Unit Gaussian: $X \sim N(\mu, \sigma^2)$ Gaussian Distribution, Unit Gaussian = N(0, 1) **Uniform Distr**: prob distr that has constant prob; X ~ uniform[a, b]; $f(x) = \frac{1}{h-a}$ [ex: toss a coin] **Joint Prob**: likelihood 2 events occurring together; p(x,y) = P(X=x,Y=y); independent: p(x,y) = P(X=x)*P(Y=y) Marg Prob: prob of event irrespective of the outcome of another var; $p(x) = \sum_{y \in Y} p(x, y)$; ex. p(X=head) = p(X=head, Y=head) + p(X=head, Y=head)tail) = .25 + .25 = .5 Bayes Rule: $p(z|x) = \frac{p(x|z)p(z)}{p(x)}$, JP: p(x,z) = p(x|z)p(z) = p(z|x)p(x); p(z): prior, p(x): marginal, p(z|x) posterior, p(x|z): conditional **Independence**: When random variables X & Z are independent; P(x,z) = P(x) * P(z), P(x|z) = P(x), P(z|x) = P(z) Chain in Prob: $P(x_n, x_{n-1}, ..., x_2, x_1) = P(x_n|x_{n-1}, ..., x_2, x_3)$..., x_2 , x_1)* $P(x_{n-1} | x_{n-2, ...}, x_2, x_1)$... $P(x_1)$; Ex. $P(x_2, x_1) = P(x_2 | x_1)P(x_1)$; $P(x_3, x_2, x_1) = P(x_3 | x_2, x_1)P(x_2 | x_1)P(x_1)$; When independent: $P(x_n, x_{n-1}, ..., x_2, x_1) = \prod_{i=1}^n P(x_i)$ **Exp Val**: the weighted average of possible values of a random variable; $E(X) = \sum X_i P(X_i)$; X = 10, 50, 100; P = .3, .5, .2; E(X) = 10*.3 + 50*.5 + 100*.2 = 48 **Point** Est = Mean Bias: exp - actual = E(X) - X Mean Squared Error (MSE) = $E(estimate - X)^2$ ED: $d(P, Q) = E(estimate - X)^2$ $\sqrt{(x_1-y_1)^2+(x_2-y_2)^2}$ ManD: right angle distance d(P, Q) = $|x_1-y_1|+|x_2-y_2|$ MinD: d(P, Q) = $(\sum_{i=1}^{n}|x_1-y_1|^r)^{\frac{1}{r}}$ where r=1: MD, r=2: ED, r=inf: max diff **HD**: compares two binary strings using xor, value is # of 1's 111xor100 = 2 JD: J(A, B) = $\frac{|A \cap B|}{|A \cup B|}$ CS: $\cos \theta = \frac{a \circ b}{||a|| * ||b||}, a \circ b = a_1 * b_1 + a_2 + a_3 * b_4 + a_4 * b_4 * b$ $b_2 ... etc$, $||a|| = \sqrt{a_1 + a_2 ... etc}$; Ex. A:I love data mining, B: like; I[10000], love[01000], like [00100], data [00010],mining[00001];A[11011],B[10111]; $\cos \theta = \frac{a \circ b}{||a|| * ||b||} = \frac{3}{4}$ Lim: ED: largest-scaled feature would dominate the others, HD: only binary measurement, MD: overestimate the distance, JD: does not work for ordered sequence, CS: does not work for semantical meaning App: ED: k-means clustering, GPS navigation, MD: A* algorithm, pac-man, HD: error detection/correction when data is transmitted over computer networks, CS: word similarity, document similarity, JD: similarity between binary vectors or sets, Z-score: outlier detection KL Div: measures the difference between two probability distributions; $D_{KL}\left(q(x) \mid |p(x)| = \int q(x) \, log \, \frac{q(x)}{p(x)}; \, D_{KL}(q \mid |p) = .5 \\ log \, \frac{.5}{.5} + .2 \\ log \, \frac{.2}{.2} + .2 \\ log \, \frac{.2}{.1} + .1 \\ log \, \frac{.1}{.2} \, \text{Entropy}: \, \text{measure a property}$ state of disorder in the universe; $H(X) = -\sum_{i=1}^k P(x_i)\log P(x_i)$; Monday: [.6, .2, .2] $H(X) = .6\log.6$ + .2log.2 + .2log.2 Rules of Exp Val: E(aX + b) = aE(X) + b, $E(X_1 + X_2) = E(X_1) + E(X_2)$ Variance: Exp of squared diff; $\sigma^2 = \text{var}(X) = \text{E}[(X-\mu)^2]$; $\sigma^2 = \text{var}(X) = \text{E}[(X-\mu)^2] = \text{E}[X^2] - (\text{E}[X])^2 \rightarrow (\text{each value} - \text{mean})^2$ **Normal**: makes on a similar scale; $X_{\text{new}} = \frac{X - X_{min}}{X_{max} - X_{min}}$ **Z-score**: $X_{new} = \frac{X - mean}{std} = \frac{X - \mu}{\sigma}$ **Co-var** for two variables X_1 and $X_2 = \text{E}[X_1X_2] - (\text{E}[X_1] * \text{E}[X_2])$; $\sigma_{12} > 0$: + cor, $\sigma_{12} < 0$: - cor **Pear Cor**: cor between two var is the covar divided by the σ of both: $\rho_{12} = \frac{\sigma_{12}}{\sigma_1 \sigma_2}$, if $\rho_{12} > 0$: + cor; =0: independent; <0: - cor; **K-Means** randomly initialize the number of clusters K; Calc the ED between i and the initial points; Assign point i to the nearest cluster with minimum distance; calc the mean value of each cluster as new central points; Recluster the points based on the new means; Repeat step 3 until there are no cluster changes Lim: sensitive to K initial points, computation is very expensive, if there are outliers it does not work well App to HC: phylogenetic tree of anim evo Lim to MIN: sensitive to noise and outliers, lim to HC: once decision combine two clusters cannot be undone Eps: max radius of the hood Adv to dbscan: separate clusters of high density vs clusters of low density, handles outliers within the dataset **Super learning**: the training data is manually labelled indicating the class of the obser, new data is classified based on the training set **Unsuper learning**: the class labels of training data is unknown, given a set of measur, obser, etc. with the aim of establishing the existence of classes or clusters in the data KNN Steps compute distance between testing point i & labeled point; choose "K" nearest neighboring points, such as 9; use majority voting to get the label, e.g. Blue Color Lim: if K is too small, sensitive to noise points; if K is too large, neighborhood may include points from other classes Adv: local classifiers, they can produce decision boundaries of arbitrary shapes