Math 050 Pre Calc Notes

1.1 *Quadratic equation*

$$\frac{x = -b \pm \sqrt{b^2 - 4ac}}{2a}$$

1.2 *Equation of a Circle*

$$r^2 = (x - h)^2 + (y - k)^2$$

1.3 *Equation of a parabola*

$$y = a(x^2 - h) + k$$

1.3 *Factoring by grouping AC mehod*

$$ax^2 + bx + c$$

Find p and q where $p * q = a * c$ *and* $p + q = b$

$$(ax^2 + pb) + (qb + c)$$

1.4 *Distance Formula*

$$Distance = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

1.5 *Law of Cosines Suppose a Triangle has angles A, B, and C with opposite sides a, b, c the Law of Cosines states the following*

$$a^2 = b^2 + c^2 - 2bc * Cos(a)$$

$$b^2 = a^2 + c^2 - 2ac * Cos(b)$$
$$c^2 = b^2 + a^2 - 2ba * Cos(c)$$

1.6 *Midpoint Formula Given two points on a line*

$$\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}$$

1.7 *Doube Angle Formulas*

$$sin2x = 2sinx * cosx$$

$$\begin{aligned} cos2x &= cos^2x - sin^2x \\ &= 1 - 2sin^2x \\ &= 2cos^2x - 1 \\ tan2x &= \frac{2tanx}{1 - tan^2x} \end{aligned}$$

1.8 *Recipricol Identities*

$$sin = \frac{1}{csc}$$

$$cos = \frac{1}{sec}$$

$$tan = \frac{1}{cot}$$

$$csc = \frac{1}{sin}$$

$$sec = \frac{1}{cos}$$

$$cot = \frac{1}{tan}$$

1.9 *Quotient Identities*

$$tan = \frac{sin}{cos}$$

$$cot = \frac{cos}{sin}$$

2.0 *Pythagorean Identities*

$$sin^2u + cos^2u = 1$$

$$tan^2u + 1 = sec^2u$$

$$cot^2u + 1 = csc^2u$$

Math 121 (Calculus I) Notes

1.1 *Limits*

If the values of $f(x)$ can be made as close as we like to L by taking values of x suffiencetly close to a but not equal to a then we write $\lim f(x) = L \; is \; x \to a$

Denotes a 2-sided limit $x \to a$

Denotes a 1-sided limit approaching from the right $x \to a^+$

Denotes a 1-sided limit approaching from the left $x \to a^-$

1.2 *Limits as $x \to +/-\infty$*

As $x \to -/+\infty$ while $n > 0$ and for any k $\lim \frac{k}{x^n} = 0$

Generally for any $n > 0$
$x \to +\infty$ $\lim(x^n) = +\infty$
$x \to -\infty$ is $+\infty$ if n is even $-\infty$ if n is odd

The limit of a polynomial as x approaches $+/-\infty$ reduces to the limit of its highest degree term.
For example $\lim(x^7 - 2x^4 + 5x^2)$ as $x \to -/+\infty = \lim(x^7)$ as $x \to -/+\infty$

1.3 *Indeterminate type $\frac{+\infty}{+\infty}, \frac{-\infty}{-\infty}, \frac{+\infty}{-\infty}, \frac{-\infty}{+\infty},$*

Method 1: Divide both the Neumerator and Denominator by the highest power x in the equation
Method 2: Reduce the Neumerator and Denominator to the highest power terms lim as $x \to +\infty$ of $\frac{1-x^7}{x^2+4} = \lim$ of $\frac{-x^7}{x^2}$

