Project 3: Two-Section Wilkinson Power Divider

James A. Morar
Department of Electrical and Computer Engineering
University of North Carolina at Charlotte
Charlotte, N.C., USA

I. Introduction

The goal of this project was to create a two-section Wilkinson power divider for a 50Ω transmission line over a band of $3GHz - 10GHz$ with an absolute maximum passband ripple of $\Gamma_m = 0.3$. This was done in HFSS using perfect electrical conductors on a $0.79375mm$ thick FR-4 substrate with $\epsilon_r = 4.4$. The rest of this paper will discuss the theory of Wilkinson power dividers, (which will henceforth be abbreviated WPD), as well as the processes and simulations used to design and test the circuit for this particular application.

II. Literature Review

For this project, three papers related to the subject were reviewed. In [1], a $4:1$ unequal WPD was designed using double-sided parallel strip lines. This method of design is useful for creating high impedance lines which are usually difficult to construct with a single trace due to how thin they become. By using both sides of a substrate, higher impedance WPDs can be milled with ease.

For more complicated designs, [2] shows a dual-band $5:1$ WPD working at $0.915GHz$ and $2.45GHz$. In this design, both sides of the substrate are utilized, as well as an Archimedean spiral for the purpose of compactness.

An interesting and more modular approach to power dividers can also be demonstrated using varactors to tune the circuit power division ratios and the center frequency. [3] uses a four-port power divider with variable capacitors and inductors to vary the amount of power sent to the output ports over a considerable range of $-14.3dB$ to $13.3dB$. The center frequency can also be adjusted separately over $750MHz$ from the frequency the circuit was designed for, with a minimal phase change of 5 degrees.

III. Theory

A WPD is a circuit designed to split the input power of a signal in half and deliver each half to a separate port. The basic WPD circuit is a 3-port reciprocal network with an input port that bifurcates into two quarter-wavelength traces of characteristic impedance $\sqrt{2}Z_0$, terminated by a resistor of $2Z_0$ which bridges both traces at the end of the quarter-wavelength.

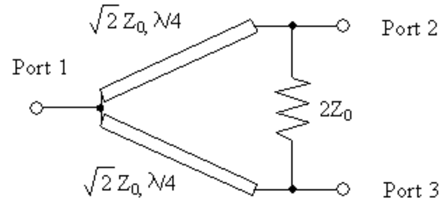

Fig. 1. Wilkinson Power Divider

Because the WPD is reciprocal, half of the power will be seen at at least one port. An input power at port 1 will result in half the power appearing at port 2 and the other half appearing at port 3. If the input port becomes port 2 or 3, then half the power will appear at port 1, and the other half will be dissipated as heat by the resistor.

A single-section WPD is narrowband, but the bandwidth can be extended by adding more sections. For a 2-section WPD design, the first step is to calculate the lower sideband, θ_l, using

$$sec(\theta_l) = cosh\left[\frac{1}{2}cosh^{-1}\left(\frac{1}{2\Gamma_m}ln\left(\frac{Z_l}{Z_0}\right)\right)\right] \tag{1}$$

where Γ_m is the desired passband ripple, and $Z_l = 2Z_0$. The next step is to equate the reflection coefficients of each section to the second-order Chebyshev polynomial, using the value of θ_l from (1):

$$\begin{aligned} &2e^{-j2\theta}[\Gamma_0 cos2\theta + \frac{1}{2}\Gamma_1] \\ &= \Gamma_m e^{-j2\theta}[2sec^2\theta_l cos^2\theta - 1] \end{aligned} \tag{2}$$

which then lets us compute the values of Z_l and Z_2 using:

$$ln(Z_1) = 2(\Gamma_0) + ln(Z_0) \tag{3}$$

and

$$ln(Z_2) = 2(\Gamma_1) + ln(Z_1) \tag{4}$$

Lastly, the two isolation resistors are calculated with

$$R2 = \frac{2Z_1Z_2}{\sqrt{(Z_1+Z_2)(Z_2 - Z_1cot^2\theta_3)}} \tag{5}$$

and

$$R1 = \frac{2Z_0R2(Z_1+Z_2)}{R2(Z_1+Z_2) - 2Z_0Z_2} \tag{6}$$

where θ_3 is the expected frequency in the passband where $\Gamma_m = 0$, and is calculated with

$$\theta_3 = cos^{-1}\left(\frac{1}{\sqrt{2}}cos\theta_l\right) \tag{7}$$

For this specific design, the final value chosen for Γ_m after many simulations was $\Gamma_m = 0.15$. The values that this produced are listed below and were close to meeting spec for this circuit.

$$Z_0 = 50.00\Omega, W_0 = 1.52mm$$
$$Z_1 = 59.00\Omega, W_1 = 1.14mm$$
$$Z_2 = 67.26\Omega, W_2 = 0.89mm$$
$$R1 = 195\Omega$$
$$R2 = 109.4\Omega$$

An ideal plot of $S11$ using these parameters is provided. It should follow a second-order Chebyshev response.

