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605.621 Algorithms

Project 2

Introduction

This project implements quicksort, heapsort, and introsort. It also analyzes their empirical run times compared to their worst case.

A git repo of the project is at: https://github.com/jmr172/algorithms/tree/master/Project_2

This project was made with Java 8. It can be run from the command line using the following from within the src directory:

```
$ javac Project2.java && java Project2 ../input/in.txt ../output/out.txt; cat ../output/out.txt
```

* in.txt is the input for the closest pairs.
  * input files with 10, 100, 1000, and 10000 data points are provided.
* out.txt contains the output of the closest pairs calculation if necessary

All my data was generated by the python script I wrote called random_data_generator.py.

To run this program, use:

```
python3 random_data_generator.py [number_of_points]
```

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Intro to Introsort

Part 1 Pseudocode and run-time analysis:

Quicksort:

// Reference: https://courses.cs.washington.edu/courses/cse373/01sp/Lect18_2up.pdf

// Java is pass by reference, so all operations are done in place on the original array

Quicksort is a divide-and-conquer approach to sorting an array. The first step is to pick an element in the array called the pivot. In our case, we always choose the last element. Next, we move every element with a value less than the pivot to the left of the pivot and every element with a value greater than the pivot to the right of the pivot. This guarantees that the pivot is in the correct position. Then we call quicksort on the sub-array that is left of the pivot and the sub-array that if right of the pivot. Since this is all done in place, there is no rebuilding step afterwards.

The partitioning step requires iteration through the entire subarray that was passed. This iteration will be for n elements the first time. It will also be for n elements when you sort the left sub-array and the right sub-array. Falling this logic, the performance is dependent on how many recursives call must be made. In the absolutely worst case, quicksort will produce always produce a sub-array that is of size 1 less than was passed to quicksort, resulting in n iterations n times which results in O(n^2). This will happen when the array is sorted in descending order. It will move the pivot to the very front of the sub-array each time, resulting in the maximum amount of recursive calls.

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```
Global int[] input

Quicksort(int x, int y):

    // Base condition to escape recursive call

    If (x > y):

        Return

    Else:

        int sorted_index = partition(x, y)

        quicksort(x, sorted_index – 1)

        quicksort(sorted_index + 1, y)

partition(int x, int y):

    // choose pivot as last element

    Int pivot = input[y]

    // keep track of where the pivot will end up

    Int index;

    For val in sub_array:

        If val < pivot:

            // move val to front

            Index++;

    // move pivot to index

    Swap(index, y)

    Return (index)
```

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Heapsort:

// Referenced a slide deck from RIT https://www.cs.rit.edu/~lr/courses/alg/student/1/heapsort.pdf

Heapsort consists first of building a max heap in O(n) time. Then we remove the root element in O(1) time and recreate the max heap in O(lg n) time. We repeat this process n times. This results in a worst case of O(n) + O(n lg n) -> O(n lg n).

```
Global int[] input

heapsort():

    // build max heap

    Heap = Build_max_heap(input)

    For (int i = input.length; I > 0; i++):

        Input[i] = heap.root

        Heap = Heapify(heap)

heapify(heap):

    if root.left > root.val:

        swap(root, root.left)

        heapify(heap)

        return

    if root.right > root.val:

        swap(root, root.right)

        heapify(heap)

        return
```

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Introsort:

// Reference: https://en.wikipedia.org/wiki/Introsort

Introsort combined quicksort, heapsort, and insertion sort. We choose a minimum array length and once the subarrays reach the threshold, insertion sort is used to sort the sub array. We use quicksort until the number of recursive calls have passed another threshold, which we then switch to heapsort. Heapsort is not recursive, so it limits the memory usage of the sorting algorithm.

Quicksort is normally O(n log n) but has a worst case of O(n^2). The worst case is caused by doing n recursive calls and making minimum progress on each. This is addressed by putting an upper limit on the number of recursive calls allowed before switching to heapsort. The worst case for heapsort is O(n log n). Insertion sort's worst case is O(n^2).

If n is less than the threshold, only insertion sort will be called, resulting in a worst case of O(n^2). That isn't very practical because that would limit it to very small data sets. When n becomes large, the complexity is dominated by quicksort and heapsort, resulting in O(n log n).

// Pseudocode references quicksort and heapsort functions from previous pseudocode

