

$$\frac{d^2 u}{dx^2} - k^2 u(x) = f(x) \quad u(x=0) = u_0, \quad u(x=L) = 0 \quad f(x) = A$$

$$\frac{u_{j-1} - 2u_j + u_{j+1}}{h^2} - k^2 u_j = f_j$$

$$u_{j-1} - (2 + k^2 h^2) u_j + u_{j+1} = h^2 f_j$$

$$\begin{bmatrix} -(2 + k^2 h^2) & 1 & 0 \\ 1 & -(2 + k^2 h^2) & 1 \\ 0 & 1 & -(2 + k^2 h^2) \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} h^2 f_1 - u_0 \\ h^2 f_2 \\ h^2 f_3 \end{bmatrix}$$

$$j=1 \quad -(2 + k^2 h^2) u_1 + u_2 = h^2 f_1 - u_0$$

$$j=N \quad u_{N-1} - (2 + k^2 h^2) u_N = h^2 f_N - u_L^{\text{po}}$$

Neumann $\frac{du}{dx} \Big|_{x=0} = V, \quad u(x=L) = 0 \quad f(x) = A$

$$\frac{u_1 - u_0}{\Delta x} = V$$

$$u_1 - u_{-1} = 2hV$$

$$j=0 \quad u_{-1} - (2 + k^2 h^2) u_0 + u_1 = h^2 f_0$$

$$-(2 + k^2 h^2) u_0 + 2u_1 = h^2 f_0 - u_1 + 2hV + O(h^3)$$