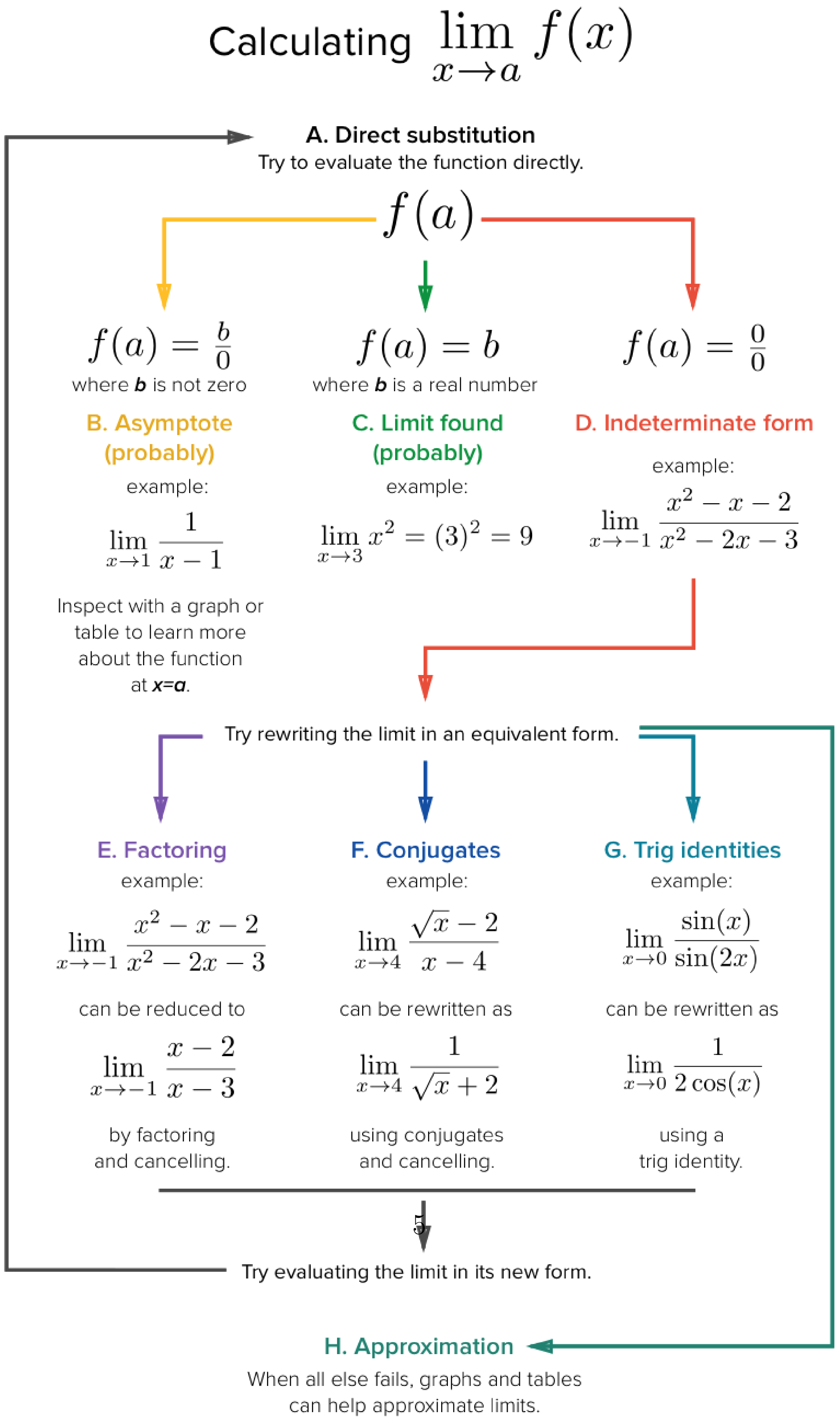

Figure 1: Chart

1.4 *Limits of Trignometric and Logarithmic Functions*
As $x \to +\infty$ or $x \to -\infty$, the values of $sin(x)$ and $cos(x)$ vary between -1 and 1 and without approaching any single real number.

$$x \to +\infty \ \lim(sin(x)) = DNE$$
$$x \to -\infty \ \lim(sin(x)) = DNE$$
$$x \to +\infty \ \lim(cos(x)) = DNE$$
$$x \to -\infty \ \lim(cos(x)) = DNE$$

Logarithmic Functions Limits
As $x \to +\infty$ e^x will increase without bound but as e is a positive number. inversely as $x \to -\infty$ e^x will approach 0

$$x \to +\infty \ \lim(e^x) = +\infty$$

$$x \to \infty \ \lim(e^x) = 0$$

Function $ln()$ is not defined for $x < 0$ so $x \to -\infty \ \lim(ln(x)) = DNE$
as $x \to +\infty$ $\lim(ln(x)) = +\infty$
as $x \to 0^+$ $\lim(ln(x)) = -\infty$

