# Project 1: Quarter-Wave Transformer

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## I. Introduction

The purpose of this project is to design a quarter-wave transformer to match a transmission line of characteristic impedance $Z_0 = 60\Omega$ to a purely real load of impedance $Z_L = 40\Omega$ at a frequency of $3GHz$. This matching is simulated in HFSS using perfect electrical conductors on a $d = 1.59mm$ tall FR-4 dielectric with $\epsilon_r = 4.4$. The remainder of this paper will discuss the theory and applications of quarter-wave transformers as well as the equations and simulation results for a design with the values described above.

## II. Literature Review

For this project, three papers related to the subject were reviewed. In [1], a quarter-wave-like transformer was used to miniaturize a power divider. As opposed to a quarter-wave transformer, a quarter-wave-like transformer serves the same matching purpose as the former except with a shorter electrical length, enabling the shrinking of microwave circuits.

A more intricate use of quarter-wave transformers is in the design of ultra-wideband filters. [2] demonstrates that a $3.1GHz - 10.6GHz$ bandpass filter can be created using two shunt quarter-wave stubs and four series quarter-wave transformers with an insertion loss below $0.86dB$.

Deviating from traditional quarter-wave transformers, a third paper [3] cites the use of tapered lines as a way to achieve wider matching. These microstrip traces, which consist of a gradual taper and then an untaper back to their original shape, are easy to manufacture and provide an experimentally verified bandwidth of just over $2GHz$.

## III. Theory

In order to match a mismatched trace of characteristic impedance $Z_0$ to a real load, $Z_L$, a transmission line with characteristic impedance $Z_1$ of length $\lambda_0/4$ must be placed between the trace and the load such that the impedance looking into the point where $Z_0$ meets $Z_1$ is equal to the characteristic impedance of $Z_0$. In other words, at the design frequency $f_0$

$$Z_{in} = Z_1 \frac{Z_L + jZ_1 \tan(\beta l)}{Z_1 + jZ_L \tan(\beta l)} = Z_0 \tag{1}$$

Where the phase constant $\beta$ and length of the added line $l$ are given by

$$\beta = 2\pi/\lambda \tag{2}$$

$$l = \lambda_0/4 \tag{3}$$

And the effective permittivity and the wavelength on a microstrip line are given by

$$\epsilon_{eff} = \frac{\epsilon_r + 1}{2} + \frac{\epsilon_r - 1}{2} \frac{1}{\sqrt{1 + 12\frac{d}{w}}} \tag{4}$$

$$\lambda = \frac{1}{f\sqrt{\mu_0 \epsilon_0 \epsilon_{eff}}} \tag{5}$$

The guided wavelength, $\lambda_0$, stays fixed while $\lambda$ varies with frequency. At $f_0$, $\lambda_0 = \lambda$, which means $\tan(\beta l) = \tan(\pi/2) = \infty$. $Z_1$ can then be solved for with

$$Z_1 = \sqrt{Z_0 Z_L} \tag{6}$$

For our design parameters, $Z_1 = \sqrt{60\Omega 40\Omega} = 48.99\Omega$ and $\lambda_0 = 54.69mm$, which means that the trace will be $\lambda_0/4 = 13.67mm$ long. To calculate the trace width, the piecewise function below is used

$$\frac{w}{d} = \begin{cases} \frac{8e^A}{e^{2A} - 2} & \frac{w}{d} < 2 \\ \frac{2}{\pi}\{B - 1 - \ln(2B - 1) & \\ + \frac{\epsilon_r - 1}{2\epsilon_r}[\ln(B - 1) + 0.39 - \frac{0.61}{\epsilon_r}]\} & \frac{w}{d} > 2 \end{cases} \tag{7}$$

where

$$A = \frac{Z_0}{60}\sqrt{\frac{\epsilon_r + 1}{2}} + \frac{\epsilon_r - 1}{\epsilon_r + 1}\left(0.23 + \frac{0.11}{\epsilon_r}\right) \tag{8}$$

$$B = \frac{377\pi}{2Z_0\sqrt{\epsilon_r}} \tag{9}$$

so that the width can be found using

$$w = d\frac{w}{d} \tag{10}$$

For this design, the width comes out to be $w = 3.14mm$.

Using these parameters, the ideal $S11$ plot is shown in Matlab:

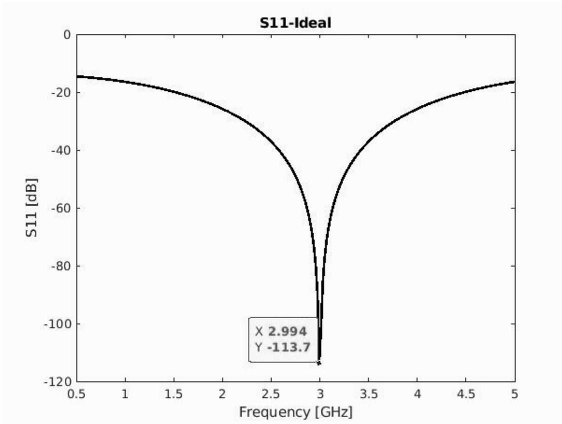


Fig. 1. Expected S11 vs Frequency

The HFSS simulations were expected to yield a plot that follows this general shape but with a much higher $S11$ and a slight offset from the design frequency of $3GHz$. (Note: in 1, in the ideal case at $3GHz$, $|\Gamma|$ becomes zero, which means that $S11$ is negative infinity - hence the next closest non-infinite point of $2.994GHz$).

