ECON 33530 - Firm Dynamics and Economic Growth
Winter 2023
Problem Set 2

In this problem set, we will replicate **a version of panel (a) of Figure 8** in NBER working paper 25756 - Akcigit and Ates (2019) - *What happened to US business dynamism?*. In theory, growth[1] and knowledge diffusion are related with an inverse-U relationship. That is, the effect of knowledge diffusion on GDP growth is not monotonic. There are two counteracting effects

1. Incentive effect
2. Decomposition effect

By the incentive effect, higher knowledge diffusion decreases firm incentives to innovate. Therefore, lower innovation causes lower growth in equilibrium. On the other hand, higher knowledge diffusion changes the sector decomposition in the economy. More precisely, it increases the number of more competitive sectors (sectors with lower productivity gaps $m = 0, 1, \dots$ between the leader and the follower) in which investment in R&D is higher due to strong escape competition effect. If the mass (number) of such sectors increases in equilibrium, aggregate growth might increase, since average innovation rate increases. In sum, these two effects are working in opposite directions.

In this exercise, we will explore the relationship between knowledge diffusion and the growth rate of GDP. Instead of solving for welfare as in Figure 8, we will focus on **annual** BGP growth rate of aggregate output.[2] In particular, we will create a figure where we have knowledge diffusion given by δ parameter on the x-axis, and corresponding equilibrium annual BGP growth rate on the y-axis.

1. Parameter values: Use initial BGP parameter values from the paper. In particular, set the following

[1] From now on, we will focus only on BGP equilibria, and hence **annual** BGP growth rates.

[2] In this class of models, growth and welfare are strongly positively correlated as one would easily guess.

values (δ is excluded)

$$
\begin{aligned}
\rho &= 0.05 \\
\gamma = \tilde{\gamma} &= 1/0.35 \\
\tau &= 0.30 \\
s &= 0.05 \\
\alpha &= 7.179 \\
\tilde{\alpha} &= 0.075 \\
\lambda &= 1.044 \\
\phi = \tilde{\phi} &= 0.0423
\end{aligned}
$$

Note that, in BGP equilibrium, parameters are constant over time. So, we do not consider any transition argument in this exercise. We will compare different BGPs with each other.

2. Create a vector for a possible set of values for δ. In particular, consider a linear space (MATLAB's linspace or Julia's range) between 0.015 and 5.0 with 15 (or more) equispaced points (including 0.015 and 5.0).

3. Keeping other parameters constant, find a numerical mapping between knowledge diffusion (δ) and annual growth rate. Hint: For every possible set of parameter values with respect to δ, solve for corresponding BGP, and calculate equilibrium annual growth rate.

4. Provide a description for annual growth rate in this kind of a continuous time growth model. Hint: Annual growth rate is a compounded growth rate of GDP. Every year is divided into subperiods. We already have an analytical expression for the period-to-period growth rate (see paper). You need to compound it up to the annual level taking into account the length of subperiods (dt's).[3]

5. Hints for solving the BGP:

   (a) Solving BGP is a fixed point problem. Consider **nested "for loops"** to obtain a solution of endogenous variables of interest. These are normalized wage rate, value functions, innovation rates, and sector distribution.[4] Note that value functions, innovation rates and sector distribution are functions of productivity gap $m = 0, 1, \ldots$.

   (b) You should restrict your state space to a finite set. Set $\bar{m} = 100$. So the state space for value functions and distribution is $\{0, 1, 2, \ldots, \bar{m} - 1, \bar{m}\}$. We can numerically represent value functions, innovation rates and sector distribution as vectors of length $\bar{m} + 1 = 101$.[5]

[3] You can report periodic growth rate rather than the annual one in your figure. However, annual growth rate is what we observe in the data.

[4] Normalized wage ω is defined as wage divided by GDP. Note that ω also corresponds to labor share in this model. Note also that following inequality always holds in equilibrium

$$0 < \omega < 1$$

[5] I prefer separating unleveled sectors from neck-and-neck sectors. So, one possibility might be to represent value functions, innovation rates and sector distribution only for the unleveled sectors as vectors of length 100. Those of neck-and-neck sectors can be considered as separate numbers rather than being included in previous vectors. In Julia, you do not need to vectorize your code for it to work faster. Therefore, having Float's do not hurt. Instead it makes your code more readable. However, putting everything into a vector is the most straightforward implementation. Both approaches are fine.

(c) Let $dt = 1/50$. That is, we divide every year into 50 subperiods.

(d) We will have nested "for loops" to solve for the BGP: One outer loop, and two separate inner loops with the same hierarchy/level.

(e) The most outer loop is a (normalized) **wage loop**. Guess a wage rate between 0 and 1, solve all other endogenous variables in the loop as will be explained below. Then find **implied wage** rate that clears the labor market. If it is close enough to initial guess, then break the loop. If not, update your guess as a weighted average of the implied wage and the initial guess. You can use 0.25 as the weight for the implied wage.

(f) Inside the wage loop, we should have two separate loops with the same hierarchy. These loops do not coincide/intersect with each other.

(g) First inner loop is the **value function iteration (VFI)**. Given a wage guess, guess the corresponding value functions for leader, follower, and neck-and-neck firms. Iterate Bellman equations to update them. If they are close enough, then break the loop and continue with the second inner loop explained below. If not, update your guess for the value function with the new one. Continue until they converge. You can use zero vectors as your initial guess. I recommend the implicit method over the explicit method.

(h) Second inner loop is the **distribution iteration**. After solving for values and corresponding innovation rates, you can get in the second loop to solve for sector distribution over gaps $m = 0, \dots, \bar{m}$. Similarly, guess the distribution. Uniform distribution works fine as an initial guess. Then using innovation rates, update the distribution. I recommend using explicit method in this loop. Continue until they converge.

(i) Last step in the wage loop after solving for value functions, innovations and distribution that correspond to the initial wage guess is to find the **implied wage** which is the wage rate that clears the labor market. In bullet point (e), I discuss how you can update the initial wage guess.

(j) Once you break the wage loop (convergence is achieved), you have the BGP for a specific set of parameters that you have chosen at the very beginning. Now, you can calculate the growth rate as well as other moments such as average markup and profit share.

(k) You can pick $1e-8$ as the tolerance for value function and distribution iterations. Similarly, you can pick $1e-4$ as the tolerance for the wage iteration.

6. Discuss your findings. Is your figure an inverse-U? Is the initial BGP value of δ in the paper on the upward sloping or the downward sloping part of the figure? Interpret your results taking into account the main conclusion of the paper regarding declining knowledge diffusion over time. Before solving the full transition equilibrium, can you use this relatively simpler figure to speculate on how the declined knowledge over time has affected the welfare since 1980?

7. Not included to the assignment: Having paid the huge sunk cost of coding up the BGP, you can try other BGP moments (not only growth) with respect to δ as well as to other parameters. That is, you can create a sensitivity table that shows percent changes in moments after a 1% increase in a single parameter. (Table D.1 at the end of the paper is the sensitivity table) With the help of such a table, you can test your intuition, at least qualitatively.