1.5 *Continuity*

A function f is said to be continuous at $x = c$ if the following is true
1.$f(c)$ is defined
2.$x \to c \lim f(x)$ exists
3.$x \to c \lim = f(c)$

Continuity On a Closed Interval (a,b) (Conditions)
1.f is continuous on (a, b)
2.f is continuous from the right at a
3.f is continuous from the left at b

Properties of continuous Functions
1. $f + g$ is continuous at c
2. $f - g$ is continuous at c
3. fg is continuous at c
4. $\frac{f}{g}$ is continuous at c if $g(c) \neq 0$ and has a discontinuity at c if $g(c) = 0$

polynomials are continuous for all real numbers

Rational functions are continuous everywhere the denominator is not 0

Calculating Limits of compositions Therom

if $x \to c \lim g(x) = L$ and if the function f is continuous at L, then $x \to c \lim f(g(x)) = f(L)$ that is,

$$\lim_{x \to c} f(g(x)) = f(\lim_{x \to c} g(x))$$

(a) If the function g is continuous at c, and the function f is continuous at g(c), then the composition f $\circ$g is continuous at c.

(b) If the function g is continuous everywhere and the function f is continuous everywhere, then the composition f $\circ$g is continuous everywhere.

Continuity of inverse functions

if f is a one to one function that is continuous at every point in its domain, then f^{-1} is continuous at every point of its domain.

Intermediate Value Theorem (IVT)

if f is continuous on a closed Interval $[a, b]$ and k is any number between $f(a)$ and $f(b)$, , then there is at least one number c in the interval such that $f(c) = k$

Special case for finding zeros of a function in an interval

if f is continuous on $[a, b]$ and $f(a)$ and $f(b)$ are non-zero and opposite in sign , then there is at least one solution of the equation $f(x) = 0$ in the interval (a, b).

1.6 *Continuity of Trignometric Functions*

$\lim_{x\to c} = sin(c)$, $\lim_{x\to c} cos(x) = cos(c)$ This impies the following:
$\lim_{x\to c} tan(x) = \lim_{x\to c} \frac{sin(x)}{cos(x)} = \frac{sin(c)}{cos(c)}$ if $cos(c) \neq 0$
$\lim_{x\to c} cot(x) = \lim_{x\to c} \frac{cos(x)}{sin(x)} = \frac{cos(c)}{sin(c)}$ if $sin(c) \neq 0$

$\lim_{x\to c} sec(x) = \lim_{x\to c} \frac{1}{cos(x)} = \frac{1}{cos(c)}$
$\lim_{x\to c} csc(x) = \lim_{x\to c} \frac{1}{sin(x)} = \frac{1}{sin(c)}$

tan cot sec and csc are continuous functions on thier domain of Definition therefore all inverse trig functions are continuous on their domain

Therom for solving Trignometric Limits
$\lim_{x\to 0}\frac{sin(x)}{x} = 1$ —and— $\lim_{x\to 0}\frac{1-cos(x)}{x} = 0$
These formulas along with the Trignometric Identities will help you solve for limits of trig functions

1.7 *Derivative and Tangent/Secant Lines*

Derivative therom 2.1.1
Suppose x_0 is in the domain of the function f the **tangent line** to the curve $y = f(x)$ at the point $P(x_0, f(x_0))$ is the line with the equation $y - f(x_0) = m_{tan} = (x - x_0)$
where

$$m_{tan} = \lim_{x\to 0}(\frac{f(x) - f(x_0))}{x - x_0})$$

provided the limit exists. This is also refered to as the tangent line to $y = f(x)$ at x_0

This formual is also expressed as the following where h is the difference between x and x_0. $h = x - x_0$

$$m_{tan} = \lim_{h\to 0}(\frac{f(x_0 + h) - f(x_0)}{h})$$

To computer average velocity

$$\frac{f(x_0 + h) - f(x_0)}{h}$$

to compute the instantanous rate of change at a point compute the

limit of the formula above when $x_0 =$ the point in question.
for example compute the instantanous rate of change at $x = 1$ for $f(x)$

$$instantanous\ velocity = \lim_{h \to 0}(\frac{f(1+h) - f(1}{h})$$

The definition of a derivative

The function $f'(x)$ read as f prime of x is defined as the following formula.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h})$$

$f'(x)$ is one way to annotate a Derivative other ways are $\frac{d}{dx}$, $\frac{dy}{dx}$, and a single dot over f (the newtonian annotation).

