Project 2: Multisection Chebyshev Transformer

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I. Introduction

The design goal of this project was to create a multisection Chebyshev transformer that would match a log-periodic antenna to a $Z_0 = 50\Omega$ microstrip transmission line from $2GHz - 10GHz$ with a maximum passband ripple of $\Gamma_m = 0.02$. This was done in HFSS using perfect electrical conductors on a $0.79375mm$ thick FR-4 substrate with $\epsilon_r = 4.4$. The conductors had a trace height of $0.03mm$. The rest of this paper will discuss the theory of Chebyshev transformers as well as the processes and simulations used to design and test the transformer for this particular application.

II. Literature Review

Three papers were reviewed for this project. In [1], a traditional Chebyshev transformer was extended to a dual-band Chebyshev tranformer. Using second-order trigonometric expressions as the argument for a Chebyshev polynomial of the first kind, the authors of this paper were able to create a transformer that has an equal-ripple response in two different bands separated by a chunk of bandwidth for which matching is rejected.

As a contrast to the first paper, article [2] keeps the same Chebyshev response but instead focuses on size, discussing a method for synthesizing $\lambda/12$ and $\lambda/16$ stepped Chebyshev transformers. These stepped transformers are much shorter in length than a traditional $\lambda/4$ transformer and are ideal for integrated microwave electronics at lower frequencies.

Unrelated to transformers but showing another application for the Chebyshev polynomial, [3] outlines a method for synthesizing Chebyshev lowpass, highpass, and band-reject filters. These filters display equal ripple characteristics in the passbands and have sharper cutoffs than other types of filters.

III. Theory

A Chebyshev transformer is an impedance matching circuit used to match a purely real load over a large band of frequencies. Since the antenna used for this project exhibited a complex impedance, only the real part could be matched.

To design a Chebyshev transformer, two things must first be known. 1 - an expression for Γ_{Total} at the input of the multisection transmission line and 2 - an expression for the n_{th} order Chebyshev polynomial where the order is equal to the number of sections in the transformer. The two equations can then be related to each other in order to solve for the characteristic impedances and dimensions of each piece.

In order to calculate Γ_{Total}, a few assumptions have to be made. First, it is assumed that the overall reflection at the input of the line is the sum total of immediate reflections that occur at characteristic impedance discontinuities. In simpler language, all the reflections where the sections change are added together. To do this, reflections that bounce more than once are disregarded. Secondly and lastly, we assume that the Z_n of each section increases monotonically as we move from the generator to the load. Given these assumptions, Γ_{Total} for an N-section transformer to a purely real Z_L comes out to be

$$\begin{aligned}\Gamma_{Total} = 2e^{-jN\theta}\{\Gamma_0 cos[N\theta] + \Gamma_1 cos[(N-2)\theta] \\ + ... + \Gamma_n cos[(N-2n)\theta] + X\}\end{aligned} \tag{1}$$

where for even N use

$$X = \frac{1}{2}\Gamma_{\frac{N}{2}}$$

or for odd N use

$$X = \Gamma_{\frac{N-1}{2cos(\theta)}}$$

This gives the first critical equation necessary to building the transformer. The second equation is the Chebyshev polynomial of the first kind. A general expression for an n_{th} order Chebyshev polynomial is characterized by

$$C_n(x) = 2xC_{n-1}(x) - C_{n-2}(x) \tag{2}$$

with the first two polynomials being

$$C_1(x) = x \tag{3}$$

$$C_2(x) = 2x^2 - 1 \tag{4}$$

This polynomial has an "equal ripple" property from $-1 < x < 1$. Outside these boundaries, the function grows exponentially. An important point to note is that this property allows a designer to maximize bandwidth by either increasing the ripple or by adding more sections while keeping the ripple the same.

Now that we have a general expression for the polynomial, the lower and upper bands must be related to -1 and 1, respectively. To do this, let $\theta_u = \beta l = \frac{\pi}{2}\frac{f_u}{f_0}$ be mapped to 1 and $\theta_l = \beta l = \frac{\pi}{2}\frac{f_l}{f_0}$ to -1. This can be achieved if the argument of the Chebyshev polynomial is set to $x = cos(\theta)sec(\theta_l)$, so that $C_n(x)$ becomes

$$C_n(cos(\theta)sec(\theta_l)) \tag{5}$$

If this is scaled by the desired ripple, Γ_m, the final equation becomes

$$\Gamma_m e^{-jN\theta} C_n(cos(\theta)sec(\theta_l)) \tag{6}$$

enabling us to set (1) = (6), collect coefficients of cos, and solve for the Γ's of each section. For this project, the antenna presented a maximum real impedance of 146Ω to a minimum 79Ω in the passband. The average of these two, 112.5Ω, was chosen to be the effective Z_L in order to get an even match at all frequencies across the band. It was determined that seven sections, and therefore a seventh-order polynomial, were needed using this equation

