Reactor Kinetics

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April 25, 2016

Point Kinetics

PKE

$$\frac{dP}{dt} = \frac{\rho - \bar{\beta}}{\Lambda} P + \sum_{i=1}^{6} \lambda_i C_i$$

$$\frac{dC_i}{dt} = \frac{\bar{\beta}_i}{\Lambda} P - \lambda_i C_i$$

Inhour Equation

Assume constant reactivity and a solution of the form

$$P(t) = P_0 e^{\omega t}$$
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$$\omega P_0 e^{\omega t} = \frac{\rho - \bar{\beta}}{\Lambda} P_0 e^{\omega t} + \sum_{i=1}^{6} \lambda_i C_{i,0} e^{\omega t}$$
$$\omega C_{i,0} e^{\omega t} = \frac{\bar{\beta}_i}{\Lambda} P_0 e^{\omega t} - \lambda_i C_{i,0} e^{\omega t}$$

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Combining the two equations results in the *inhour* equation

$$\rho = \Lambda\omega + \sum_{i=1}^{6} \frac{\bar{\beta}_i \omega}{\omega + \lambda_i}$$

There are 7 roots of the inhour equation (they are the eigenvalues of the PKE *system matrix*:

$$\rho = \Lambda\omega_j + \sum_{i=1}^{6} \frac{\bar{\beta}_i \omega_j}{\omega_j + \lambda_i}, \quad j = 1, 2, \ldots, 7$$

Roots of the Inhour Equation

There are 7 roots of the inhour equation (they are the eigenvalues of the PKE *system matrix*:

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This means we have a solution of the form

$$P(t) = \sum_{j=0}^{6} a_j e^{\omega_j t}$$

Six of the roots will always be negative

$$\omega_j < -\lambda_j, \quad j = 1, 2, \ldots, 6$$

The root ω_0 may be positive or negative.

$$P(t) = a_0 e^{\omega_0 t} + \sum_{j=1}^{6} a_j e^{-|\omega_j| t}$$

Asymptotically ($t >> 0$) we have

$$P(t) \approx a_0 e^{\omega_0 t} \sim e^{t/T}$$

where the (asymptotic) period is defined as

$$T = \omega_0^{-1}$$

Subcritical Limit

As $\rho \to -\infty$

$$T \to \lambda_1^{-1} \approx 80 \text{ seconds (U-235)}$$

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This means that

$$T = 0.1\rho^{-1} \text{ seconds} \approx \frac{1}{4}\frac{\bar{\beta}}{\rho} \text{ minutes}$$

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This means that

$$T = \frac{\Lambda}{\rho - \bar{\beta}} \approx 10^{-2} \left(\frac{\rho}{\bar{\beta}} - 1 \right)^{-1}$$