Problem Set 2

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Session L03

Collaboration

For the following problems I worked alone using only the course materials from this term. Owen Trueblood

L03

Problem 1

Collaboration: On problem 1, I collaborated with no one, and received help from no one.

Proof.

By induction.

Define:

$r(x)$ = set of all numbers where
1) the digits are equal to one another AND
2) the number of digits is x

for example: $r(3) = \{111, 222, 333, 444, 555, 666, 777, 888, 999\}$

The induction hypothesis is $P(n) ::= \{n \in N, Q \in r(3^n) \mid 3^n$ divides evenly into Q$\}$

In the base case $P(0)$, $3^n \implies 3^0 \implies 1$. Because 1 divides every number with no remainder, $P(0)$ is true.

If X stands in for the digit (1, 2, ..., or 9) repeated in Q, then

$$\sum_{i=1}^{3^n} X = (3^n)X \tag{1}$$

Because 3 divides a number evenly IFF it divides the sum of its digits, and the previous equation shows that the sum of the digits has 3^n as a factor, $P(n+1) = 3^{n+1}|Q$ will always be true. Expounding on that argument, 3^n as a factor forces the truth of $P(n+1)$ because for n $>$ 0 it has 3 as a factor. The base case is true, and $P(n) \implies P(n+1)$ is true, so by induction P(n) is proven. $\diamond$