NCRM NATIONAL CENTRE FOR RESEARCH METHODS

Social Research Methods Leeds
Interdisciplinary Centre

Adjustment Methods for Measurement Error

Jose Pina-Sánchez

Albert Varela

Adjustment Methods

- We should always aim to improve data collection processes to avoid measurement error
- When that is not possible, we can (and should) adjust its impact
  - This enhances the rigour of our research
  - And allows us to analyse data that would otherwise be too dubious

Adjustment Methods

- Sometimes we can adjust the impact of measurement error directly
  - We can do so in some simple settings, where we can anticipate its impact

Adjustment Methods

- Sometimes we can adjust the impact of measurement error directly
  - We can do so in some simple settings, where we can anticipate its impact
- Ex.1, the effect of self-reported anxiety on life satisfaction (both of them subject to classical errors)
  - the reliability ratio can be derived by repeating the interview for a subsample of participants,
  - which can then be used to adjust the expected bias (assuming a simple linear model),

$$\widehat{\beta}^* = \widehat{\beta}\left(\frac{\sigma_X^2}{\sigma_X^2 + \sigma_U^2}\right)$$

Adjustment Methods

- Sometimes we can adjust the impact of measurement error directly
  - We can do so in some simple settings, where we can anticipate its impact
- Ex.1, the effect of self-reported anxiety on life satisfaction (both of them subject to classical errors)
  - the reliability ratio can be derived by repeating the interview for a subsample of participants,
  - which can then be used to adjust the expected bias (assuming a simple linear model),

$$\widehat{\beta}^* = \widehat{\beta}\left(\frac{\sigma_X^2}{\sigma_X^2 + \sigma_U^2}\right)$$

- Ex.2, the effect of immigration on crime recorded by the police (systematic multiplicative errors)
  - the under-recording can be estimated using victimisation surveys,
  - and we can adjust the estimate of interest accordingly (assuming a linear model),

$$\widehat{\beta}^* = \widehat{\beta}/\bar{U}$$

Adjustment Methods

- When we can't trace out the impact of measurement error algebraically we need to use adjustment methods

Adjustment Methods

- When we can't trace out the impact of measurement error algebraically we need to use adjustment methods
- Most adjustment methods require additional forms of data
  - Multiple reflective indicators (latent variable models)
  - Instrumental variables (two stage processes)
  - A validation subsample (multiple imputation)
  - Repeated observations (regression calibration)

Adjustment Methods

- When we can't trace out the impact of measurement error algebraically we need to use adjustment methods
- Most adjustment methods require additional forms of data
  - Multiple reflective indicators (latent variable models)
  - Instrumental variables (two stage processes)
  - A validation subsample (multiple imputation)
  - Repeated observations (regression calibration)
- Question: Could you use any of these methods for the measurement problems you have encountered in your research?
  - Validation and repeated observations are hard to find when you rely on secondary data

Adjustment Methods

- We will focus on methods that can be used without additional data
  - SIMEX (Cook & Stefanski, 1994)
  - Simulations (*RCME* Pina-Sánchez et al., 2022)
  - Bayesian adjustments (Gustaffson, 2003)
- All we need is an intuition of the form and prevalence of the measurement error

Estimating Measurement Error

- We can estimate the form and prevalence of measurement error in a given variable using different sources
- Question: Any ideas?

Estimating Measurement Error

- We can estimate the form and prevalence of measurement error in a given variable using different sources
- Question: Any ideas?
  - More valid estimates from the same population

    E.g. Comparing crime rates from police statistics to victimisation surveys we can ascertain systematic errors in the former

Estimating Measurement Error

- We can estimate the form and prevalence of measurement error in a given variable using different sources
- Question: Any ideas?
  - More valid estimates from the same population

    E.g. Comparing crime rates from police statistics to victimisation surveys we can ascertain systematic errors in the former
  - Measurement error studies from the literature

    E.g. A test-retest mental health assessment conducted in a different country (Biemer et al., 2004)

Estimating Measurement Error

- We can estimate the form and prevalence of measurement error in a given variable using different sources
- Question: Any ideas?
  - More valid estimates from the same population