```
Global int[] input

introsort(int x, int y, recursive_count):

    // build max heap

    If ( y – x < insertion_limit) :

        Insertion_sort(x, y)

    elif (recursive_count == 0):

        heapsort()

    else:

        int sorted_index = partition(x, y)

        introsort(x, sorted_index – 1, recursive_count - 1)

        introsort(sorted_index + 1, y, recursive_count - 1)
```

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Runtime analysis:

Below is a table of the number of comparisons on n sized data sets sorted in descending vs random order:

	Random order					
Input Size	10	100	500	1000	5000	10000
Quicksort	19	645	5102	10963	68702	164619
Heapsort	33	629	4266	9579	59599	129123
Introsort	18	668	5505	11626	72224	172393
n log n	33	664	4483	9966	61439	132877
n^2	100	10000	250000	1000000	25000000	1E+08

	Descending order					
Input Size	10	100	500	1000	5000	10000
Quicksort	45	4950	124750	499500	12497500	49995000
Heapsort	26	566	3926	8816	55936	121696
Introsort	45	1617	11698	26575	175584	381275
n log n	33	664	4483	9966	61439	132877
n^2	100	10000	250000	1000000	25000000	1E+08

All three sorting algorithms run in O(n log n) on random data, but this clearly falls apart when the data is sorted in descending order. This is the worst case for quicksort, and it always takes 0.5*n^2 comparisons. Introsort also takes a small performance hit on descending order data due to utilizing quicksort at first, but it avoids losing the asymptotic relationship to O(n log n) even in this case. This point is illustrated in the following graphs:

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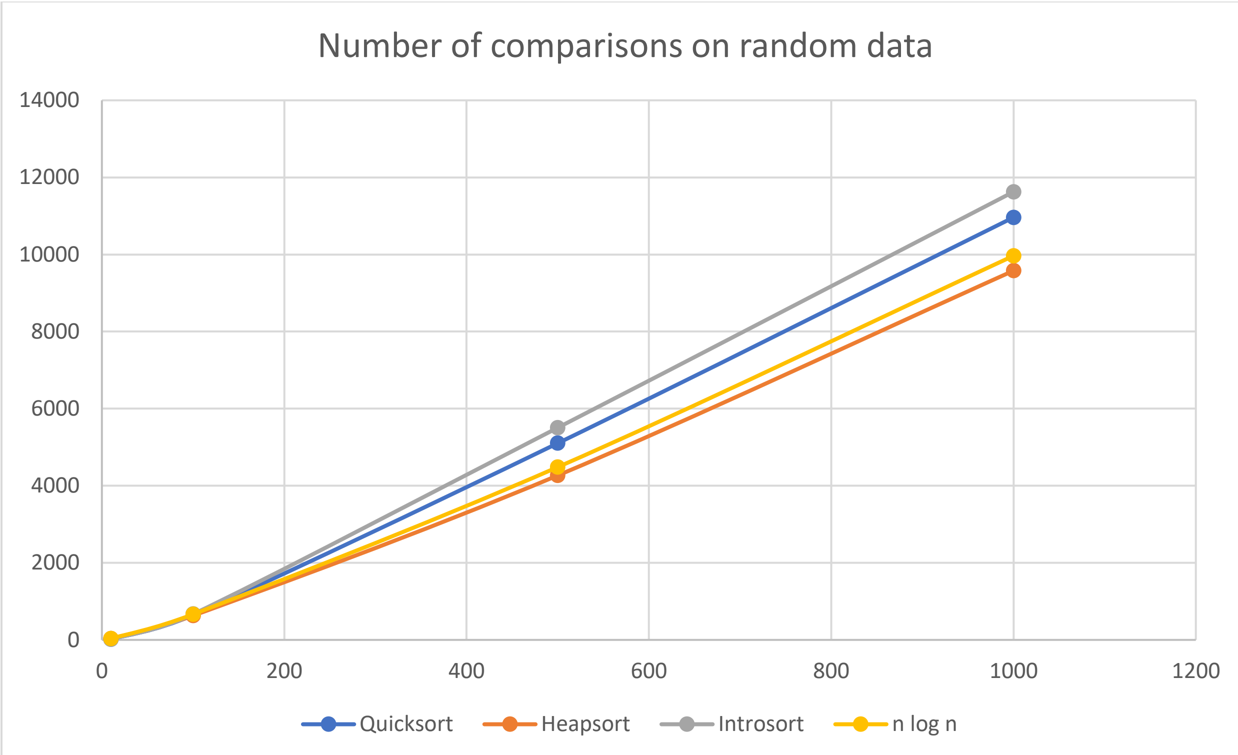
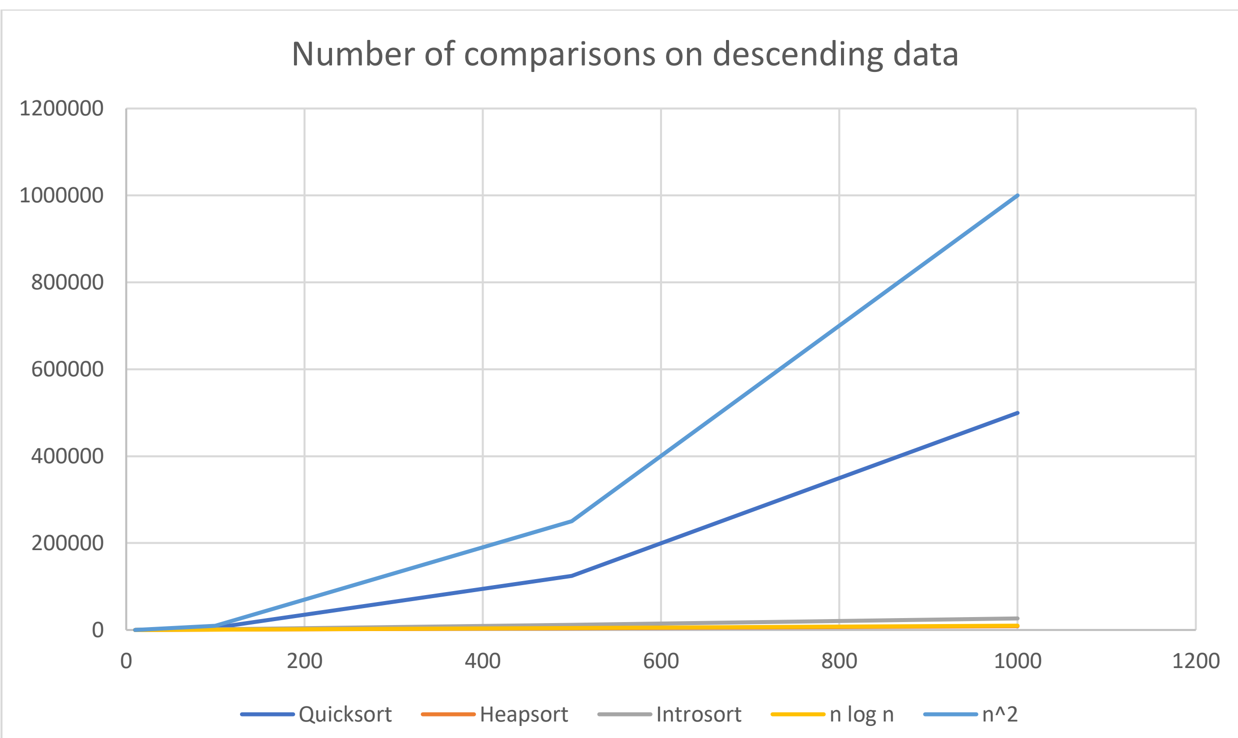


