PROBLEM SET 1
COVER SHEET

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Session L03

COLLABORATION:

For the following problems, I worked alone using only course materials from this term.

Owen Trueblood, L03
Problem Set 1

Problem 1

COLLABORATION On problem 1, I collaborated with no one, and received help from Alex Willisson and Robert Sloan, and referred to wolframalpha.com, wikipedia.org, mathisfun.com.

Proof. The proof is by case analysis. There are ~~two direct cases:~~ three cases:

1) $\log_7 n$ is irrational
2) $\log_7 n$ is rational, but not an integer
3) $\log_7 n$ is rational, and an integer

Because these cases cover all possibilities, and it is only necessary to show $\log_7 n$ falls into case 1 or 3 (either integer or irrational), it is sufficient to show that case 2 is impossible:

A rational number can be expressed as $\frac{m}{p}$, where m and p are integers. Assuming $\log_7 n$ is rational, it equals some $\frac{m}{p}$:
$\log_7 n = \frac{m}{p} \longrightarrow 7^{\log_7 n} = 7^{m/p} \longrightarrow n = 7^{m/p} = (7^{1/p})^m$
~~Because 7 is a prime number, it does not have~~
7 is a prime number, so it cannot have any integer roots, because they would constitute factors other than one and itself. Therefore $7^{1/p}$ cannot be an integer, and if $\frac{m}{p}$ is not an integer then $n = 7^{m/p}$, derived above, states that n is not an integer, which is a contradiction.
This shows that case 2 is not possible, so $\log_7 n$ must be either irrational or an integer. □