    E.g. Comparing crime rates from police statistics to victimisation surveys we can ascertain systematic errors in the former
  - Measurement error studies from the literature

    E.g. A test-retest mental health assessment conducted in a different country (Biemer et al., 2004)
  - Replicate a coding exercise for a subsample of cases

    E.g. Manually review a subsample of automatically classified offenders' ethnicity based on their name

Estimating Measurement Error

- We can estimate the form and prevalence of measurement error in a given variable using different sources
- Question: Any ideas?
  - More valid estimates from the same population

    E.g. Comparing crime rates from police statistics to victimisation surveys we can ascertain systematic errors in the former
  - Measurement error studies from the literature

    E.g. A test-retest mental health assessment conducted in a different country (Biemer et al., 2004)
  - Replicate a coding exercise for a subsample of cases

    E.g. Manually review a subsample of automatically classified offenders' ethnicity based on their name
  - Interviews with survey interviewers, experts (e.g. practitioners), or individuals from the target population

Estimating Measurement Error

- We can estimate the form and prevalence of measurement error in a given variable using different sources
- Question: Any ideas?
  - More valid estimates from the same population

    E.g. Comparing crime rates from police statistics to victimisation surveys we can ascertain systematic errors in the former
  - Measurement error studies from the literature

    E.g. A test-retest mental health assessment conducted in a different country (Biemer et al., 2004)
  - Replicate a coding exercise for a subsample of cases

    E.g. Manually review a subsample of automatically classified offenders' ethnicity based on their name
  - Interviews with survey interviewers, experts (e.g. practitioners), or individuals from the target population
  - Our own educated guess as subject experts

Sensitivity Analysis

- Such estimates should be taken as highly uncertain
  - ‘Gold standard’ measures are rarely perfect
  - Problems of transportability with studies using different samples/populations
  - Subjective nature of qualitative methods
  - Researcher bias

Sensitivity Analysis

- Such estimates should be taken as highly uncertain
  - 'Gold standard' measures are rarely perfect
  - Problems of transportability with studies using different samples/populations
  - Subjective nature of qualitative methods
  - Researcher bias
- We should undertake multiple adjustments
  - Using a range of plausible values, as opposed to assuming we know the form and prevalence of measurement error mechanism/s perfectly

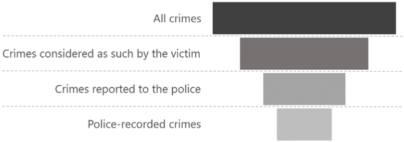
Sensitivity Analysis

- Such estimates should be taken as highly uncertain
  - 'Gold standard' measures are rarely perfect
  - Problems of transportability with studies using different samples/populations
  - Subjective nature of qualitative methods
  - Researcher bias
- We should undertake multiple adjustments
  - Using a range of plausible values, as opposed to assuming we know the form and prevalence of measurement error mechanism/s perfectly
- We will not obtain a single 'adjusted' finding
  - Rather, we will seek to assess how 'sensitive' or robust our findings are under different scenarios
  - This is known as sensitivity analysis

Simulations

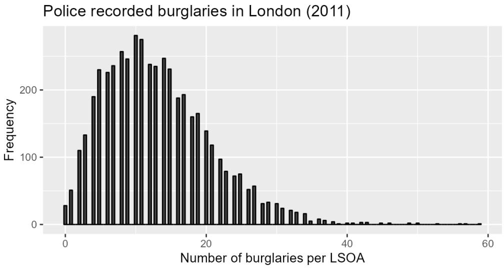
- The idea is to use our understanding of the measurement error process to recreate the original variable
- Then repeat the analysis using the 'adjusted' variable
  - Ideally for a range of measurement error scenarios
- Examples:
  - The reporting rate of burglaries has fluctuated between 40% to 60% in England and Wales (Pina-Sánchez et al., 2022)
  - Men report an average 14 lifetime opposite-sex partners, women report 7 (Mitchell et al., 2019)