$$N = \frac{cosh^{-1}[\frac{1}{2\Gamma_m} ln(\frac{Z_L}{Z_0})]}{cosh^{-1}(sec(\theta l))} \tag{7}$$

such that in this instance (1) now becomes

$$\begin{aligned} \Gamma_{Total} = 2e^{-j7\theta}\{&\Gamma_0 cos(7\theta) + \Gamma_1 cos(5\theta) \\ &+ \Gamma_2 cos(3\theta) + \Gamma_3 cos(\theta)\} \end{aligned} \tag{8}$$

and the seventh-order Chebyshev polynomial from (6) is

$$\begin{aligned} 0.02e^{-j7\theta}&[42cos(\theta)sec^3(\theta_l) - 7cos(\theta)sec(\theta_l) \\ &- 70cos(\theta)sec^5(\theta_l) + 35cos(\theta)sec^7(\theta_l) \\ &+ 14cos(3\theta)sec^3(\theta_l) - 35cos(3\theta)sec^5(\theta_l) \\ &+ 21cos(3\theta)sec^7(\theta_l) - 7cos(5\theta)sec^5(\theta_l) \\ &+ 7cos(5\theta)sec^7(5\theta_l) + cos(7\theta)sec^7(\theta_l)] \end{aligned} \tag{9}$$

Keeping in mind that $\Gamma_1 = \Gamma_L$, $\Gamma_2 = \Gamma_7$, and so on, the calculated reflection coefficients produced by setting these two equations equal lets the characteristic impedance of each section be solved with

$$Z_n = e^{2\Gamma + ln(Z_{n-1})} \tag{10}$$

With this information and the use of a microstrip calculator, the final dimensions of the design turn out to be

$Trace$	Γ_n	Z_n	L_n	W_n
T0	XX	50Ω	$6.85mm$	$1.518mm$
T1	0.0255	52.62Ω	$6.87mm$	$1.393mm$
T2	0.0419	57.22Ω	$6.91mm$	$1.205mm$
T3	0.0616	64.72Ω	$6.97mm$	$0.959mm$
T4	0.0737	75.00Ω	$7.04mm$	$0.710mm$
T5	0.0737	86.92Ω	$7.12mm$	$0.507mm$
T6	0.0616	98.31Ω	$7.18mm$	$0.369mm$
T7	0.0419	106.91Ω	$7.22mm$	$0.291mm$

The ideal $S11$ plot that these parameters produce shows the Chebyshev response with its equal ripple characteristic:

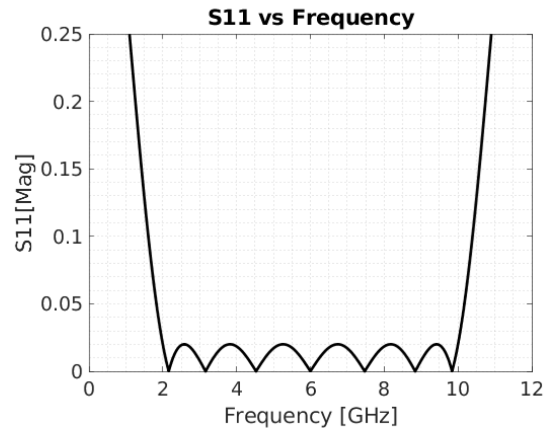

Fig. 1. Expected S11 vs Frequency

As we will see from simulations, the actual results deviate slightly but still conform to the general shape of the function.

IV. Simulation

Two separate designs were made to test this multisection transformer. The first one is displayed below. It contains a series of traces, whose geometries come from the table in section III, laid out on an FR-4 slab, and are then terminated in a lumped RLC port with the chosen impedance of 112.5Ω.

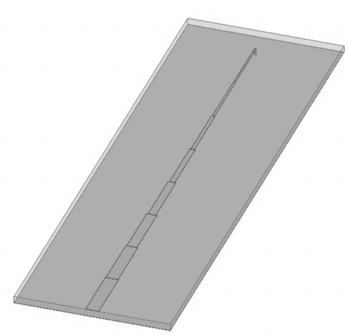

Fig. 2. Chebyshev Transformer - Lumped RLC Termination

The $S11$ results for this layout somewhat follow an ideal plot, with a bit of skew and uneveness of ripple, as well as the presence of five humps as opposed to six. Regardless, the reflections are at an acceptably low level:

