

Grad Project - José Ossorio

Part 1

Portfolio Optimization Using Quantum Computing

Introduction

Currently available quantum computers are small and not capable of solving many industry problems. Nevertheless, quantum computing (QC) is a promising technology that is expected to bring exponential speedups to certain areas of science and engineering. As a software engineer, I am interested in the application of quantum computers to current and future industry problems, and how to integrate this technology with classical computing in order to produce effective software applications. Despite the limited processing power of current quantum processors, there are still areas of industry that can benefit from QC right now.

In the past few years, QC applications have transitioned from being a hope for the future to a reality in some promising fields, such as quantum chemistry, simulations, and operations research. Among the applications of QC, portfolio optimization promises to deliver great value to the industry. A portfolio is a collection of assets that interact with each other and have potential returns. According to Orús et al. in their paper *Quantum computing for finance: Overview and prospects*, an optimal portfolio will pick the best subset of assets to maximize the return according to a desired risk. This also indicates that for a given return there is a portfolio that minimizes the risk.

In order to model this problem, we need to take into consideration each possible asset, as well as the interactions that occur among them. With the addition of assets to the set of possible options, the difficulty of finding an optimal solution quickly increases. As mentioned in chapter four of the book *Portfolio optimization: Applications in quantum computing*, "The optimization techniques traditionally used to solve this problem are so-called classical optimization techniques that rely on mathematically well-defined gradient based or descent/directional indications that must be well defined and constrained". Such heuristics provide a way to solve combinatorial optimization problems using classical computers, but may lead to suboptimal results and poor runtime.

The portfolio optimization problem can be modeled as a quadratic unconstrained binary optimization (QUBO) problem. It is quadratic because of the interaction between the variables of the model; it is unconstrained since there are no limitations in terms of the total cost of the portfolio (although one could formulate a different problem and include this). It is also binary, since we only care about each asset being present or not in the optimal solution. Recent advances in this area have shown that quantum computers are a good fit for such problems. According to Orús et al., demonstrated speedups in solving optimization problems using QC can be described as follows: "when the number of classical bits needed to specify the input data is increased, the number of operations needed to run the quantum algorithm increases slower than the best known classical alternative". Moreover, QC startup D-Wave provides powerful quantum computers designed specifically to solve optimization problems. According to their website, "to solve a problem on quantum samplers, you formulate the problem as an objective function, usually in Ising or QUBO format. Low energy states of the objective function represent good solutions to the problem." This means that if we can formulate our problem accordingly (using a QUBO or an Ising model formulation) we

can find optimal solutions using a quantum computer. The D-Wave computers make use of a QC paradigm called quantum annealing, but this is not the only algorithm used to solve optimization problems.

Hybrid quantum-classical approaches to solving optimization problems include the Variational Quantum Eigensolver (VQE) and the Quantum Approximate Optimization Algorithm (QAOA). These two algorithms are designed for universal quantum processors and are built using parametrized quantum circuits. These circuits have a set of variational parameters which need to be optimized to minimize the problem's cost function. The optimization process consists of multiple iterations of the algorithm. The circuit runs on quantum processors and according to the results after each run, classical techniques are used to optimize the parameters for the next iteration. Hybrid approaches such as this one have proven to be very effective at taking advantage of the limited capabilities of quantum computers and utilizing them to solve the part of the problem that benefits from quantum processing while solving other parts using classical techniques, such as gradient descent, that work well on classical hardware.

I find this application area to be very exciting, since it allows us to make use of current quantum computers to generate cost-effective solutions to hard problems. It is a great opportunity to show that quantum computers are useful in real world problems and that if we pair them together with classical computers, we can build efficient and effective software applications that could benefit society. In this project, I will document my findings after researching different approaches to solving the portfolio optimization problem. For this first part, I will briefly describe some of the different approaches I came across while doing a literature search.

Quantum annealing

In the review paper by Orús et al., it is stated that the most prominent approach to solving optimization problems involving quantum computers is using an adiabatic algorithm. In adiabatic QC, one must first map the optimization problem to a function known as a Hamiltonian. The Hamiltonian encodes the cost function to be minimized, which is equivalent to finding the ground state of the Hamiltonian function. The state of the energy function is slowly evolved (in the case of D-Wave, this is done by applying an external magnetic field to the superconducting qubits) until it reaches the ground state of our desired Hamiltonian. At this point, a measurement of the system has a high probability of returning the correct answer to our problem. This process is known as quantum annealing.

Reverse quantum annealing

In the paper titled *Reverse quantum annealing approach to portfolio optimization problems*, Venturelli et al. attempted a hybrid quantum-classical method that implements a reverse annealing protocol. In this study, the researchers used the D-Wave Quantum Annealer 2000Q to sample various portfolio optimization problems and compare the reverse annealing protocol to the traditional forward quantum annealing algorithm. They found that their approach is 100 times faster on average.

According to the authors of the same study, the theory of reverse annealing is still being investigated. For this particular experiment described in the paper, a classical computer will use a 'greedy search' algorithm to seed the quantum annealer with a possible (but not necessarily optimal) solution, i.e., a local minimum of the objective function. Then, the reverse annealing protocol starts, pausing at one point to then continue with forward annealing and measurement. It is highlighted in the study that "the quality of the initial state is likely to influence dramatically the reverse annealing process.

Using quantum walks to optimize portfolios

In a recent study by Slate et al., the researchers evaluated the performance of a newly developed Quantum Walk Optimization Algorithm (QWOA) and compared it to approaches that use the QAOA algorithm. Similar to other hybrid approaches, the problem is solved iteratively, adjusting the parameters each time according to a cost function through the use of classical algorithms. The authors found that using the QWOA the search space was reduced by a significant factor, allowing the algorithm to arrive at an optimal solution using fewer iterations.

A real-world application

Quantum computers have yet to deliver much of what the theory has promised us. Implementing a quantum computer capable of solving hard problems is no trivial task, but as it has been shown in the different studies mentioned in this document, obtaining useful results with current quantum technology is possible. In fact, the Spanish company Multiverse has been working with the bank BBVA to create optimized portfolios using quantum computers. The company devised an algorithmic approach using the D-Wave hybrid solver service to generate portfolios based on a set of constraints and real financial data. Ultimately, this shows that current quantum computers can already be used (together with classical computers) to achieve valuable results over classical approaches.

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