

Using Rejection Regions for a z-Test for a Mean μ

In Words

1. State the claim mathematically and verbally. Identify the null and alternative hypotheses.
2. Specify the level of significance.
3. Sketch the sampling distribution.
4. Determine the critical value(s).
5. Determine the rejection region(s).
6. Find the standardized test statistic.
7. Make a decision to reject or fail to reject the null hypothesis.
8. Interpret the decision in the context of the original claim.

In Symbols

- State H_0 and H_a .
- Identify α .
- Use invNorm or Table
- $$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} \text{ or if } n \geq 30$$
- use $\sigma \approx s$.
- If z is in the rejection region, reject H_0 .
Otherwise, fail to reject H_0 .

Section 7.1 Objectives

- State a null hypothesis and an alternative hypothesis
- Identify type I and type II errors and interpret the level of significance
- Determine whether to use a one-tailed or two-tailed test and find a *p*-value
- Make and interpret a decision based on the results of a statistical test
- Write a claim for a hypothesis test

You Try

A medical research team is investigating the benefits of a new surgical treatment. One of the claims is that the mean recovery time for patients after the new treatment is less than 96 hours. How would you write the null and alternative hypotheses if :

- (1) You are on the research team and want to support the claim?
 - (2) You are on the opposing team and want to reject the claim?
 - (3) Discuss meaning & risk of Type I error (Rejecting H_0 when it's true)
 - (4) Discuss meaning & risk of Type II error (Fail to reject H_0 when it's false)
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Steps for Hypothesis Testing

1. State the claim mathematically and verbally. Identify the null and alternative hypotheses.

$$H_0: ? \quad H_a: ?$$

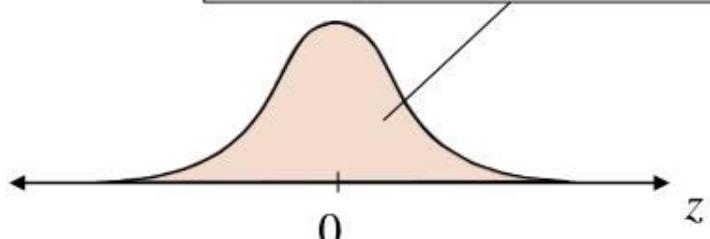
2. Specify the level of significance.

$$\alpha = ?$$

3. Determine the standardized sampling distribution and draw its graph.

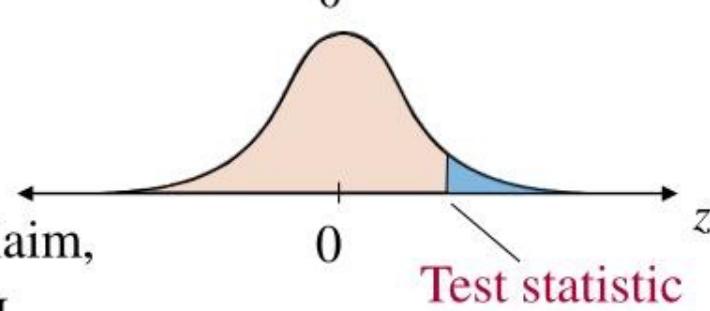
This sampling distribution is based on the assumption that H_0 is true.

4. Calculate the test statistic and its standardized value. Add it to your sketch.



5. Find the P -value.

6. If P -value \leq the level of significance then write a statement to interpret the decision in terms of the context of the claim, and Reject H_0 , otherwise Fail to reject H_0



Two-tailed Test

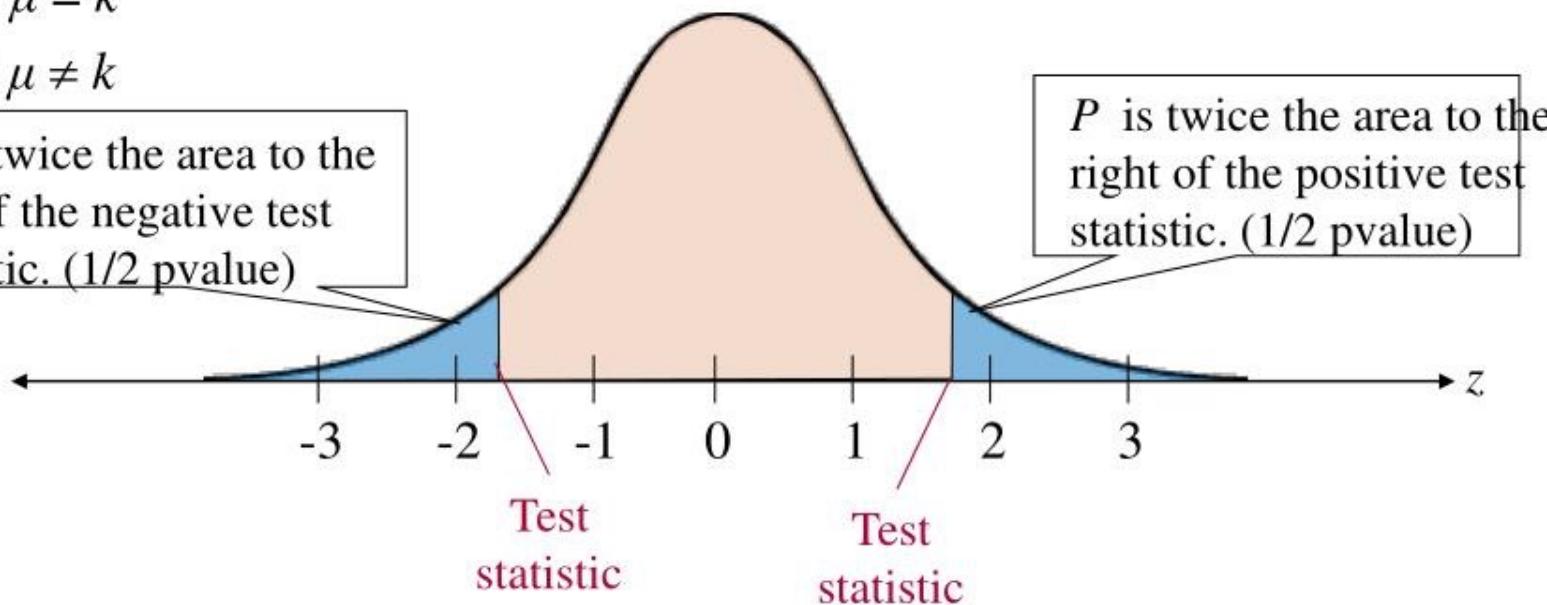
- The alternative hypothesis H_a contains the not equal inequality symbol (\neq). Each tail has an area of $\frac{1}{2}P$.

