

## Problem Set 2 for G.042

## Problem 2

COLLABORATION: On problem 2 I collaborated with no one, and received help from no one.

a)  $A = \{1, 2\}$   $B = \{3, 4\}$   $C = \{5, 6\}$   $D = \{7, 8\}$  does not work

$$L: A \cup B = \{1, 2, 3, 4\} \quad C \cup D = \{5, 6, 7, 8\}$$

$$(A \cup B) \times (C \cup D):$$

	1	2	3	4
5	0	0	0	0
6	0	0	0	0
7	0	0	0	0
8	0	0	0	0

16 ordered pairs

$$R: A \times C \quad B \times D$$

	1	2
3	0	0
4	0	0

	5	6
7	0	0
8	0	0

← 4 unique ordered pairs each  
so  $(A \times C) \cup (B \times D)$  has 8 elements

The # of elements in  $L$  and  $R$  does not match, so the theorem fails because  $L \neq R$ .

b) The "either's" in "iff either  $x \in A$  or  $x \in B$ , and either  $y \in C$  or  $y \in D$ " are a mistake, because it is possible for  $x$  to exist in both  $A$  and  $B$ , and for  $y$  to exist in both  $C$  and  $D$ .

c) Also, the lines "iff  $x \in A \cup B$  and  $y \in C \cup D$ " are a mistake due to the non-commutativity and non-associativity of the cartesian product. The commutativity and associativity of the union operator is transferred when it should not be. ← iff  $(x, y) \in L$

c) For example, if  $x$  is in  $A$  then in  $R$  it will never have a chance to form a pair with a  $y$  from  $D$  because of the grouping in  $R$ . But in  $L$  an  $x$  in  $A$  can form a pair with  $D$ . This is why  $R \subseteq L$  must be the case.  $\square$