Otto HW1

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1 Question 2

1.1 Derivation

We want to prove that equation 1 holds:

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{2c}{-b \mp \sqrt{b^2 - 4ac}} \tag{1}$$

when $a \neq 0$ and $c \neq 0$.

Multiplying the left side of the equation by:

$$\frac{-b \mp \sqrt{b^2 - 4ac}}{-b \mp \sqrt{b^2 - 4ac}} \tag{2}$$

gives:

$$\frac{b^2 - (b^2 - 4ac)}{2a(-b - \sqrt{b^2 - 4ac})} \tag{3}$$

When taking the top of the plus/minus and minus/plus respectively. Simplifying the top gives:

$$\frac{4ac}{2a(-b - \sqrt{b^2 - 4ac})} \tag{4}$$

Which nicely simplifies into our desired equation:

$$\frac{2c}{-b - \sqrt{b^2 - 4ac}} \tag{5}$$

This is the right side of equation 1 when choosing the plus/minus appropriately.

1.2 Answer

When $-b + \sqrt{b^2 - 4ac}$ gets close to 0 the computer does not give a precise answer to what the roots are. Using either method of finding the roots causes one to be the exact answer and the other to be wrong. This is because the computer can't handle the necessary number of significant figures to get the exact answer. Combining both methods circumvents this problem by never asking the computer to deal with too many significant digits.