                    A[e,im2var[row][col]] = 1
                    b[e] = gradient + bg_img[bg_ul[0] + row,bg_ul[1] + col + 1]
                     A[e,im2var[row][col+1]] = 1
                    A[e,im2var[row][col]] = -1
                    b[e] = gradient
                e += 1
                #Left Neighbor
                s_gradient = object_img[row,col] - object_img[row,col - 1]
               t_gradient = bg_img[row,col] - bg_img[row,col - 1]
               gradient = max(abs(s_gradient),abs(t_gradient))
                if (im2var[row][col - 1] == -1):
                    A[e,im2var[row][col]] = 1
                    b[e] = gradient + bg_img[bg_ul[0] + row,bg_ul[1] + col - 1]
                else:
                     A[e,im2var[row][col-1]] = 1
                    A[e,im2var[row][col]] = -1
                    b[e] = gradient
                e += 1
               #Upper Neighbor
                s_gradient = object_img[row,col] - object_img[row- 1,col]
                t_gradient = bg_img[row,col] - bg_img[row - 1,col]
               gradient = max(abs(s_gradient),abs(t_gradient))
                if (im2var[row - 1][col] == -1): #on the border
                    A[e,im2var[row][col]] = 1
                    b[e] = gradient + bg_img[bg_ul[0] + row - 1,bg_ul[1] + col]
                else:
                    A[e,im2var[row - 1][col]] = 1
                    A[e,im2var[row][col]] = -1
                    b[e] = gradient
                e += 1
```

```
v = scipy.sparse.linalg.lsqr(A.tocsr(),b)[0]
   fusion = object_img * object_mask
    counter = 0
   for index in range(len(x_pixels)):
     col = y_pixels[index]
     row = x_pixels[index]
     fusion[row][col] = v[counter]
     counter += 1
   plt.figure()
   plt.imshow(fusion)
   for index in range(len(x_pixels)):
     col = y_pixels[index]
     row = x_pixels[index]
     bg_img[bg_ul[0] + row,bg_ul[1] + col] = fusion[row][col]
   return bg_img
im_mix = np.zeros(background_img.shape)
im_mixr = mixed_blend(object_img[:,:,0], object_mask, background_img[:,:,0].copy(), bg_ul)
im_mixg = mixed_blend(object_img[:,:,1], object_mask, background_img[:,:,1].copy(), bg_ul)
\label{lem:mixb} im\_mixb = mixed\_blend(object\_img[:,:,2], object\_mask, background\_img[:,:,2].copy(), bg\_ul)
im_mix = cv2.merge((im_mixr, im_mixg, im_mixb))
plt.figure()
plt.imshow(im_mix)
```

- → Bells & Whistles (Extra Points)
- → Color2Gray (20 pts)

```
def color2gray(img):
    pass
```

→ Laplacian pyramid blending (20 pts)

```
def laplacian_blend(object_img, object_mask, bg_img, bg_ul):
    # feel free to change input parameters
    nass
```

▼ More gradient domain processing (up to 20 pts)