Simulations: Under-recorded Crime

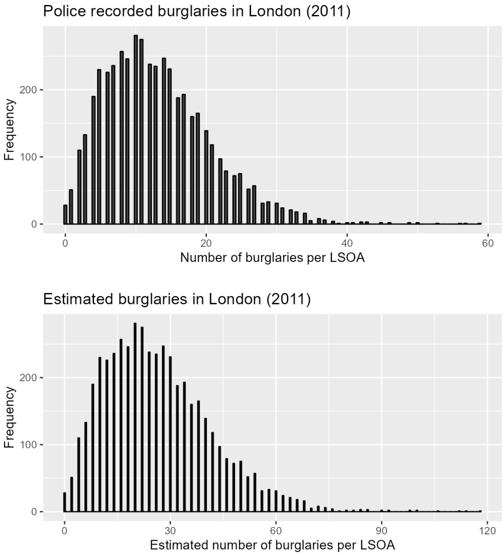


- We formalise the above intuition into a measurement model
  - $X^* = X \cdot U$ with $U \sim N(0.5, \sigma_U)$
- We rearrange the measurement model and substitute to adjust the error-prone variable
  - $\widehat{X} = X^*/0.5$

Simulations: Underrecorded Crime



Simulations: Underrecorded Crime



Simulations: Lifetime Partners

- A slightly more complex measurement error mechanism
- If we assume the true number of partners is in the middle (i.e. men overreport as much as women underreport)
  - We have the following measurement error model

$$\begin{cases} X^* = X \cdot U_1; & \text{if Z=man} \\ X^* = X \cdot U_2; & \text{if Z=woman} \end{cases}$$

  - And the adjusted variable

$$\begin{cases} \widehat{X} = X^*/1.33; & \text{if Z=man} \\ \widehat{X} = X^*/0.66; & \text{if Z=woman} \end{cases}$$

  - With the 33% worked out for men as: $14/(14-(7/2)) = 1.33$ and similarly for women: $7/(7+(7/2)) = 0.66$

Simulations

- Simulations represent a direct and simple approach to adjusting measurement error
  - Making them an intuitive, parsimonious and transparent method

Simulations

- Simulations represent a direct and simple approach to adjusting measurement error
  - Making them an intuitive, parsimonious and transparent method
- They can be applied to any kind of analysis
  - Focus on adjusting the error-prone variable, which can then be used anywhere we want
  - Many other adjustment methods can only be used in specific outcome models, or estimation methods

Simulations

- Simulations represent a direct and simple approach to adjusting measurement error
  - Making them an intuitive, parsimonious and transparent method
- They can be applied to any kind of analysis
  - Focus on adjusting the error-prone variable, which can then be used anywhere we want
  - Many other adjustment methods can only be used in specific outcome models, or estimation methods
- They are also remarkably flexible in that they can mimic a wide range of forms of measurement error and misclassification
  - Gallop & Weschle, 2019
- One exception being random errors
  - Even if we know the magnitude of the error mechanism, we will not be able to estimate each true value

SIMEX

- A simulation-based, but indirect, approach to adjusting for measurement error
  - Simulates increasing layers of measurement error, to trace out its impact
  - Then extrapolates to retrieve the true finding, when no measurement error is present

SIMEX

- A simulation-based, but indirect, approach to adjusting for measurement error
  - Simulates increasing layers of measurement error, to trace out its impact
  - Then extrapolates to retrieve the true finding, when no measurement error is present
- The SIMulation-EXtrapolation algorithm
  - Assuming $Y = \alpha + \beta X^* + \epsilon$, and $X^* = X + U$

SIMEX

- A simulation-based, but indirect, approach to adjusting for measurement error
  - Simulates increasing layers of measurement error, to trace out its impact
  - Then extrapolates to retrieve the true finding, when no measurement error is present
- The SIMulation-EXtrapolation algorithm
  - Assuming $Y = \alpha + \beta X^* + \epsilon$, and $X^* = X + U$

1. Generate new variables with increasing levels of measurement error, $X_k^*(\lambda_k) = X^* + \sqrt{(\lambda_k)}U$, with $\lambda_k = (0.5, 1, 1.5, 2)$