$$H_0: \mu = k$$

$$H_a: \mu \neq k$$

P is twice the area to the left of the negative test statistic. (1/2 pvalue)

P is twice the area to the right of the positive test statistic. (1/2 pvalue)



Example: University publicizes that the proportion of its students who graduate in 4 years is 82%.

$$H_0: p = .82$$

$$H_a: p \neq .82$$

Example1: Testing μ with a t-test

A used car dealer says that the mean price of a 2005 Honda Pilot LX is at least \$23,900. You suspect this claim is incorrect and find that a random sample of 14 similar vehicles has a mean price of \$23,000 and a standard deviation of \$1113. Is there enough evidence to reject the dealer's claim at $\alpha = 0.05$? Assume the population is normally distributed. (*Kelley Blue Book*)

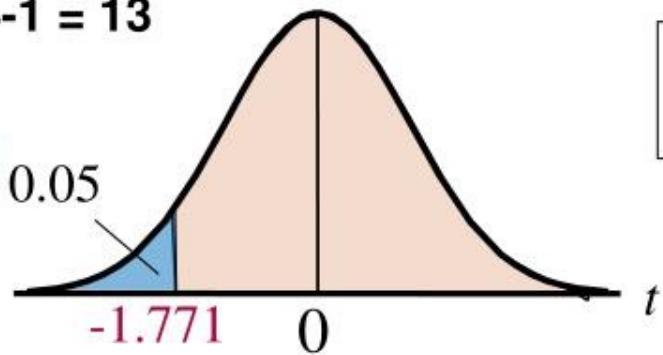
$$H_0: \mu \geq 23,900$$

$$H_a: \mu < 23,900$$

$$\alpha = .05$$

$$d.f. = 14 - 1 = 13$$

Rejection Region



$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{23,000 - 23,900}{1113/\sqrt{14}} \approx 3.026$$

(Negative)

Decision: Reject H_0
T-value is inside the rejection region

At the 0.05 level of significance, there is enough evidence to reject the claim that the mean price of a 2005 Honda Pilot LX is at least \$23,900

Using Rejection Regions to Test the Mean

Rejection region (or critical region)

- The range of values for which the null hypothesis is not probable.
- If a test statistic falls in this region, the null hypothesis H_0 is rejected.
- A critical value z_0 separates the rejection region from the non-rejection region.

Finding Critical Values in a Normal Distribution

1. Specify the level of significance α .
2. Decide whether the test is left-, right-, or two-tailed.
3. Find the critical value(s) z_0 . If the hypothesis test is
 - a. *left-tailed*, find the z -score that corresponds to an area of α ,
 - b. *right-tailed*, find the z -score that corresponds to an area of $1 - \alpha$,
 - c. *two-tailed*, find the z -score that corresponds to $\frac{1}{2}\alpha$ and $1 - \frac{1}{2}\alpha$.
4. Sketch the standard normal distribution. Draw a vertical line at each critical value and shade the rejection region(s).

Rejection regions for 2-tailed test
 $\alpha = .05$

$$\frac{1}{2}\alpha = 0.025$$

$$1 - \alpha = 0.95$$

$$\frac{1}{2}\alpha = 0.025$$

$$-z_0 = -1.96$$

$$z_0 = 1.96$$

z 19

Ch7: Hypothesis Testing (1 Sample)

- 7.1 Introduction to Hypothesis Testing
- 7.2 Hypothesis Testing for the Mean (σ known)
- 7.3 Hypothesis Testing for the Mean (σ unknown)
- 7.4 Hypothesis Testing for Proportions

Hypothesis

- A tentative assumption (sometimes a ‘guess’)

Stating a Hypothesis

Null hypothesis

- A statistical hypothesis
- Statement of equality (\leq , $=$, or \geq).
- Denoted H_0
- Read “H subzero” or “H naught.”

Alternative Hypothesis

(Complementary to Null Hypothesis)

- A statement of inequality ($>$, \neq , or $<$).
- Must be true if H_0 is false.
- Denoted H_a read “H sub-a.”