Derivatives do not always exist at every point in the domain of a functions.
A function is said to be differentiable at x_0 if the limit exists.
if f is differentiable at each point on an open Interval (a, b) we say *it is differentiable on* (a, b)
functions are also not differentiable on corner points or points of vertical tangency.
if $f(x)$ is differentiable at a point it must be continuous at that point.
If $f(x)$ is not continuous it is not differentiable.

One sided derivatives are also defined and we can that a function is is differentiable on a closed Interval $[a, b]$ is it is differentiable at every point in the Interval and and the appropriate one sided derivative exists for each end point $f'_+(a)$ and $f' - (b)$ respectivly.
These deratives are defined by the normal formula with the modification as $h \to 0^+$ and $h \to 0^-$ respectivly.

Finding the equation to a tangent line at point $x_0, f(x_0)$ Given the function $f(x)$ substitute your values into the following equation.

$$y - f(x_0) = f'(x_0) * (x - x_0)$$

where $x_0 =$ your given value of x

1.8 *Techniques of differention*

The derivative of a constant function is 0 (since slope of $f(x) = 2$) would be 0)

Derivatives of a power: if n is a positive Integer.
$\frac{d}{dx}[x^n] = nx^{n-1}$ for example if $f(x) = x^3$ then $f'(x) = 3x^2$

To differentiate a power function decrease the constant exponent by one and multiply the resulting power function by the original exponent. Square roots can also be seen as power functions as $\frac{1}{\sqrt{x}} = x^{-\frac{1}{2}}$

Constant Multiple Rule
if f is differentiable at x and c is any real number then cf is also differentiable at x.

$$\frac{d}{dx}[cf(x)] = c\frac{d}{dx}[f(x)]$$

$f'(cf(x)) = \lim_{h\to 0} \frac{cf(x+h)-cf(x)}{h} = c\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$
The Constant factor can be moved through the derivative sign.

Sum and Diff of Derivatives
The sum of the Derivatives is the Derivative of the sum
$f'(0) = 7$, $g'(0) = 6$ then $(f+g)'(0) = 13$
Same goes for difference

Product and Quotient Rules
The Derivative of a product of derivatives is the sum of function one times derivative two and function two times derivative one.

$$\frac{d}{dx}[f(x)g(x)] = f(x)\frac{d}{dx}[g(x)] + g(x)\frac{d}{dx}[f(x)]$$

The derivetive of a Quotient of derivatives is the difference of the functions in the same format as products over the derivative of the divisors squared.

$$\frac{d}{dx}[\frac{f(x)}{g(x)}] = \frac{g(x)\frac{d}{dx}[f(x)] - f(x)\frac{d}{dx}[g(x)]}{[g(x)]^2}$$

Derivatives of Trignometric and Logarithmic Functions

$\frac{d}{dx}[sin(x)] = cos(x)$ and $\frac{d}{dx}[cos(x)] = -sin(x)$

$$\frac{d}{dx}[tan(x)] = sec^2(x)$$

$$\frac{d}{dx}[sec(x)] = sec(x) * tan(x)$$

$$\frac{d}{dx}[cotan(x)] = -csc^2(x)$$

$$\frac{d}{dx}[csc(x)] = -csc(x) * cotan(x)$$

The Derivative of e^x is e^x and the derivative of $ln(x)$ is $\frac{1}{x}$

$\frac{d}{dx}[e^x] = e^x$ and $\frac{d}{dx}[ln(x)] = \frac{1}{x}$

The Chain Rule
The derivative of a composition of functions can be expressed as The derivative of the first function evaluated at the value of the second

function, times the derivetive of the second function.

$\frac{d}{dx}[f(g(x))] = f'(g(x)) * g'(x)$
This is refered to as the chain rule.