SIMEX

- A simulation-based, but indirect, approach to adjusting for measurement error
  - Simulates increasing layers of measurement error, to trace out its impact
  - Then extrapolates to retrieve the true finding, when no measurement error is present
- The SIMulation-EXtrapolation algorithm
  - Assuming $Y = \alpha + \beta X^* + \epsilon$, and $X^* = X + U$

1. Generate new variables with increasing levels of measurement error, $X^*_k(\lambda_k) = X^* + \sqrt{(\lambda_k)}U$, with $\lambda_k = (0.5, 1, 1.5, 2)$
2. Re-estimate the outcome model using X^*_k, which produces increasingly biased estimates, $\hat{\beta}^*_k = \beta_1 \frac{S^2_X}{S^2_X + (1+\lambda_k)S^2_U}$

SIMEX

- A simulation-based, but indirect, approach to adjusting for measurement error
  - Simulates increasing layers of measurement error, to trace out its impact
  - Then extrapolates to retrieve the true finding, when no measurement error is present
- The SIMulation-EXtrapolation algorithm
  - Assuming $Y = \alpha + \beta X^* + \epsilon$, and $X^* = X + U$

1. Generate new variables with increasing levels of measurement error, $X_k^*(\lambda_k) = X^* + \sqrt{(\lambda_k)}U$, with $\lambda_k = (0.5, 1, 1.5, 2)$
2. Re-estimate the outcome model using X_k^*, which produces increasingly biased estimates, $\hat{\beta}_k^* = \beta_1 \frac{S_X^2}{S_X^2 + (1+\lambda_k)S_U^2}$
3. Steps 1 and 2 are repeated to obtain $\overline{\hat{\beta}_k^*}$, reduce simulation error

SIMEX

- A simulation-based, but indirect, approach to adjusting for measurement error
  - Simulates increasing layers of measurement error, to trace out its impact
  - Then extrapolates to retrieve the true finding, when no measurement error is present
- The SIMulation-EXtrapolation algorithm
  - Assuming $Y = \alpha + \beta X^* + \epsilon$, and $X^* = X + U$

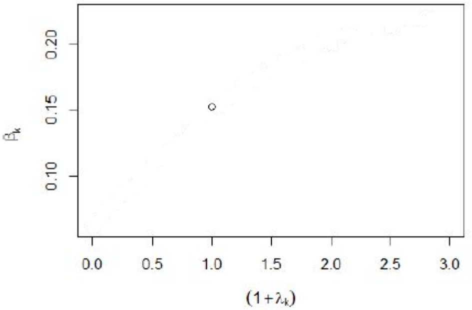
1. Generate new variables with increasing levels of measurement error, $X_k^*(\lambda_k) = X^* + \sqrt{(\lambda_k)}U$, with $\lambda_k = (0.5, 1, 1.5, 2)$
2. Re-estimate the outcome model using X_k^*, which produces increasingly biased estimates, $\hat{\beta}_k^* = \beta_1 \frac{S_X^2}{S_X^2 + (1+\lambda_k)S_U^2}$
3. Steps 1 and 2 are repeated to obtain $\overline{\hat{\beta}_k^*}$, reduce simulation error
4. $\overline{\hat{\beta}_k^*}$ and λ_k can now be paired and their relationship estimated

SIMEX

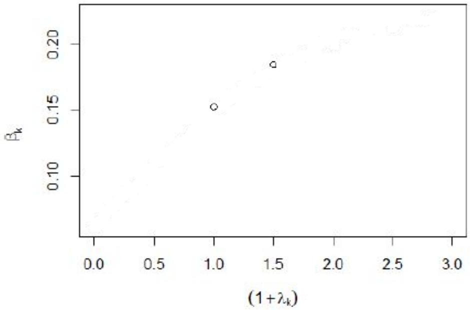
- A simulation-based, but indirect, approach to adjusting for measurement error
  - Simulates increasing layers of measurement error, to trace out its impact
  - Then extrapolates to retrieve the true finding, when no measurement error is present
- The SIMulation-EXtrapolation algorithm
  - Assuming $Y = \alpha + \beta X^* + \epsilon$, and $X^* = X + U$