- Translate ***the claim*** made about the population parameter from a verbal statement to a mathematical statement, then write its complement. Assume $\mu = k$ & examine the sampling distribution on the basis of this assumption.

$$H_0: \mu \leq k$$

$$H_a: \mu > k$$

OR

$$H_0: \mu \geq k$$

$$H_a: \mu < k$$

OR

$$H_0: \mu = k$$

$$H_a: \mu \neq k$$

Example1: A university publicizes: Proportion students who graduate in 4 years is 82%. $H_0: p = .82$
 $H_a: p \neq .82$

Example2: A water faucet manufacturer announces: Mean flow rate of their faucet is less than 2.5 gal/min $H_0: \mu \geq 2.5$
 $H_a: \mu < 2.5$

Example3: A cereal company says: Mean weight of box is more than 20 oz $H_0: \mu \leq 20$
 $H_a: \mu > 20$

Example2: Hypothesis Testing Using *P*-values

You think that the average franchise investment information shown in the graph is incorrect, so you randomly select 30 franchises and determine the necessary investment for each. The sample mean investment is \$135,000 with a population standard deviation of \$30,000. Is there enough evidence to support your claim at $\alpha = 0.05$? Use a *P*-value.

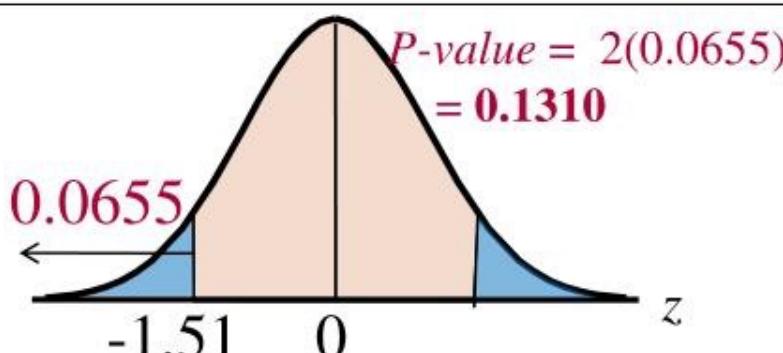


$$\begin{aligned}H_0: \mu &= 143,260 \\H_a: \mu &\neq 143,260\end{aligned}$$

$$\alpha = .05$$

$$\begin{aligned}z &= \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} \\&= \frac{135,000 - 143,260}{30,000 / \sqrt{30}} \\&= -1.51\end{aligned}$$

If *P*-value $\leq \alpha$
Then REJECT H_0



$$\text{Normalcdf}(-10000, -.151) = .0655$$

Decision: $.1310 > .05$
So, FAIL TO REJECT H_0

TI 83/84
Stat-Tests
1: Z-Test

At the 5% level of significance, there is not sufficient evidence to conclude the mean franchise investment is different from \$143, 260.

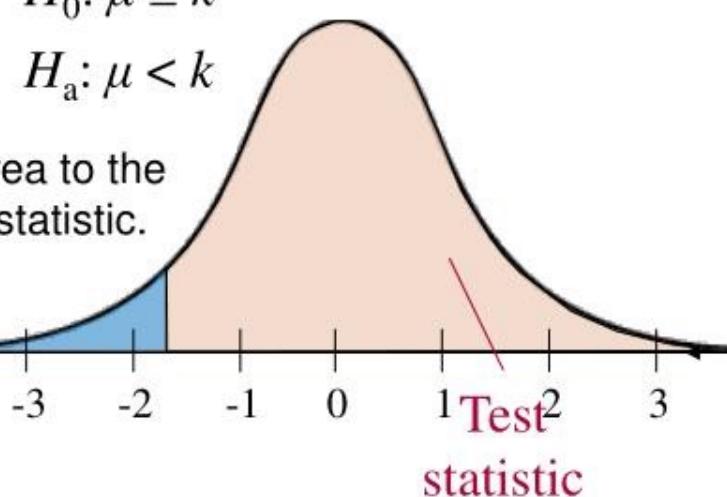
3 Types of Hypothesis Tests

Left Tailed Test: The alternative hypothesis H_a contains the less-than inequality symbol ($<$).

$$H_0: \mu \geq k$$

$$H_a: \mu < k$$

P-value area to the left of the statistic.



A water faucet manufacturer announces that the mean flow rate of a certain type of faucet is less than 2.5 gallons per minute.

$$H_0: \mu \geq 2.5$$

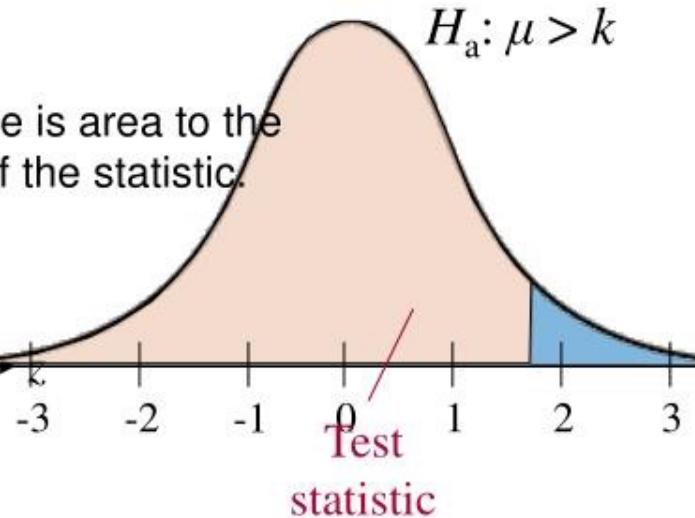
$$H_a: \mu < 2.5$$

Right Tailed Test: The alternative hypothesis H_a contains the less-than inequality symbol ($>$).

$$H_0: \mu \leq k$$

$$H_a: \mu > k$$

P-value is area to the right of the statistic.



A cereal company says: Mean weight of box is more than 20 oz.

$$H_0: \mu \leq 20$$

$$H_a: \mu > 20$$

Example: Interpreting a Decision



You perform a hypothesis test for the following claim. How should you interpret your decision if you reject H_0 ? If you fail to reject H_0 ?