Logarithmic differention
Remebmer to use all properties of logs when Calculating derivatives of Logarithmic functions
for $x > 0$

$$\frac{d}{dx}[ln(x)] = \frac{1}{x}$$
$$\frac{d}{dx}[ln(|x|)] = \frac{1}{x}$$
$$\frac{d}{dx}[log_b(x)] = \frac{1}{x * ln(b)}$$
$$\frac{d}{dx}[b^x] = b^x * ln(b)$$
$$\frac{d}{dx}[e^x] = e^x$$
$$\frac{d}{dx}[b^{u(x)}] = b^{u(x)} * ln(b) * u'(x)$$

Using the definitions above we can use a Technique called Logarithmic differention to take the log of both sides of an equation and the absolute value where necessary to Calculate the derivetive more succintly

1.9 *Derivitives of inverse functions*

From the definition of inverse functions and derivatives
$f'(x)^{-1} = \frac{1}{f'(f(x)^{-1})}$

The derivative of the inverse is equal to one over the derivetive of the function evaluated at the inverse of the funtion.

Derivatives of Trignometric Functions

$$sin^{-1}(x) = \frac{1}{\sqrt{1-x^2}}$$
$$cos^{-1}(x) = -\frac{1}{\sqrt{1-x^2}}$$
$$tan^{-1}(x) = \frac{1}{1+x^2}$$

2.0 *Linear Approximation*

for values of x near x0 we can approximate values of f(x) using the definition of the local Linear Approximation function evaluated at a given x_0

$$L(x) = f(x_0) + f'(x_0)(x - x_0)$$

Differentials

Definition of dy

$$dy = f'(x)dx$$

dy is the change in y as you move along the tangent line with respect to dx aka Δx

Definition of dx

$$\Delta x = dx$$

dx is the "run" of the tangent line at the point where slope $= f'(x)$

as Δx approches 0 $\Delta y \approx f'(x)\Delta x$ or if $\Delta x = dx$ then $\Delta y \approx f'(x)dx = dy$

2.1 *Related Rates*

A Strategy for Solving Related Rates Problems

Step 1. Assign letters to all quantities that vary with time and any others that seem relevant to the problem. Give a definition for each letter.

Step 2. Identify the rates of change that are known and the rate of change that is to be found. Interpret each rate as a derivative.

Step 3. Find an equation that relates the variables whose rates of change were identified in Step 2. To do this, it will often be helpful to draw an appropriately labeled figure that illustrates the relationship.

Step 4. Differentiate both sides of the equation obtained in Step 3 with respect to time to produce a relationship between the known rates of change and the unknown rate of change.

Step 5. After completing Step 4, substitute all known values for the rates of change and the variables, and then solve for the unknown rate of change.

2.2 *L' Hopitals Rule*

for indeterminate forms $\frac{0}{0}$ and $\frac{\infty}{\infty}$ let $f(x) = Neumerator$ and $g(x) = denominator$ if $\frac{f'(x)}{g'(x)}$ exists then $\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)}$

More simply

$$\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)}$$

for $\frac{0}{0}$ and $\frac{\infty}{\infty}$

2.3 *Analyzing Functions*

Increasing and decreasing at a point/interval

if $f'(x) > 0$ at every point in (a, b) then $f(x)$ is increasing on interval $[a, b]$

if $f'(x) < 0$ at every point in (a, b) then $f(x)$ is decreasing on interval $[a, b]$

if $f'(x) = 0$ at every point in (a, b) then $f(x)$ is constant on interval $[a, b]$

Concavity

if $f''(x) > 0$ at every point in (a, b) then $f(x)$ is concave up on interval $[a, b]$

if $f''(x) < 0$ at every point in (a, b) then $f(x)$ is concave down on interval $[a, b]$

Informal definitions
$f'(x) > 0 =$ Increasing
$f'(x) < 0 =$ Decreasing
$f'(x) = 0 =$ Constant
$f''(x) > 0 =$ Concave up
$f''(x) < 0 =$ Concave down

Inflection point = point on the graph where concavity changes. To find inflection points find candidate points that are the zeros of the given functions

Relative Extrema = relative maxima or minama
Minima = lowest point on a Interval

Maxima = highest point on an Interval

in short a peak or valley in a function
candidate points for Extrema are points where the $f'(x) = 0, DNE$

If concavity changes at a point you also have a Extrema at that point or if $f''(x) > 0$ and $f'(x) = 0$ then you have a relative Minima at x.

Likewise if $f''(x) < 0$ and $f'(x) = 0$ then you have a relative Maxima at x