1. Generate new variables with increasing levels of measurement error, $X_k^*(\lambda_k) = X^* + \sqrt{(\lambda_k)}U$, with $\lambda_k = (0.5, 1, 1.5, 2)$
2. Re-estimate the outcome model using X_k^*, which produces increasingly biased estimates, $\hat{\beta}_k^* = \beta_1 \frac{S_X^2}{S_X^2 + (1+\lambda_k)S_U^2}$
3. Steps 1 and 2 are repeated to obtain $\overline{\hat{\beta}_k^*}$, reduce simulation error
4. $\overline{\hat{\beta}_k^*}$ and λ_k can now be paired and their relationship estimated
5. $\hat{\beta}_{SIMEX}$ can now be calculated by extrapolating to $\lambda_k = -1$

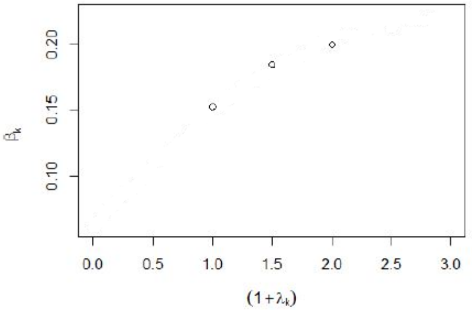
SIMEX



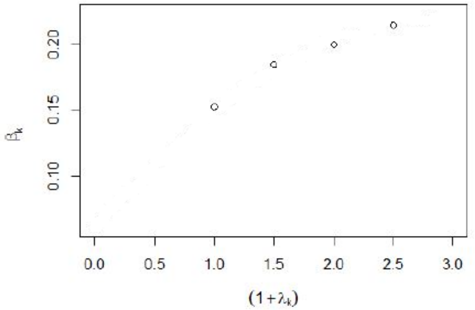
SIMEX



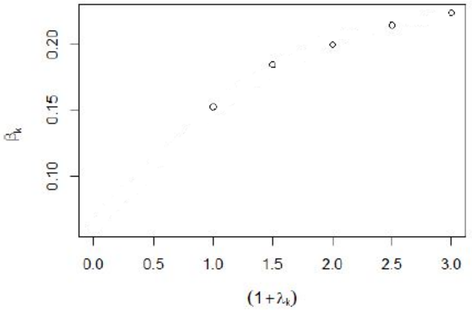
SIMEX



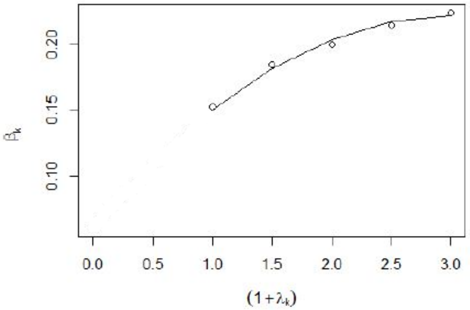
SIMEX



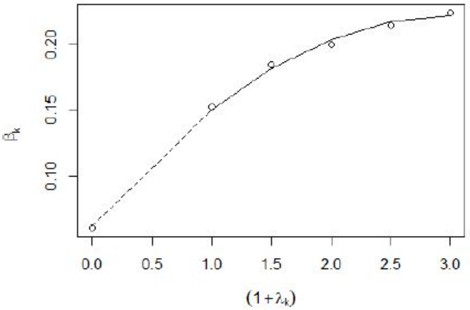
SIMEX



SIMEX



SIMEX



SIMEX

- The quality of the adjustment depends on:
  - The accuracy with which we define the measurement error mechanism
  - Choosing the right extrapolation function

SIMEX

- The quality of the adjustment depends on:
  - The accuracy with which we define the measurement error mechanism
  - Choosing the right extrapolation function
- A very flexible approach
  - Works for all kinds of outcome models
  - An R package (simex) with built-in commands to explore general cases (e.g. classical errors, misclassification)
  - New packages exploring other measurement error forms (e.g. multiplicative errors)
  - Not perfectly flexible though, we can only explore pre-established measurement error forms
  - And explore the impact of measurement error when the variable affected is the predictor of interest