2. H_a (**Claim**): *Consumer Reports* states that the mean stopping distance (on a dry surface) for a Honda Civic is less than 136 feet.

Solution:

- The claim is represented by H_a .
- H_0 is “the mean stopping distance...is greater than or equal to 136 feet.”
- If you reject H_0 , you should conclude “there is enough evidence to support *Consumer Reports*’ claim that the stopping distance for a Honda Civic is less than 136 feet.”
- If you fail to reject H_0 , you should conclude “there is not enough evidence to support *Consumer Reports*’ claim that the stopping distance for a Honda Civic is less than 136 feet.”

Hypothesis Tests

Hypothesis test

- A process that uses sample statistics to test a claim about the value of a population parameter.
- **Example:**

Claim: An automobile manufacturer advertises that its new hybrid car has a mean mileage of 50 miles per gallon.

Test the Claim: A sample would be taken. If the sample mean differs enough from the advertised mean, you can decide the advertisement is wrong.

Statistical hypothesis

- A statement, or claim, about a population parameter.
- Need a pair of hypotheses (called H_0 and H_a)
 - one that represents the claim (**Mean mileage = 50**)
 - the other, its complement (**Mean mileage \neq 50**)
- When one of these hypotheses is false, the other must be true.

Using *P*-values for a z-Test for Mean μ

Use when: Population is normal and σ is known, or any sample when $n \geq 30$.

Test statistic is the sample mean \bar{x} => **Standardized test statistic** is $z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$

In Words

In Symbols

- | | |
|---|--|
| 1. State the claim mathematically and verbally.
Identify the null and alternative hypotheses. | State H_0 and H_a . |
| 2. Specify the level of significance. | Identify α . |
| 3. Determine the standardized test statistic.
(Note: s can be used for σ if $n \geq 30$) | $z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$ |
| 4. Find the area that corresponds to z . | normalcdf() or Std. Norm table |
| 5. Find the <i>P</i> -value (area in left/right tail(s)) | |
| 6. Make a decision to reject or fail to reject the null hypothesis. | If <i>P</i> -value $\leq \alpha$ Reject H_0
Otherwise, fail to reject H_0 . |
| 7. Interpret the decision <u>in the context of the original claim</u> . | |

Example3: Testing with Rejection Regions

Employees in a large accounting firm claim that the mean salary of the firm's accountants is less than that of its competitor's, which is \$45,000. A random sample of 30 of the firm's accountants has a mean salary of \$43,500 with a population standard deviation \$5200. At $\alpha = 0.05$, test the employees' claim.

$$H_0: \mu \geq 45000$$

$$H_a: \mu < 45000$$

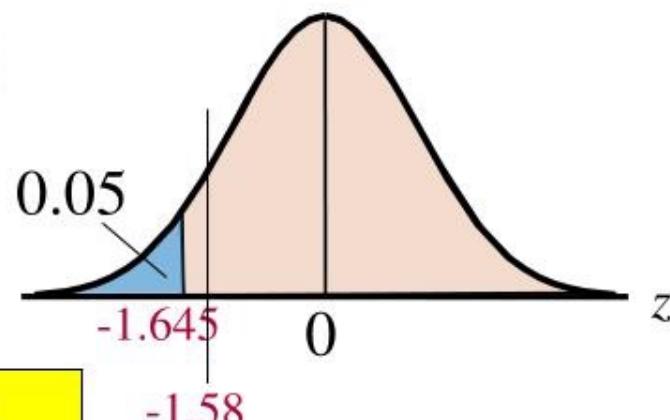
$$\alpha = .05$$

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{43,500 - 45,000}{5200/\sqrt{30}} \\ = -1.58$$

invNorm (.05) = -1.645
(Rejection region z-boundary)

-1.58 is not in the rejection region
So, **FAIL TO REJECT H_0**

At the 5% level of significance, there is not sufficient evidence to support the employee's claim that the mean salary is less than 45000.



7.2 Hypothesis Testing (σ known)

Finding a P-value

After determining the hypothesis test's standardized test statistic and the test statistic's corresponding area, do one of the following to find the P -value.

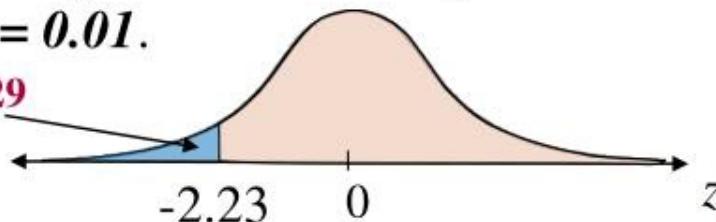
- a. Left-tailed test, $P = (\text{Area in left tail})$.
- b. Right-tailed test, $P = (\text{Area in right tail})$.
- c. Two-tailed test, $P = 2(\text{Area in tail of test statistic})$.

