1.0 Introduction

Well Ordering Principle: Every nonempty set S of non-negative integers contains a least element; that is there is osme integer a in S such that $a \leq b$ for all b's belonging to S

Theorem 1.1: Archimedian property. If a and b are any positive integers, then there exists a positive integer n such that $na \geq b$.

Theorem 1.2 First Principle of Finite Induction. Let S be the set of positive integers.
(a) The integer 1 belongs to S
(b) Whenever the integer k is in S, the next integer $k+1$ must also be in S

Theorem 1.2 Second Principle of Finite Induction. Let S be the set of positive integers.
(a) The integer 1 belongs to S
(b') If k is a positive integer such that $1, 2, \cdots, k$ for $k \in S$, then $k+1$ must also be in S.

Thus S is the set of all positive integers.

Binomial Theorem

$\binom{n}{k} = \frac{n!}{k!(n-k)!}$

Canceling either $k!$ or $(n-k)!$ yields

$\frac{n(n-1)\cdots(k+1)}{(n-k)!}$ or $\frac{n(n-1)\cdots(n-k+1)}{k!}$

If $k=0$ or $k=1$ then we have $\binom{n}{0} = \binom{n}{n} = 1$

$$\binom{n}{k} + \binom{n}{k-1} = \binom{n+1}{k}$$

Pascals Triangle

Rows of pascals triangle are built by $(a+b)^n$.
$(a+b)^1 = a+b$
$(a+b)^2 = a^2+2ab+b^2$
$(a+b)^3 = a^3+3a^2b+3ab^2+b^3$
$(a+b)^4 = a^4+5a^3b+6a^2b^2+4ab^3+b^4$
When $a=b=1$ the following triangle is built
1 1
1 2 1
1 3 3 1
1 4 6 4 1

The binomial expansion takes the form $(a+b)^n = \binom{n}{0} a^n + \binom{n}{1} a^{n-1}b + \binom{n}{2} a^{n-2}b^2 + \cdots + \binom{n}{n-1} ab^{n-1} + \binom{n}{n} b^n$

or $(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k}b^k$

1.1 Chapter 2

Pythagoreans were pretty weird and attached tons of religious connotations to numbers.
The number 1 represents reason
The number 2 stood for man
The number 3 stood for woman
4 stood for justice since it is the first number that is the product of equals
5 was for marriage because it formed the union of 2 and 3 (man and woman)

All sums $1 + \cdots + n$ are actually triangular numbers.