Bayesian Adjustments

- The most flexible approach
  - Can be used in any outcome model to adjust for any form of measurement error
  - Overcomes the limitations of simulations-based approaches (e.g. simulating classical errors, or multiplicative errors affecting count-duration data)

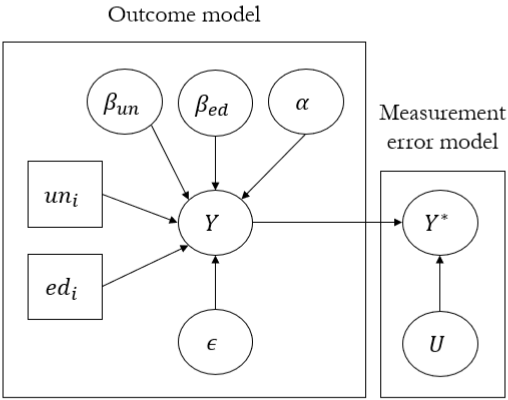
Bayesian Adjustments

- The most flexible approach
  - Can be used in any outcome model to adjust for any form of measurement error
  - Overcomes the limitations of simulations-based approaches (e.g. simulating classical errors, or multiplicative errors affecting count-duration data)
- We specify both an outcome and a measurement model
  - The former reflects the substantive relationship that we want to estimate
  - The latter can reflect any form of measurement error that we can express algebraically

Bayesian Adjustments

- The most flexible approach
  - Can be used in any outcome model to adjust for any form of measurement error
  - Overcomes the limitations of simulations-based approaches (e.g. simulating classical errors, or multiplicative errors affecting count-duration data)
- We specify both an outcome and a measurement model
  - The former reflects the substantive relationship that we want to estimate
  - The latter can reflect any form of measurement error that we can express algebraically
- These two (or more) models are estimated simultaneously
  - Using Markov chain Monte Carlo (MCMC) methods
  - We obtain a 'posterior distribution' for each estimate included in our models
  - This reflects the probability distribution of an estimate given the models we are using, the data that we observe, and any prior knowledge we might want to include

Bayesian Adjustments

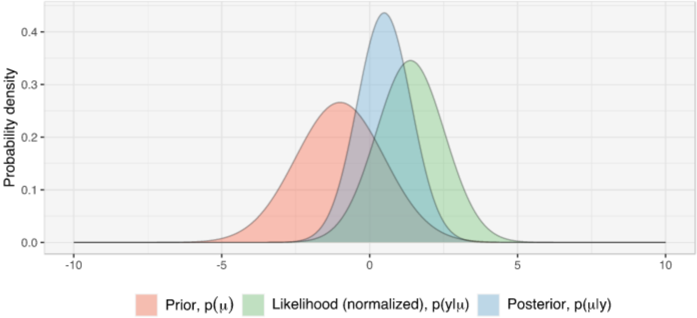


Bayesian Statistics: Overview

- **Bayesian Statistics** is a framework for statistical inference that combines prior knowledge with observed data to make probabilistic predictions.
- The key elements include:
  - **Prior Distribution:** Represents beliefs about parameters before observing data.
  - **Likelihood Function:** Describes the probability of observing the data given the parameters.
  - **Posterior Distribution:** The updated distribution of parameters after considering the data.

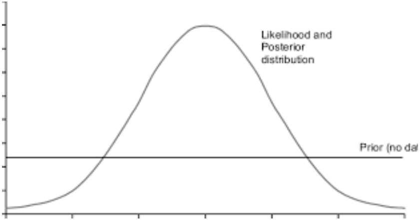
Adjustments
Sensitivity Analysis
Simulations
SIMEX
Bayesian Adjustments

The posterior distribution as a combination of the likelihood function and prior distribution



Source: Bolstad (2018)

When using diffuse priors the posterior distribution equals the likelihood function