If $P\text{-value} \leq \alpha$
Then REJECT H_0

Example (Left Tail)

Find the P -value for a left-tailed hypothesis test with a test statistic of $z = -2.23$. Decide whether to reject H_0 if the level of significance is $\alpha = 0.01$.

$$P = 0.0129$$



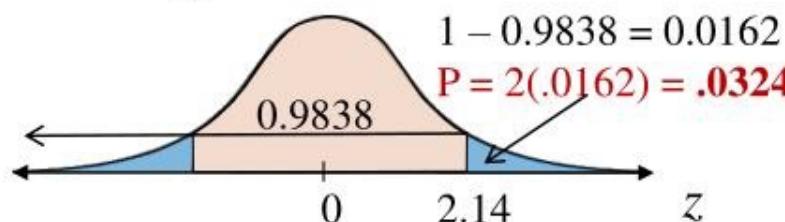
Because $0.0129 > 0.01$, you should **fail to reject H_0**

Example (Two-Tail)

Find the P -value for a two-tailed hypothesis test with a test statistic of $z = 2.14$. Decide whether to reject H_0 if the level of significance is $\alpha = 0.05$.

$$1 - 0.9838 = 0.0162$$

$$P = 2(0.0162) = .0324$$



Because $0.0324 < 0.05$, you should **reject H_0**

Example2: Testing μ with a t-test

An industrial company claims that the mean pH level of the water in a nearby river is 6.8. You randomly select 19 water samples and measure the pH of each. The sample mean and standard deviation are 6.7 and 0.24, respectively. Is there enough evidence to reject the company's claim at $\alpha = 0.05$? Assume the population is normally distributed.

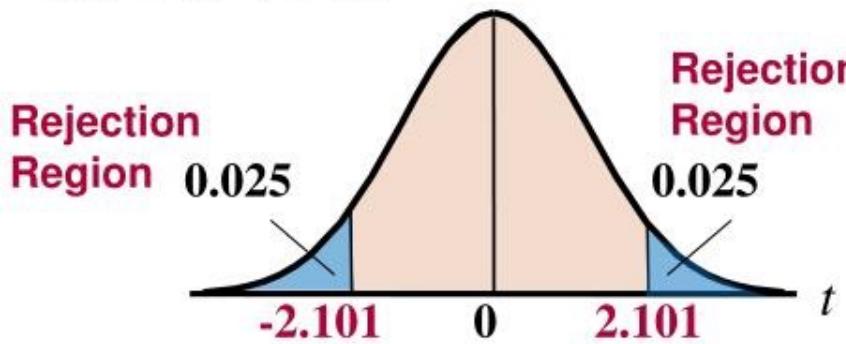
$$H_0: \mu = 6.8$$

$$H_a: \mu \neq 6.8$$

$$\alpha = .05$$

$$d.f. = 19 - 1 = 18$$

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{6.7 - 6.8}{0.24/\sqrt{19}} \approx -1.816$$



Decision: Fail to Reject H_0
T-value NOT inside the rejection region

At the 0.05 level of significance, there is not enough evidence to reject the claim that the mean pH is 6.8.

Example: Identifying Type I and Type II Errors

The USDA limit for salmonella contamination for chicken is 20%. A meat inspector reports that the chicken produced by a company exceeds the USDA limit. You perform a hypothesis test to determine whether the meat inspector's claim is true. When will a type I or type II error occur? Which is more serious? (*U.S. Department of Agriculture*)

Let p = proportion of chicken contaminated.

$$H_0: p \leq .2$$

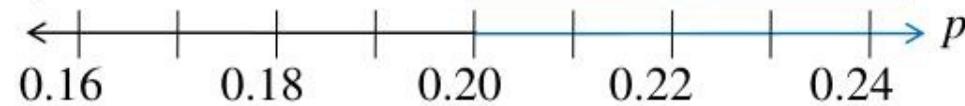
$$H_a: p > .2$$

Chicken meets
USDA limits.

$$H_0: p \leq 0.20$$

Chicken exceeds
USDA limits.

$$H_A: p > 0.20$$



α = Probability of Type I error
 β = Probability of Type II error

Usually we set α low (.10, .05, .01)

Type I error: Rejecting H_0 when it is true.

The actual proportion of contaminated chicken is less than or equal to 0.2, but you decide to reject H_0 .

Risk: Might create a health scare and hurt the sales of chicken producers who were actually meeting the USDA limits.

Type II error: Failing to reject H_0 when it is false.

The actual proportion of contaminated chicken is greater than 0.2, but you do not reject H_0 .

Risk: Allows chicken exceeding limits to be sold resulting in sickness or death.

Types of Errors

- No matter which hypothesis represents the claim, always begin the hypothesis test **assuming that the equality condition in the null hypothesis is true.** At the end of the test, one of two decision will be made (based on a sample):
 - Reject the null hypothesis***
 - Fail to reject the null hypothesis***

Possibility of Wrong Decision!

A **type I error** occurs if the null hypothesis is rejected when it is true.

A **type II error** occurs if the null hypothesis is not rejected when it is false.

Comparable to U.S. legal system:

- The defendant is '**assumed innocent**' (H_0)
- Until '**proven guilty**' (H_a).

Burden of proof lies with prosecution.

If evidence is not strong enough then no conviction. “Not guilty” verdict does not prove that a defendant is innocent. The evidence needs to be conclusive beyond a reasonable doubt.

The system assumes that more harm is done by :

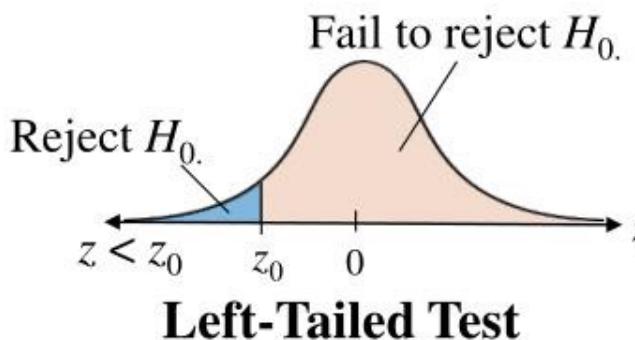
Type I error: Convicting the innocent rather than by

Type II error : Not convicting the guilty.

