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Social Research Methods Leeds
Interdisciplinary Centre



# Adjustment Methods for Measurement Error

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# Adjustment Methods

- We should always aim to improve data collection processes to avoid measurement error
- When that is not possible, we can (and should) adjust its impact
  - This enhances the rigour of our research
  - And allows us to analyse data that would otherwise be too dubious

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- Ex.1, the effect of self-reported anxiety on life satisfaction (both of them subject to classical errors)
  - the reliability ratio can be derived by repeating the interview for a subsample of participants,
  - which can then be used to adjust the expected bias (assuming a simple linear model),

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- Ex.2, the effect of immigration on crime recorded by the police (systematic multiplicative errors)
  - the under-recording can be estimated using victimisation surveys,
  - and we can adjust the estimate of interest accordingly (assuming a linear model),

$$\widehat{\beta}^* = \widehat{\beta}/\bar{U}$$

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  - Multiple reflective indicators (latent variable models)
  - Instrumental variables (two stage processes)
  - A validation subsample (multiple imputation)
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- Question: Could you use any of these methods for the measurement problems you have encountered in your research?
  - Validation and repeated observations are hard to find when you rely on secondary data

# Adjustment Methods

- We will focus on methods that can be used without additional data
  - SIMEX (Cook & Stefanski, 1994)
  - Simulations (*RCME* Pina-Sánchez et al., 2022)
  - Bayesian adjustments (Gustaffson, 2003)
- All we need is an intuition of the form and prevalence of the measurement error

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  - Interviews with survey interviewers, experts (e.g. practitioners), or individuals from the target population
  - Our own educated guess as subject experts

# Sensitivity Analysis

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  - ‘Gold standard’ measures are rarely perfect
  - Problems of transportability with studies using different samples/populations
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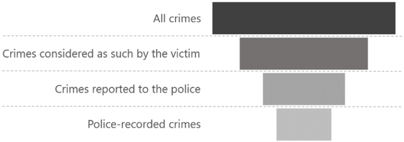
- We should undertake multiple adjustments
  - Using a range of plausible values, as opposed to assuming we know the form and prevalence of measurement error mechanism/s perfectly
- We will not obtain a single ‘adjusted’ finding
  - Rather, we will seek to assess how ‘sensitive’ or robust our findings are under different scenarios
  - This is known as sensitivity analysis

# Simulations

- The idea is to use our understanding of the measurement error process to recreate the original variable
- Then repeat the analysis using the 'adjusted' variable
  - Ideally for a range of measurement error scenarios
- Examples:
  - The reporting rate of burglaries has fluctuated between 40% to 60% in England and Wales (Pina-Sánchez et al., 2022)
  - Men report an average 14 lifetime opposite-sex partners, women report 7 (Mitchell et al., 2019)

# Simulations: Under-recorded Crime




- We formalise the above intuition into a measurement model
  - $X^* = X \cdot U$ with $U \sim N(0.5, \sigma_U)$
- We rearrange the measurement model and substitute to adjust the error-prone variable
  - $\widehat{X} = X^*/0.5$

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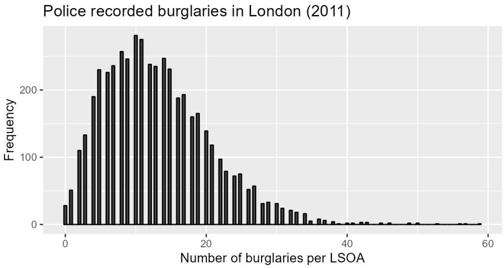

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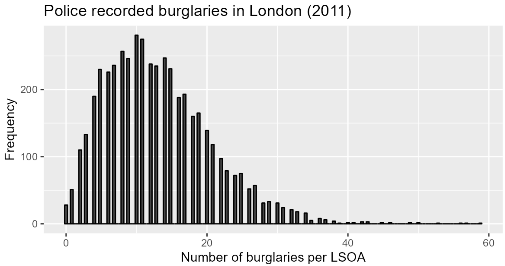


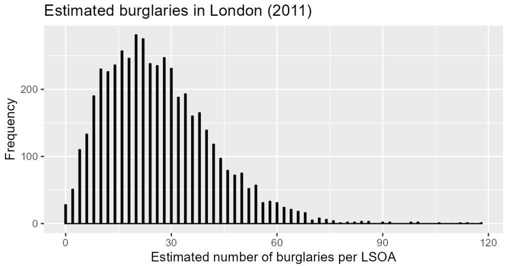

# Simulations: Lifetime Partners

- A slightly more complex measurement error mechanism
- If we assume the true number of partners is in the middle (i.e. men overreport as much as women underreport)
  - We have the following measurement error model

$$\begin{cases} X^* = X \cdot U_1; & \text{if Z=man} \\ X^* = X \cdot U_2; & \text{if Z=woman} \end{cases}$$

  - And the adjusted variable

$$\begin{cases} \widehat{X} = X^*/1.33; & \text{if Z=man} \\ \widehat{X} = X^*/0.66; & \text{if Z=woman} \end{cases}$$

  - With the 33% worked out for men as: $14/(14-(7/2)) = 1.33$ and similarly for women: $7/(7+(7/2)) = 0.66$

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  - Many other adjustment methods can only be used in specific outcome models, or estimation methods
- They are also remarkably flexible in that they can mimic a wide range of forms of measurement error and misclassification
  - Gallop & Weschle, 2019
- One exception being random errors
  - Even if we know the magnitude of the error mechanism, we will not be able to estimate each true value