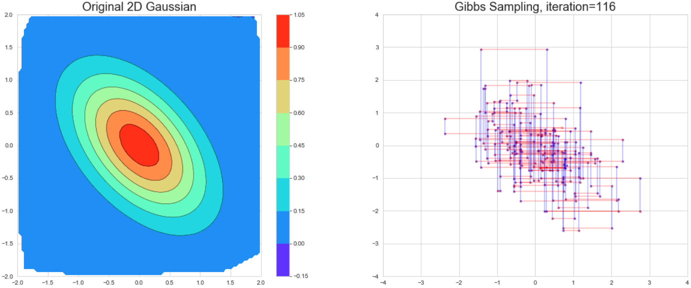


Source: Ghazoul and McAllister (2003)

Markov Chain Monte Carlo (MCMC) Estimation

- **MCMC** is a class of algorithms used for sampling from complex probability distributions.
- In Bayesian statistics, MCMC is employed to draw samples from the posterior distribution.
- **Metropolis-Hastings Algorithm** and **Gibbs Sampling** are common MCMC techniques.
- MCMC estimation allows us to explore the parameter space and approximate the posterior distribution without requiring explicit solutions.

Gibbs sampling algorithm approximating a Gaussian distribution



Source: Dey (2020)

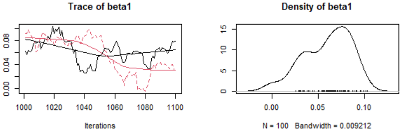
Convergence and Diagnostic Checks

- **Convergence** is crucial in MCMC methods to ensure the sampled values accurately represent the target distribution.
- **Traceplots** and **Gelman-Rubin diagnostic** are commonly used to assess convergence.
- **Burn-in Period:** Initial samples often discarded to mitigate the impact of the starting point.
- Adequate convergence ensures reliable inference from the posterior distribution.

Evaluating Convergence

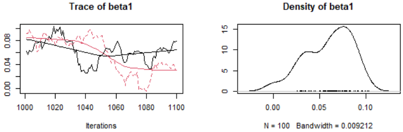
Evaluating Convergence

MCMC chains that have not converged

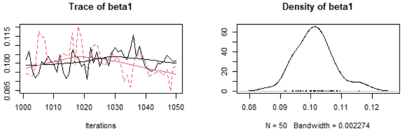


Evaluating Convergence

MCMC chains that have not converged

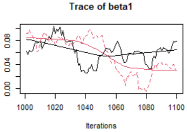


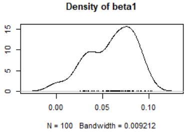
MCMC chains that have converged but the sample is not big enough



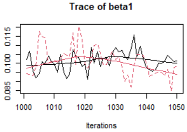
Evaluating Convergence

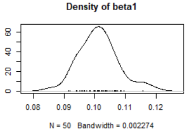
MCMC chains that have not converged



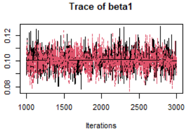


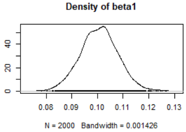
MCMC chains that have converged but the sample is not big enough





MCMC chains that have converged and can estimate the posterior distribution precisely





Bayesian Adjustments

- Can be expanded in many different ways
  - Informative priors, we can incorporate any subjective knowledge we possess about any of the parameters to be estimated
  - Can adjust for measurement error and missing data simultaneously

Bayesian Adjustments

- Can be expanded in many different ways
  - Informative priors, we can incorporate any subjective knowledge we possess about any of the parameters to be estimated
  - Can adjust for measurement error and missing data simultaneously
- Probably the method with the steepest learning curve
  - To really exploit the full flexibility of Bayesian methods we need to use Bayesian software (e.g. Stan, JAGS)
  - And learn more about Bayesian inference

NATIONAL CENTRE FOR RESEARCH METHODS

comprehensive training in research methods

www.ncrm.ac.uk

UKRI Economic and Social Research Council
UNIVERSITY OF Southampton
MANCHESTER 1824 The University of Manchester

NATIONAL CENTRE FOR RESEARCH METHODS

Social Research Methods Leeds
INTERDISCIPLINARY CENTRE

Evaluation Form

www.ncrm.ac.uk/surveys/hub