Decision Rule Based on Rejection Region

To use a rejection region to conduct a hypothesis test, calculate the standardized test statistic, z . If the standardized test statistic

1. is in the rejection region, then reject H_0 .
2. is *not* in the rejection region, then fail to reject H_0 .



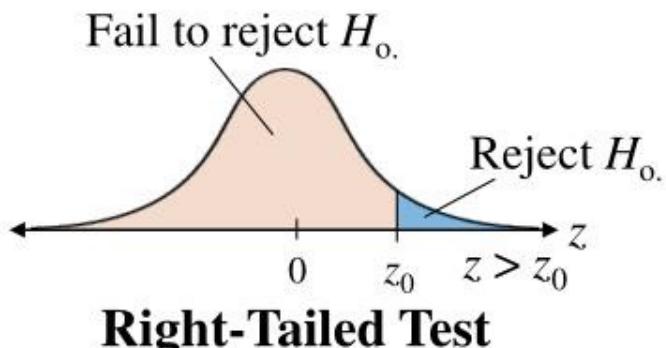
Fail to reject H_0

z_0

z_0

z_0

Left-Tailed Test

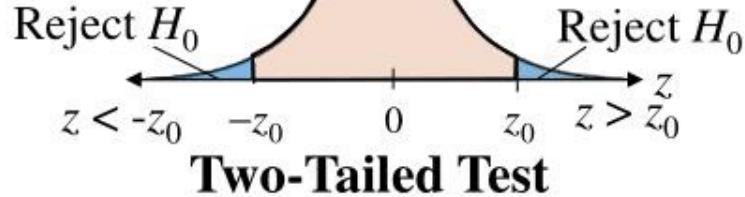


Fail to reject H_0

Reject H_0

0

Right-Tailed Test



Two-Tailed Test

Example 3: Using *P*-values & Technology

The American Automobile Association claims that the mean daily meal cost for a family of four traveling on vacation in Florida is \$118. A random sample of 11 such families has a mean daily meal cost of \$128 with a standard deviation of \$20. Is there enough evidence to reject the claim at $\alpha = 0.10$? Assume the population is normally distributed. (*Adapted from American Automobile Association*)

$$H_0: \mu = 118$$

$$H_a: \mu \neq 118$$

$$\alpha = .10$$

$$d.f. = 11 - 1 = 10$$

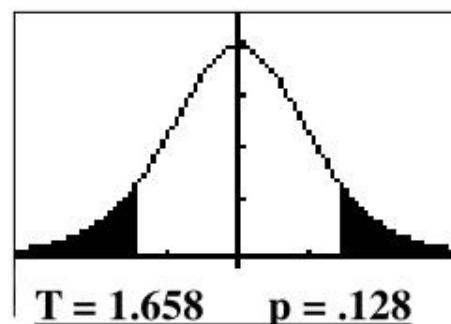
TI-83/84 set up:

T-Test
Inpt: Data **STATS**
 μ_0 : 118
 \bar{x} : 128
 S_x : 20
 n : 11
 μ : **<μ₀** **>μ₀**
Calculate Draw

Calculate:

T-Test
 $\mu \neq 118$
 $t = 1.658312395$
 $p = .1282459922$
 $\bar{x} = 128$
 $S_x = 20$
 $n = 11$

Draw:



Decision: Fail to Reject H_0

P-value: $.1664 > \alpha = .10$

At the 0.10 level of significance, there is not enough evidence to reject the claim that the mean daily meal cost for a family of four traveling on vacation in Florida is \$118.

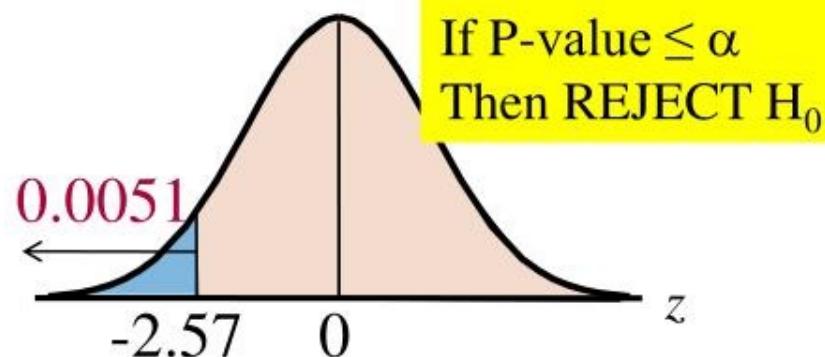
Example1: Hypothesis Testing Using *P*-values

In an advertisement, a pizza shop claims that its mean delivery time is less than 30 minutes. A random selection of 36 delivery times has a sample mean of 28.5 minutes and a population standard deviation of 3.5 minutes. Is there enough evidence to support the claim at $\alpha = 0.01$? Use a *P*-value.

$$\begin{aligned}H_0: \mu &\geq 30 \text{ min} \\H_a: \mu &< 30 \text{ min}\end{aligned}$$

$$\alpha = .01$$

$$\begin{aligned}z &= \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} \\&= \frac{28.5 - 30}{3.5 / \sqrt{36}} \\&= -2.57\end{aligned}$$



$$\text{Normalcdf } (-10000, -.2.57) = .0051 \text{ [P-value]}$$

Decision: $.0051 < .01$
So, **REJECT H_0**

At the 1% level of significance, you have sufficient evidence to conclude the mean delivery time is less than 30 minutes.