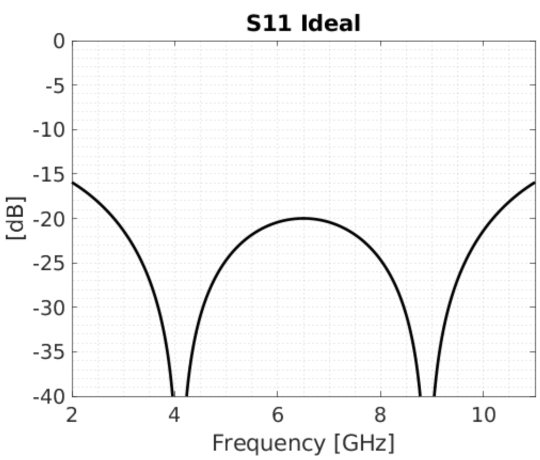

Fig. 2. Desired S11 Response

IV. Simulation

The design of the WPD was completed and simulated in HFSS. All traces used were perfect electrical conductors, and the isolation resistors were modeled as lumped RLC components. As stated in the introduction, the substrate was an FR-4 epoxy slab with $\epsilon_r = 4.4$. Ports $1, 2,$ and 3 are also terminated by waveports. A picture of the design with added measurements is included below. Port 1 is on the bottom, port 2 is on the top left, and port 3 is on the top right. Because the design is symmetrical about the Y-axis and to avoid clutter, only half of the dimensions are labeled.

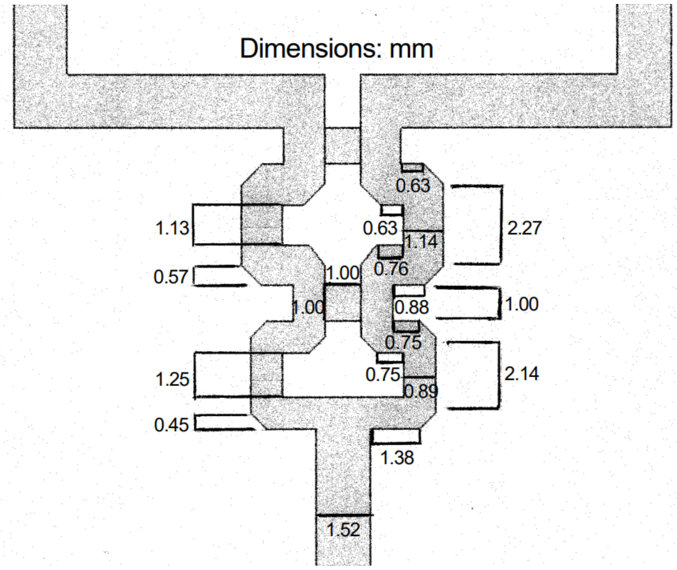

Fig. 3. Physical Circuit Layout

The equivalent schematic of this design is also provided:

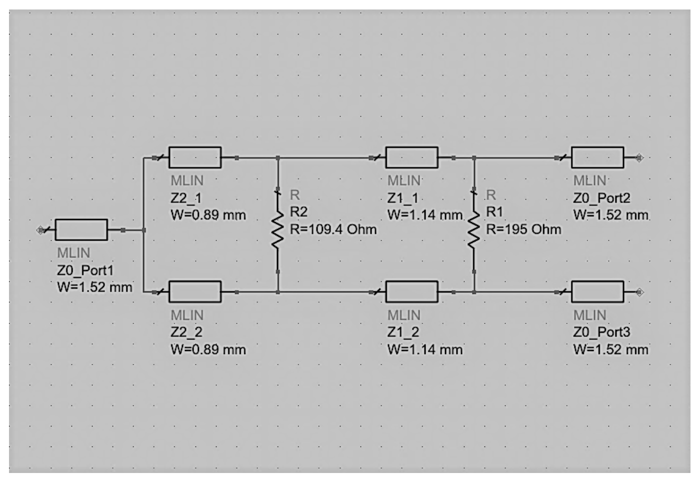

Fig. 4. Schematic Equivalent

The S-parameters resulting from simulations are grouped into three distinct plots:

$$S11, S21, S31$$
$$S12, S22, S32$$
$$S13, S23, S33$$

In the first plot, we can see that $S21$ and $S31$ overlap and lie very close to $-3dB$ in the passband, which is the ideal response for a perfect WPD. In this case, $S21$ and $S31$ vary from $-3.5dB$ to $-4dB$ in the lower sideband and reach $-5dB$ at $9GHz$. As for $S11$, it mimics the Chebyshev response, but reaches $-10dB$ too soon at about $8.5GHz$. An interesting point to note is that although the WPD was designed at a center frequency of $6.5GHz$, there is a frequency shift of at least $-0.5GHz$ and as much as $-1GHz$ for the response of all the S-parameters.