## IV. SIMULATION

The design of the transformer was completed and simulated in HFSS. It consists of a $10mm$ x $2.21mm$ $60\Omega$ trace abutted to a $13.67mm$ x $3.14mm$ $49.99\Omega$ trace which is then terminated in a $40\Omega$ lumped port that is buried in the dielectric. The dielectric extends $5mm$ beyond the termination to contain the electric fields and is comprised of a $28.67mm$ x $40mm$ x $1.59mm$ FR-4 epoxy slab with $\epsilon_r = 4.4$. The final physical layout is pictured below:

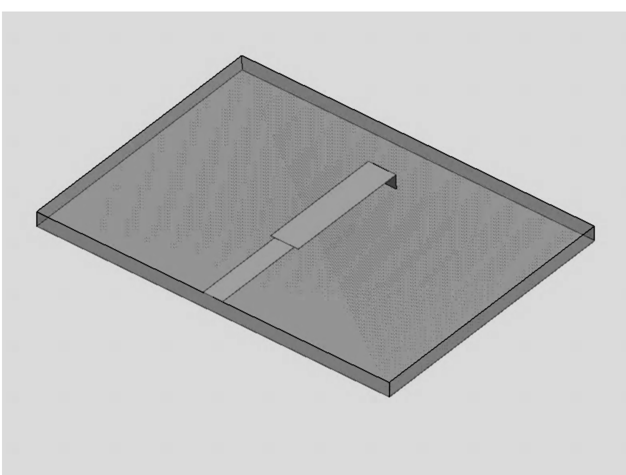

Fig. 2. The Quarter-Wave Transformer

The simulation which was carried out on 2 behaved as expected. The plot below - exported from HFSS and imported into Matlab - shows a depression slightly offset to the left of $3GHz$ with an $S11$ of $-34dB$ at the design frequency, which is around $80dB$ higher than the ideal Matlab simulation but still very practical and indicative of a good match:

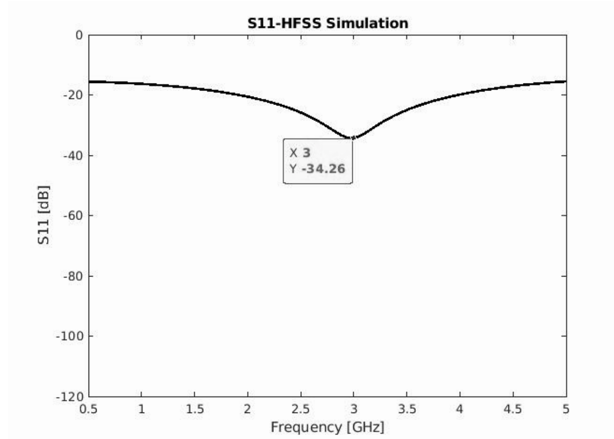


Fig. 3. HFSS S11 vs Frequency

## V. CONCLUSION

The design of this impedance transformer met specifications. It was demonstrated that a $60\Omega$ trace could be matched to a $40\Omega$ load with an $S11$ of $-34dB$ at the design frequency of $3GHz$. The absolute minima in the simulation occured a few megahertz below the target frequency, but that margin of error was expected due to minute imprefections introduced by the dielectric and the abrupt trace width change at the interface where the transmission line meets the transformer which produces a small reflection among other things. If desired, the transformer could have been trimmed a sliver shorter to push the minima right on top of the target frequency, but this went beyond the scope of the design and would amount to splitting hairs. While there are much better and more sophisticated ways of designing impedance transformers such as the tapered lines talked about in section II, the quarter-wave transformer is a necessary development in microwave design proficiency and a good place to start for those keen on developing their skills in distributed analysis.

## REFERENCES

[1] S. Korananan, P. Janpugdee, and D. Torrungrueng, "Miniaturization of power dividers using quarter-wave-like transformers (qwlts)," pp. 1–2, 2017.

[2] M. Uhm, K. Kim, and D. S. Filipovic, "Ultra-wideband bandpass filters using quarter-wave short-circuited shunt stubs and quarter-wave series transformers," *IEEE Microwave and Wireless Components Letters*, vol. 18, no. 10, pp. 668–670, 2008.

[3] T. Urakami and Y. Kusama, "A study on design of microstrip linear tapered line impedance transformer using fft," pp. 917–919, 2020.