**TI 83/84
Stat-Tests
1: Z-Test**

Making a Decision

Decision Rule Based on P -value

- Compare the P -value with α .
 - If $P \leq \alpha$, then reject H_0 .
 - If $P > \alpha$, then fail to reject H_0 .

	Claim	
Decision	Claim is H_0	Claim is H_a
Reject H_0	There is enough evidence to reject the claim	There is enough evidence to support the claim
Fail to reject H_0	There is not enough evidence to reject the claim	There is not enough evidence to support the claim

7.4 Hypothesis Testing for a Proportion

z-Test: Use when binomial distribution & $np \geq 5$ and $nq \geq 5$

Test statistic: \hat{p} Standardized test statistic is z .
$$z = \frac{\hat{p} - \mu_{\hat{p}}}{\sigma_{\hat{p}}} = \frac{\hat{p} - p}{\sqrt{pq/n}}$$

-
1. State the claim mathematically and verbally.
Identify the null and alternative hypotheses.
State H_0 and H_a .
 2. Specify the level of significance.
Identify α .
 3. Sketch the sampling distribution.
 4. Determine any critical value(s).
Use Table 5 in Appendix B.
 5. Determine any rejection region(s).
 6. Find the standardized test statistic.
$$z = \frac{\hat{p} - p}{\sqrt{pq/n}}$$
 7. Make a decision to reject or fail to
reject the null hypothesis & interpret the
decision in the context of the claim.
If z is in the rejection region,
reject H_0 .



Example: Interpreting a Decision

You perform a hypothesis test for the following claim. How should you interpret your decision if you reject H_0 ? If you fail to reject H_0 ?

1. H_0 (Claim): A university publicizes that the proportion of its students who graduate in 4 years is 82%.

- The claim is represented by H_0 .
- If you reject H_0 , you should conclude “there is sufficient evidence to indicate that the university’s claim is false.”
- If you fail to reject H_0 , you should conclude “there is insufficient evidence to indicate that the university’s claim (of a four-year graduation rate of 82%) is false.”

Example2: Hypothesis Test for Proportions

The Pew Research Center claims that more than 55% of U.S. adults regularly watch their local television news. You decide to test this claim and ask a random sample of 425 adults in the United States whether they regularly watch their local television news. Of the 425 adults, 255 respond yes. At $\alpha = 0.05$ is there enough evidence to support the claim?

- Verify that $np \geq 5$ and $nq \geq 5$. $np = 425(0.55) = 234$ and $nq = 425(0.45) = 191$

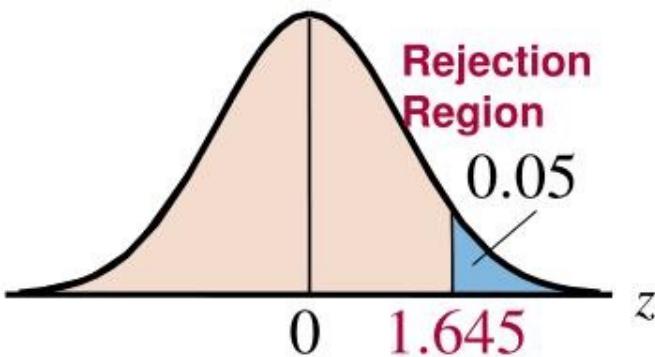
$$H_0: p \leq .55$$

$$H_a: p > .55$$

$$\alpha = .05$$

$$z = \frac{\hat{p} - p}{\sqrt{pq/n}} = \frac{255/425 - 0.55}{\sqrt{(0.55)(0.45)/425}} \\ = 2.07$$

TI 83/84
Stat-Tests
5:1-PropZTest



Decision: Reject H_0
z-value is Inside the rejection region

At the 5% level of significance, there is enough evidence to support the claim that more than 55% of U.S. adults regularly watch their local television news.

Statistical Tests

- After stating the null and alternative hypotheses and specifying the level of significance (α) a random sample is taken from the population and sample statistics are calculated.
- The statistic that is compared with the parameter in the null hypothesis is called the **test statistic**.

Population parameter	Test statistic	Standardized test statistic
μ	\bar{x}	z (Section 7.2 $n \geq 30$) t (Section 7.3 $n < 30$)
p	\hat{p}	z (Section 7.4)

P-value (or probability value)

- The probability, if the null hypothesis is true, of obtaining a sample statistic with a value as extreme or more extreme than the one determined from the sample data. (Depends on the nature of the test).

***t*-Test for a Mean μ ($n < 30$, σ Unknown)**

Use when: Population is ‘nearly’ normal, σ is unknown, and $n < 30$.

Test statistic is the sample mean \bar{x} => **Standardized test statistic** is $t = \frac{\bar{x} - \mu}{s/\sqrt{n}}$

1. State the claim.
Identify the null and alternative hypotheses. State H_0 and H_a .
2. Specify the level of significance.
Identify α .
3. Identify the degrees of freedom and sketch the sampling distribution.
 $d.f = n - 1$.
4. Determine any critical value(s).
Use Table 5 in Appendix B.
InvT()
5. Determine any rejection region(s).
6. Find the standardized test statistic.
$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}}$$
7. Make a decision to reject or fail to reject the null hypothesis.
If t is in the rejection region, reject H_0 .
Otherwise, fail to reject H_0 .
8. Interpret the decision in the context of the original claim.