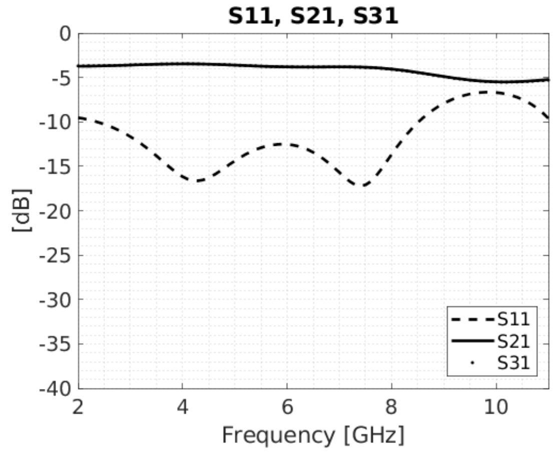

Fig. 5. Simulation S11, S21, and S31

In the next plot, $S12$ follows the exact same pattern as $S21$ and $S31$ in the last plot. The reflection at port two, $S22$, should be as low as possible and should imitate the response for $S11$ and $S33$. In this case, $S22$ reaches $-10dB$ early at $9GHz$ instead of the desired frequency of $10GHz$. For $S32$, the values are all beneath $-10dB$ across the band. Ideally, port 2 should be completely isolated from port 3 and vice-versa, meaning that the value of $S32$ should be as low as possible.

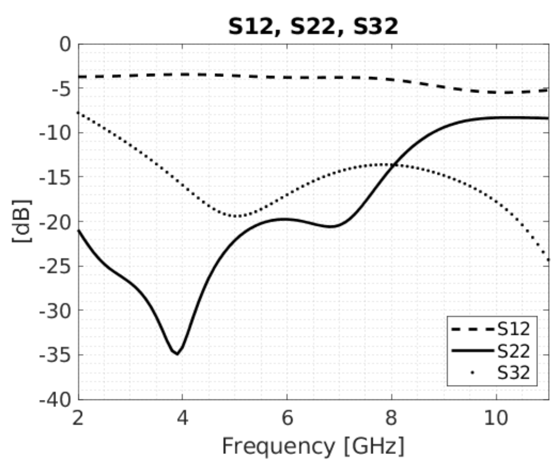

Fig. 6. Simulation S12, S22, and S32

The final plot of the response at port 3 is an identical copy of the previous response at port 2 due to the symmetry of the circuit.

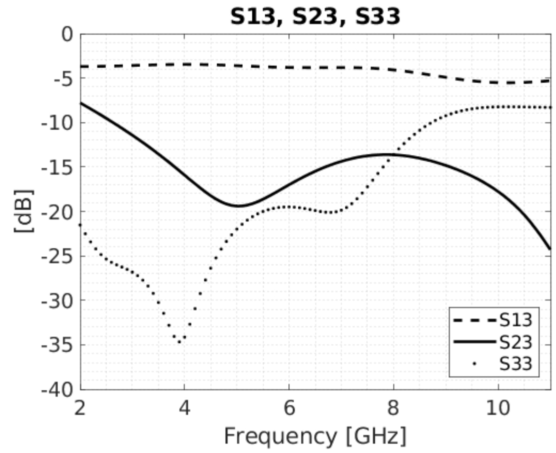

Fig. 7. Simulation S13, S23, and S33

V. Conclusion

The design of the WPD was close, but did not meet all specifications. There was a frequency shift of around $-1GHz$ for all the scattering parameters which made the lower cutoff $2GHz$ instead of $3GHz$. This caused $S11$, $S22$, and $S33$ to reach $-10dB$ at a lower frequency than desired. This shift also contributed to more loss at higher frequencies for $S12$, $S13$, $S31$, and $S21$. This could be due to the coupling of the electric fields between the traces or the lengths and widths of the traces. Some improvements that could be made would be in the adjustment of the widths and lengths of the traces, perhaps even using integer multiples of $\lambda/4$ to create more space between traces so that field lines don't interfere with the signal.

References

[1] W. Huang, W. Ruan, and F. Tan, "A miniaturized 4:1 unequal wilkinson power divider using artificial transmission lines and double-sided parallel-strip lines," *International journal of antennas and propagation*, vol. 2017, pp. 1–7, 2017.

[2] S.-Y. Yin, J.-L. Li, and S.-S. Gao, "Compact dual-band five-way wilkinson power divider," *Electronics letters*, vol. 53, no. 13, pp. 866–868, 2017.

[3] J. Kim, J.-R. Yang, and J. Oh, "Reconfigurable hybrid matrix-based power divider with variable power-dividing ratios and frequencies," *Electronics letters*, vol. 56, no. 17, pp. 889–891, 2020.

[4] D. M. Pozar, *Microwave engineering; 3rd ed.* Hoboken, NJ: Wiley, 2005. [Online]. Available: https://cds.cern.ch/record/882338