Vertical algorithm

$$given:$$
$$\Delta X=1$$
$${}_{cp}e_1^1=f(0)=5$$
$${}_{cp}e_2^3=59$$
$${}_{cp}e_3^2=36$$
$${}_{cp}e_4^2={}_d te=18$$

$$solve\,for\,f(x):$$

$$d_{max}+1=te_{level}$$
$$te_{level}=4$$
$$d_{max}=3$$
$$[{}_{cp}e_1^1,{}_{cp}e_1^2,{}_{cp}e_1^3,{}_{cp}e_1^4]=[5,{}_{cp}e_1^2,{}_{cp}e_1^3,{}_{cp}e_1^4]$$
$$[{}_{cp}e_2^1,{}_{cp}e_2^2,{}_{cp}e_2^3]=[{}_{cp}e_2^1,{}_{cp}e_2^2,59]$$
$$[{}_{cp}e_3^1,{}_{cp}e_3^2]=[{}_{cp}e_3^1,36]$$
$$[{}_{cp}e_4^1]=[18]$$
$${}_{cp}e_i^L=\sum_{i=L-1}^{d_{max}}{}_i e_L^n$$
$${}_d te={}_d e_{d+1}^n=c_d\prod_{i=1}^{d}\Delta X(i)$$
$${}_{3cp}e_1^1=\sum_{i=0}^{3}{}_i e_1^1={}_0e_1^1+{}_1e_1^1+{}_2e_1^1+{}_3e_1^1=c_0+{}_1e_1^1+{}_2e_1^1+{}_3e_1^1$$
$${}_{3cp}e_2^3=\sum_{i=1}^{3}{}_i e_2^3={}_1e_2^3+{}_2e_2^3+{}_3e_2^3={}_1te+{}_2e_2^3+{}_3e_2^3$$
$${}_{3cp}e_3^2=\sum_{i=2}^{3}{}_i e_3^2={}_2e_3^2+{}_3e_3^2={}_2te+{}_3e_3^2$$
$${}_{3cp}e_4^1=\sum_{i=3}^{3}{}_i e_4^1={}_3e_4^1={}_3te$$

$${}_{3cp}e_4^1={}_3e_4^1={}_3te$$
$$18={}_3te$$
$$18=c_2\prod_{i=1}^{3}1(i)$$
$$18=c_3\,3!$$
$$\frac{18}{6}=c_3$$
$$3=c_3$$

$$f_3(x)=3\mathrm{x}^3$$
$$[f_3(0),f_3(1),f_3(2),f_3(3)]=[0,3,24,81]$$

$$[0,3,24,81]$$
$$[3,21,57]$$
$$[18,36]$$
$$[18]$$
$${}_3e_1^1=0$$
$${}_3e_2^3=57$$
$${}_3e_3^2=36$$
$${}_3e_4^1=18$$
$${}_{3cp}e_3^2={}_2te+{}_3e_3^2$$
$$36={}_2te+36$$
$$0=c_2$$

$${}_2e_1^1=0$$
$${}_2e_2^3=0$$
$${}_2e_3^2=0$$

$${}_{3cp}e_1^1={}_1te+{}_2e_2^3+{}_3e_2^3$$
$$59={}_1te+0+57$$
$$2=c_1\prod_{i=1}^{1}1(i)$$
$$2=c_1$$

$$f(x)=3\mathrm{x}^3+2\mathrm{x}+c_0$$
$$f(0)=0+0+c_0$$
$$5=c_0$$
$$f(x)=3\mathrm{x}^3+2\mathrm{x}+5$$