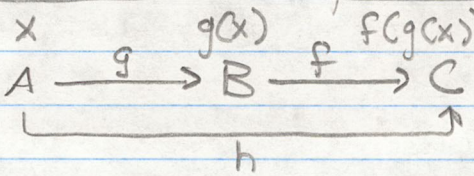
Owen Trueblood

Problem Set 2 for 6.042

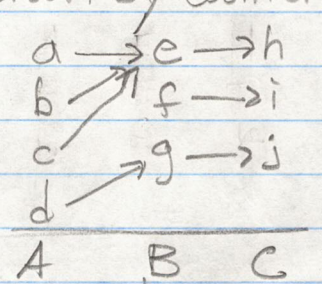
Problem 3

COLLABORATION: On problem 3 I collaborated with no one, and received help from no one.



a) Proof. $h : A \to C$ and $f : B \to C$ have the same codomain. Surjectivity specifies ≥ 1 arrow going into each element of the codomain. Because the codomain is shared, surjectivity on one function must be shared by the other. □

b) Disproof. By counterexample:

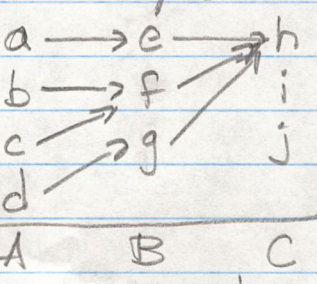


h is surjective here because ≥ 1 arrow leads to each element of its codomain, C.

g is not surjective because < 1 arrow leads to one of the elements of its codomain (f). Therefore it is not true that if h is surjective g must be too.

c) Proof. By the argument in (a), a constraint on arrows going into a codomain will be shared by all of the functions who map to that codomain. Therefore if h is injective, f must be too.

d) Disproof. By counterexample.



- h is injective because its codomain, C, has ≤ 1 arrow going in.
- f is total because its domain, B, has ≥ 1 arrow going out
- g is not injective, because its codomain, B, has > 1 arrows going in.

∴ h being injective and f being total does not mean g must be injective.