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2. Re-estimate the outcome model using $X^*_k$, which produces increasingly biased estimates, $\hat{\beta}^*_k = \beta_1 \frac{S^2_X}{S^2_X + (1+\lambda_k)S^2_U}$
3. Steps 1 and 2 are repeated to obtain $\overline{\hat{\beta}^*_k}$, reduce simulation error
4. $\overline{\hat{\beta}^*_k}$ and $\lambda_k$ can now be paired and their relationship estimated
5. $\hat{\beta}_{SIMEX}$ can now be calculated by extrapolating to $\lambda_k = -1$

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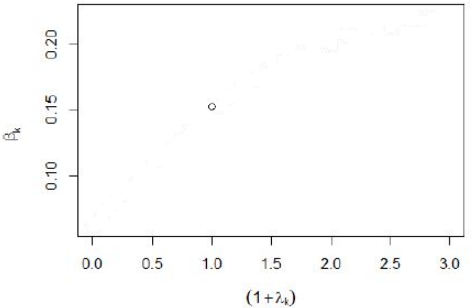

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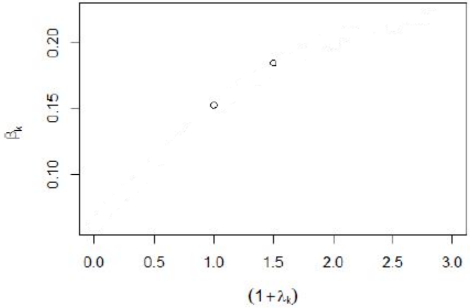

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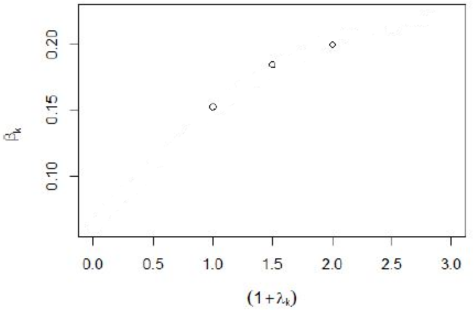

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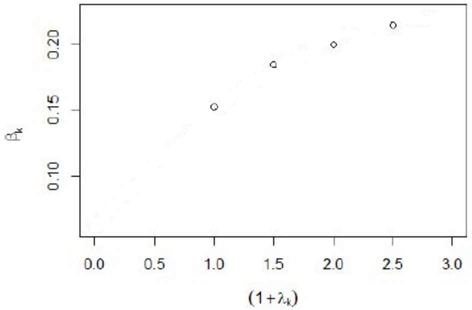

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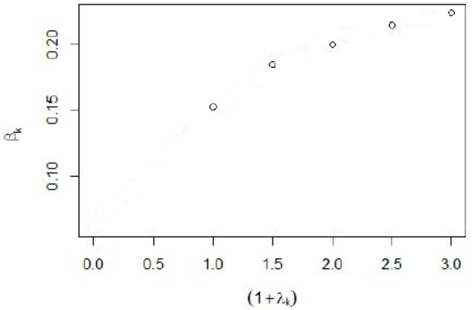

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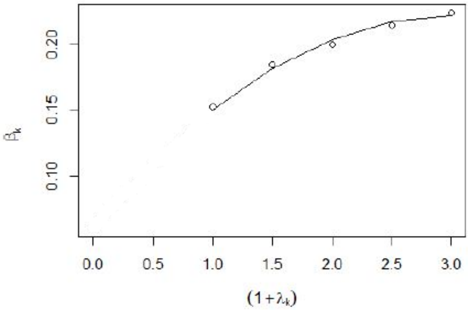

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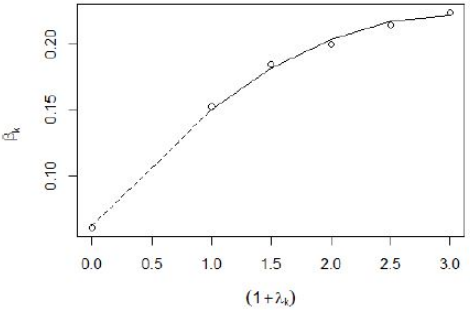

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- The quality of the adjustment depends on:
  - The accuracy with which we define the measurement error mechanism
  - Choosing the right extrapolation function
- A very flexible approach
  - Works for all kinds of outcome models
  - An R package (simex) with built-in commands to explore general cases (e.g. classical errors, misclassification)
  - New packages exploring other measurement error forms (e.g. multiplicative errors)
  - Not perfectly flexible though, we can only explore pre-established measurement error forms
  - And explore the impact of measurement error when the variable affected is the predictor of interest

## Bayesian Adjustments

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- These two (or more) models are estimated simultaneously
  - Using Markov chain Monte Carlo (MCMC) methods
  - We obtain a 'posterior distribution' for each estimate included in our models
  - This reflects the probability distribution of an estimate given the models we are using, the data that we observe, and any prior knowledge we might want to include

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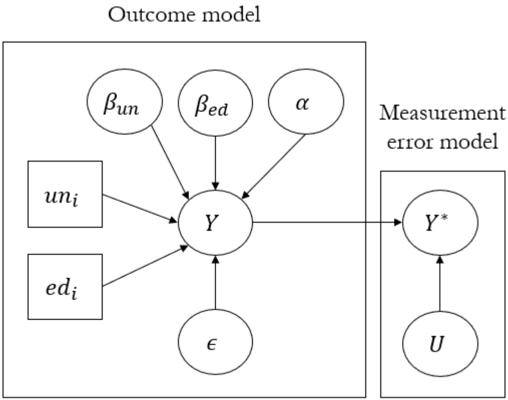

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- Probably the method with the steepest learning curve
  - To really exploit the full flexibility of Bayesian methods we need to use Bayesian software (e.g. Stan, JAGS)
  - And learn more about Bayesian inference

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