7-3 Hypothesis Testing (σ unknown) t-Distribution

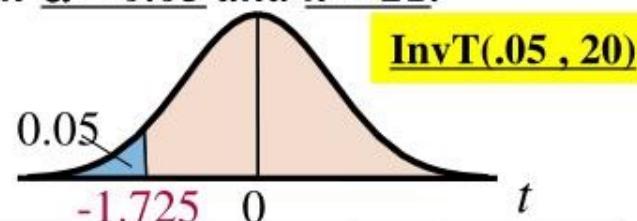
Finding Critical Values using t-Distribution

1. Identify α . Level and degrees of freedom d.f. = $n - 1$.
2. Find the critical value(s) using Table 5 in Appendix B in the row with $n - 1$ degrees of freedom. If the hypothesis test is
 - a. left-tailed, use “One Tail, α ” column with a negative sign,
 - b. right-tailed, use “One Tail, α ” column with a positive sign,
 - c. two-tailed, use “Two Tails, α ” column with a negative and a positive sign.

InvT()

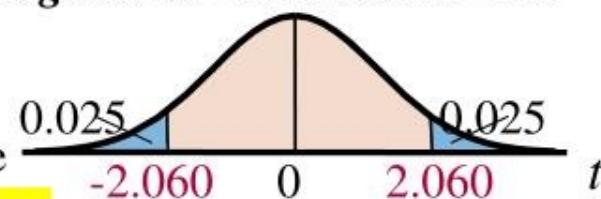
Example1:Find the critical value t_0 for a left-tailed test given $\alpha = 0.05$ and $n = 21$.

- The degrees of freedom are d.f. = $n - 1 = 21 - 1 = 20$.
- Look at $\alpha = 0.05$ in the “One Tail, α ” column.
- Because the test is left-tailed, the critical value is negative.



Example2:Find the critical values t_0 & $-t_0$ for a two-tailed test given $\alpha = 0.05$ and $n = 26$.

- The degrees of freedom are d.f. = $n - 1 = 26 - 1 = 25$.
- Look at $\alpha = 0.05$ in the “Two Tail, α ” column.
- Test is two-tailed, so 1 critical value is negative & 1 is positive



InvT(.025 , 24)

Example1: Hypothesis Test for Proportions

Zogby International claims that 45% of people in the United States support making cigarettes illegal within the next 5 to 10 years. You decide to test this claim and ask a random sample of 200 people in the United States whether they support making cigarettes illegal within the next 5 to 10 years. Of the 200 people, 49% support this law. At $\alpha = 0.05$ is there enough evidence to reject the claim?

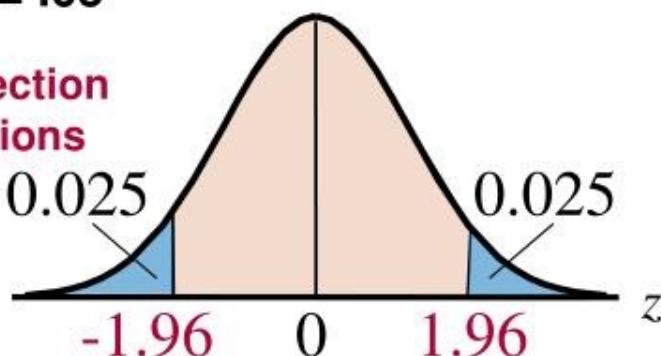
- Verify that $np \geq 5$ and $nq \geq 5$. $np = 200(0.45) = 90$ and $nq = 200(0.55) = 110$

$$H_0: p = .45$$

$$H_a: p \neq .45$$

$$\alpha = .05$$

Rejection Regions



$$z = \frac{\hat{p} - p}{\sqrt{pq/n}} = \frac{0.49 - 0.45}{\sqrt{(0.45)(0.55)/200}}$$
$$\approx 1.14$$

Decision: Fail to Reject H_0
z-value NOT inside the rejection region

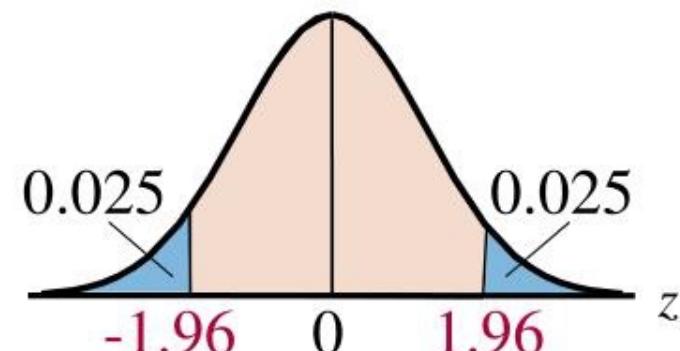
At the 5% level of significance, there is not enough evidence to reject the claim that 45% of people in the U.S. support making cigarettes illegal within the next 5 to 10 years.

Example4: Testing with Rejection Regions

The U.S. Department of Agriculture reports that the mean cost of raising a child from birth to age 2 in a rural area is \$10,460. You believe this value is incorrect, so you select a random sample of 900 children (age 2) & find the mean cost is \$10,345 with a population standard deviation \$1540. At $\alpha = 0.05$, is there enough evidence to conclude that the mean cost is different from \$10,460? (*Adapted from U.S. Department of Agriculture Center for Nutrition Policy and Promotion*)

$$\begin{aligned} H_0: \mu &= 10,460 \\ H_a: \mu &\neq 10,460 \\ \alpha &= .05 \end{aligned}$$

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{10,345 - 10,460}{1540/\sqrt{900}} = -2.24$$



invNorm (.025) = -1.96 and invNorm (.975) = 1.96
(Rejection region z-boundaries)

Decision: -2.24 is inside the rejection region, So, **REJECT H_0**

At the 5% level of significance, you have enough evidence to conclude the mean cost of raising a child from birth to age 2 in a rural area is significantly different from 10,460