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Math 201 Section 001– Quiz 4

1. Consider the vector space of $\mathbb{P}_3$ of all polynomials with degrees at most 3. Let $\mathcal{B} = \{t, t+1, t^2 + 1, t^3\}$.

   (a) Show that $\mathcal{B}$ is linearly independent, and hence $\mathcal{B}$ is a basis of $\mathbb{P}_3$.

   (b) Let $p(t) = 4t^3 - 3t^2 + 5t - 6$. Find the coordinates $[p]_{\mathcal{B}}$ of p relative to the basis $\mathcal{B}$.

Answers on the next two pages.

Question 1 (a)

$B = \{t, t+1, t^2+1, t^3\}$

linearly independent if

$c_1(t) + c_2(t+1) + c_3(t^2+1) + c_4(t^3) = 0$

only when $c_1 = c_2 = c_3 = c_4 = 0$

$c_1 t + c_2 t + c_2 + c_3 t^2 + c_3 + c_4 t^3 = 0$

$$\left.\begin{aligned} (c_2 + c_3) &= 0 \\ t(c_1 + c_2) &= 0 \\ t^2(c_3) &= 0 \\ t^3(c_4) &= 0 \end{aligned}\right\} \Rightarrow \begin{bmatrix} 0 & c_2 & c_3 & 0 \\ c_1 & c_2 & 0 & 0 \\ 0 & 0 & c_3 & 0 \\ 0 & 0 & 0 & c_4 \end{bmatrix} \rightarrow$$

$$\rightarrow \cdot \begin{bmatrix} 1 \\ t \\ t^2 \\ t^3 \end{bmatrix} = 0$$

So if the above matrix is linearly independent then B is indepedent. Row reduce to prove.

$$\begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \rightarrow$$

$$\rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Reduces to a 4x4 square matrix with a pivot in every column and a unique solution to zero so it is independent

Question 1 (b)

$$P_B = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$ and $P(t) = P_B [P(t)]_B$

So solving the above equation gives us $[P(t)]_B$

vector ~~Matrix~~ of $P(t) = \begin{bmatrix} -6 \\ 5 \\ -3 \\ 4 \end{bmatrix}$

$$\left[\begin{array}{cccc|c} 0 & 1 & 1 & 0 & -6 \\ 1 & 1 & 0 & 0 & 5 \\ 0 & 0 & 1 & 0 & -3 \\ 0 & 0 & 0 & 1 & 4 \end{array}\right] \rightarrow \left[\begin{array}{cccc|c} 1 & 1 & 0 & 0 & 5 \\ 0 & 1 & 1 & 0 & -6 \\ 0 & 0 & 1 & 0 & -3 \\ 0 & 0 & 0 & 1 & 4 \end{array}\right] \rightarrow$$

$$\left[\begin{array}{cccc|c} 1 & 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 0 & -3 \\ 0 & 0 & 1 & 0 & -3 \\ 0 & 0 & 0 & 1 & 4 \end{array}\right] \rightarrow \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 8 \\ 0 & 1 & 0 & 0 & -3 \\ 0 & 0 & 1 & 0 & -3 \\ 0 & 0 & 0 & 1 & 4 \end{array}\right]$$

so $[P(t)]_B = \begin{bmatrix} 8 \\ -3 \\ -3 \\ 4 \end{bmatrix}$