From the graph, it is clear that on random data, they all asymptotically track n log n. However, for descending data both linearly and logarithmically:



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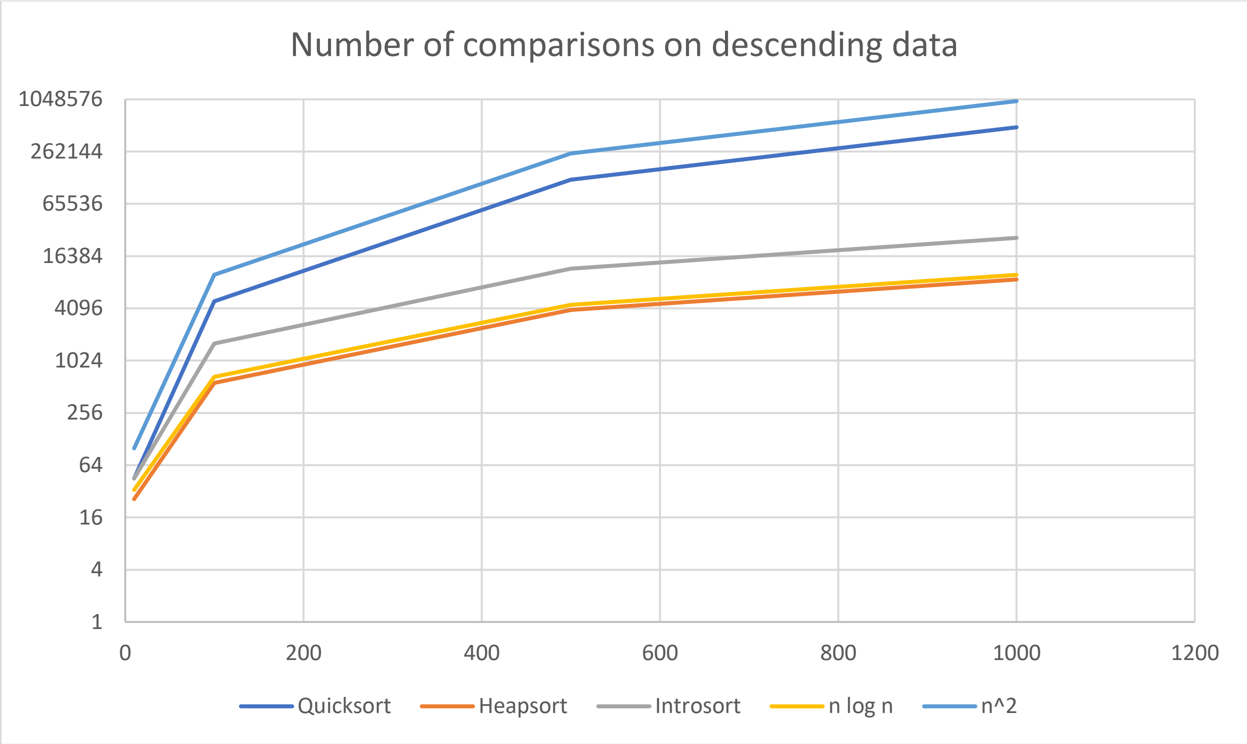

Quicksort falls away from the others and begins asymptotically tracking n^2. The graph illustrates that introsort does use more comparisons than heapsort in this case, but it does still track n log n.

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Median-of-Three Partitioning

Part 1 Pseudocode for median-of-three in the partition function

Median-of-three is the strategy of choosing a pivot that is the median out of the first value in the array, the last value in the array, and the middle value in the array. This would replace the int pivot = input[y] line of the partition function.