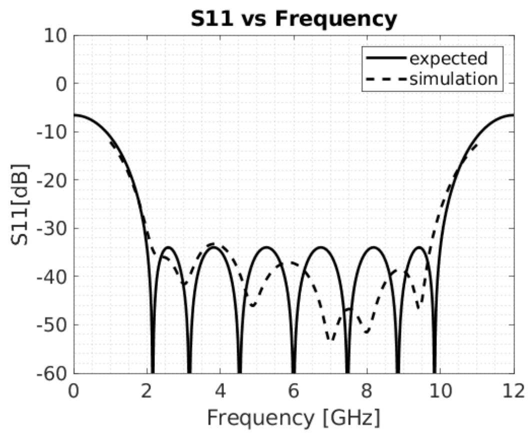

Fig. 3. HFSS S11 vs Frequency

Because the decibel plot above visually skews the response, a magnitude plot is included which shows a much more well-behaved ripple, with a peak ripple magnitude of 0.0217 at around $3.85GHz$ - just slightly above the desired 0.02 magnitude that the transformer was designed for.

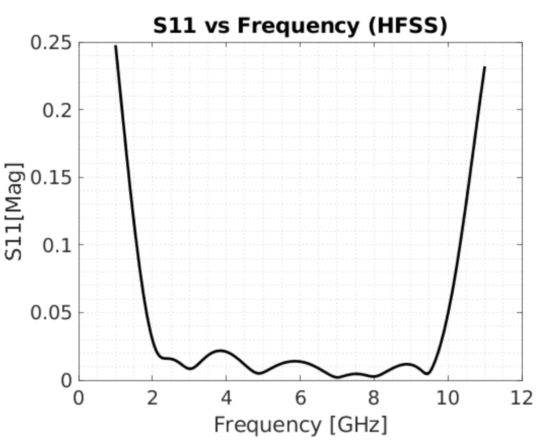

Fig. 4. HFSS S11 vs Frequency - Magnitude

In the following layout, the RLC lumped port was replaced by a wave port and a trace of $Z = 112.5\Omega$ with width $W = 0.249mm$ and length $L = 7.25mm$ to model the load:

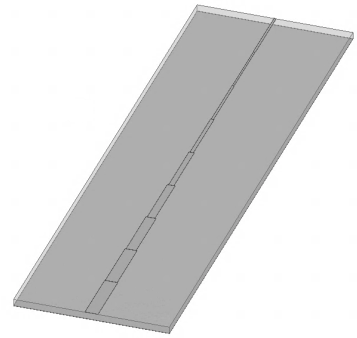

Fig. 5. Chebyshev Transformer - Trace and Wport Termination

The result of this design has an insertion loss of $-0.5dB$ at $2GHz$ and a maximum of $-2.14dB$ at $10GHz$, as well as an $S11$ below $-20dB$ throughout the passband:

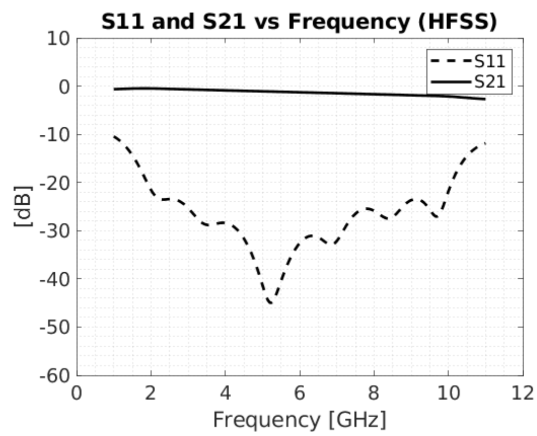

Fig. 6. HFSS S21 vs Frequency

V. Conclusion

For the most part, the design of this transformer met specifications. From $2GHz$ - $10GHz$, $S11$ stayed well below $-10dB$. Γ_m also stayed below 0.02 with the exception of 0.0217 at around $3.85GHz$. While not necessarily a detriment to the design, one peculiarity is that the number of ripples in the HFSS simulations came out to be five instead of six as the ideal plot in Matlab predicted. One reason for this could be that the extremely thin traces at the end of the transformer behaved like one trace instead of two.

References

[1] G. Castaldi, V. Fiumara, and I. M. Pinto, "A dual-band chebyshev impedance transformer," *Microwave and optical technology letters*, vol. 39, no. 2, pp. 141–145, 2003.

[2] P. Van Der Walt, "Short-step-stub chebyshev transformers impedance," *IEEE transactions on microwave theory and techniques*, vol. 34, no. 8, pp. 863–868, 1986.

[3] F. Xiao, "Direct synthesis techniques for general chebyshev filters: lowpass, highpass, and bandstop cases: Direct synthesis techniques for general chebyshev filters," *International journal of circuit theory and applications*, vol. 44, no. 3, pp. 584–601, 2016.

[4] D. M. Pozar, *Microwave engineering; 3rd ed.* Hoboken, NJ: Wiley, 2005. [Online]. Available: https://cds.cern.ch/record/882338