```
partition(int x, int y):

    // choose pivot as median of three

    Int pivot = median(input[x], input[y], input[y-x])

    swap(y, median_val)

    // keep track of where the pivot will end up

    Int index;

    For val in sub_array:

        If val < pivot:

            // move val to front

            Index++;

    // move pivot to index

    Swap(index, y)

    Return (index)
```

Median-of-three partitioning adds a constant time operation to every partition function call, but prevents the worst case scenario of always choosing the largest element. Since the worst case is elinimated, the runtime of median-of-three is O(n log n).

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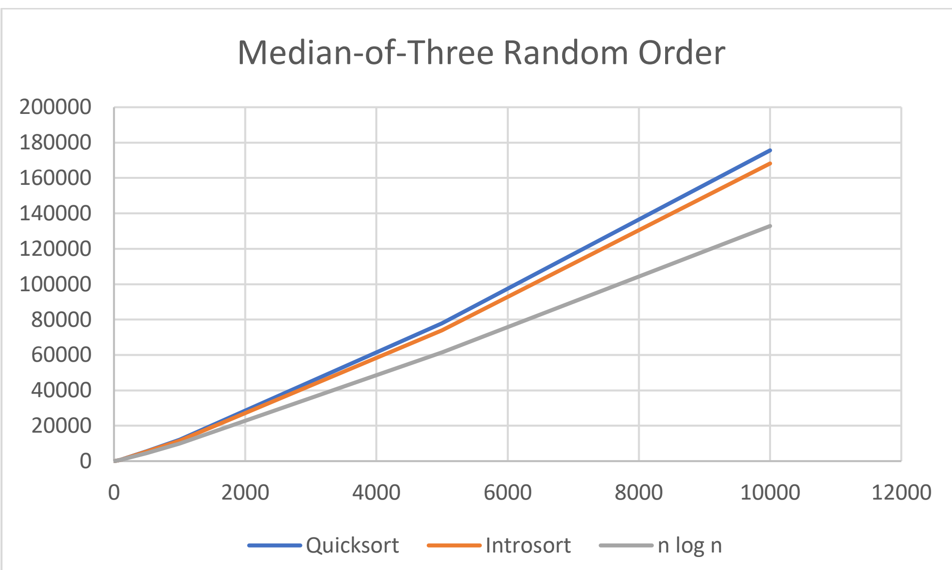
Runtime analysis:

	Random order					
Input Size	10	100	500	1000	5000	10000
Quicksort	25	784	5661	12056	77795	175617
Introsort	18	662	5378	11516	73926	168211
n log n	33	664	4483	9966	61439	132877
n^2	100	10000	250000	1000000	25000000	1E+08

	Descending order					
Input Size	10	100	500	1000	5000	10000
Quicksort	43	1571	19503	66259	1306385	5040571
Introsort	45	1325	10431	24079	163755	359214
n log n	33	664	4483	9966	61439	132877
n^2	100	10000	250000	1000000	25000000	1E+08

Quicksort and introsort took a small performance loss in the average case as shown by the random order tests, but both improved when ran on lists sorted in descending order. Specifically, quicksort was taking 0.5*n^2 comparisons and it gained a factor of 10 increase in performance. In practice, it is still quite a bit above n log n as illustrated in the following graphs.



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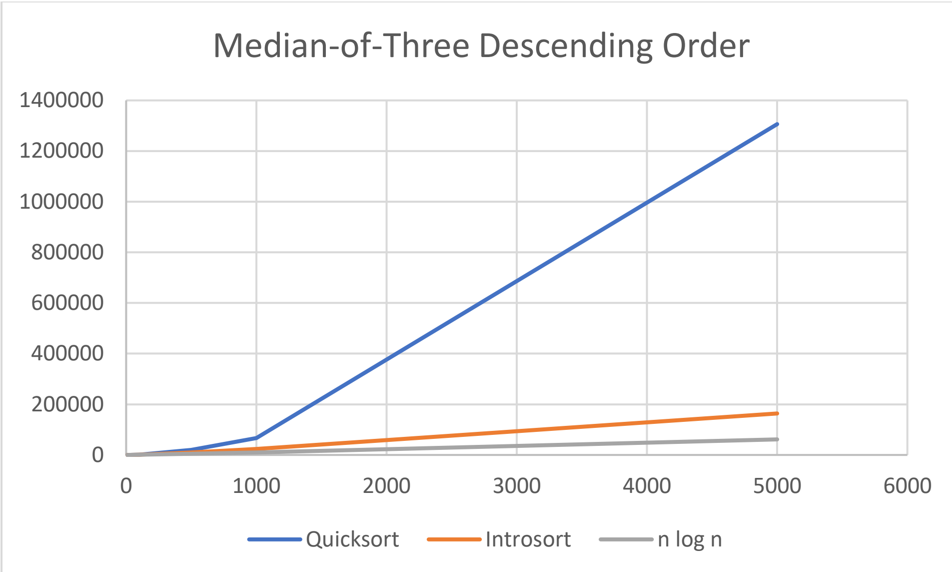

In conclusion, quicksort outperforms introsort when operating on random data, but introsort dominates quicksort when operating on descending order data with or without the median-of-three implementation.