~~TSR~~ HW # 9.

1. $g_1(\underline{x}) = m_1+m_2+m_4+m_5-5h \le 0$ $\quad \beta_1 = 6.2446$

$\rightarrow m_1+m_2+m_4+m_5-250 \le 0$ $\quad \hat{\alpha}_1 = (-0.6915, -0.5051, -0.1267, -0.3812, -0.3244)$

$\underline{u}_1^* = (-4.3178, -3.1540, -0.7911, -2.3804, -2.0277)^T$ $\quad P_{f_1} = 2.1243 \times 10^{-10}$

$\underline{x}_1^* = (62.5465, 62.5465, 85.3752, 62.5465, 62.5465)^T$

$g_2(\underline{x}) = m_2+2m_3+m_4-5v \le 0$ $\quad \beta_2 = 4.5523$

$\rightarrow m_2+2m_3+m_4-300 \le 0$ $\quad \hat{\alpha}_2 = (-0.4090, -0.5640, -0.6367, -0.3303, 0)$.

$\underline{u}_2^* = (-1.8621, -2.5678, -2.8984, -1.5038, 0)^T$ $\quad P_{f_2} = 2.6528 \times 10^{-6}$

$\underline{x}_2^* = (101.7221, 80.9974, 69.0026, 80.9974, 101.7221)^T$

$g_3(\underline{x}) = m_1+2m_3+2m_4+m_5-5h-5v \le 0$ $\quad \beta_3 = 3.3939$

$\rightarrow m_1+2m_3+2m_4+m_5-550 \le 0$ $\quad \hat{\alpha}_3 = (-0.6093, -0.2603, -0.5503, -0.4545, -0.2272)$

$\underline{u}_3^* = (-2.0682, -0.8834, -1.8675, -1.5422, -0.7712)^T$.

$\underline{x}_3^* = (97.6551, 109.9241, 88.6744, 88.6744, 97.6551)^T$ $\quad P_{f_3} = 3.4452 \times 10^{-4}$.

Failure of the frame can be described, letting $E_1: g_1(\underline{x}) \le 0$, $E_2: g_2(\underline{x}) \le 0$, $E_3: g_3(\underline{x}) \le 0$, $E_1 \cup E_2 \cup E_3$ (series system)

$$\therefore P_f = P\left(\bigcup_{i=1}^{3} E_i\right) = P\left(\bigcup_{i=1}^{3} g(\underline{x}) \le 0\right) \overset{FORM}{\cong} P\left(\bigcup_{i=1}^{3} \beta_i - \hat{\alpha}_i \underline{u} \le 0\right).$$

$$= P\left(\bigcup_{i=1}^{3} \beta_i \le Z_i\right) = 1 - P\left(\bigcup_{i=1}^{3} Z_i \le \beta_i\right) \quad \left(Z_i = \hat{\alpha}_i \underline{u} \sim N(0, 1^2)\right)$$

and $\underline{Z} = \begin{pmatrix} Z_1 \\ Z_2 \\ Z_3 \end{pmatrix} \sim N(0, \underline{R})$ and $\underline{R} = \left[\hat{\alpha}_i \cdot \hat{\alpha}_j^T\right]$

$$\therefore \underline{R} = \begin{bmatrix} 1 & 0.7743 & 0.8695 \\ 0.7743 & 1 & 0.8966 \\ 0.8695 & 0.8966 & 1 \end{bmatrix} \qquad \therefore P_f = 1 - \Phi_3(\beta_1, \beta_2, \beta_3; \underline{R})$$

$\therefore P_f = 0.0003446 < 0.02644$ (used mvncdf.m)

It is lower. Probability derived is conditional probability of ~~when~~ failure when $h=50$ and $v=60$ are given. But original probability is just probability of system failure's FORM approximation. So design point by conditional probability is much far from origin comparing to design point by FORM approximation of system failure. Therefore, giving much smaller value of probability.
This difference of design point is due to elimination of uncertainty in h and v.

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2.

(a) Maximum likelihood estimate of μ_s can be computed by

$\mu_s = \arg\max L(\underline{x};\mu_s)$ where $\underline{x}$ is observed data.

and $L(\underline{x};\mu_s) \propto P(\underline{x}|\theta) = P(\underline{x}|\mu_s)$ assumming each observations independent, $L(\underline{x};\mu_s) \propto P(x_1|\mu_s) \times P(x_2|\mu_s) \times \cdots \times P(x_t|\mu_s)$

getting logarithm dosen't change their inequality relationship.

$$\therefore \mu_s = \arg\max \left\{ \log \Phi\left(\frac{x_1-\mu_s}{45}\right) + \log\Phi\left(\frac{x_2-\mu_s}{45}\right) + \cdots + \log\Phi\left(\frac{x_t-\mu_s}{45}\right) \right\}.$$

$\therefore \mu_s^{MLE} \approx 153.01$

(b) As $R \sim N(\ )$ and $S \sim N(\ )$ $R-S \sim N(300-\mu_s, \sqrt{30^2+45^2}^2)$

$$P(R-S \le 0) = P\left(Z \le \frac{\mu_s-300}{\sqrt{30^2+45^2}}\right) = \Phi\left(\frac{\mu_s-300}{\sqrt{30^2+45^2}}\right)$$

$$\therefore \beta = -\frac{\mu_s-300}{\sqrt{30^2+45^2}} \qquad \therefore \beta(\mu_s^{MLE}) \doteq 2.7178.$$

(c) $n=5$, $\bar{x} = 153.0120$ $\sigma^2 = 1960.5271$ $\mu_\mu' = 170$, $\sigma_\mu^{2\,\prime} = 85^2$

$\therefore \mu_\mu'' = 153.8865$ $\sigma_\mu^{2\,\prime\prime} = 371.9210$ $\therefore \sigma_\mu'' = 19.2853.$

$(\sigma^2 = (44.2778)^2 \doteq 1960.5271)$

(d) $\therefore \mu_s \sim N(153.8865, 19.2853^2)$

$$\tilde{P}_f = E_\theta[P_f(\theta)] = \int_{-\infty}^{\infty} P_f(\mu_s)\cdot f_{\mu_s}(\mu_s)\,d\mu_s = \int_{-\infty}^{\infty} \Phi\left(\frac{\mu_s-300}{\sqrt{30^2+45^2}}\right)\varphi\left(\frac{\mu_s-153.8865}{19.2853}\right)d\mu_s$$

$\therefore \tilde{P}_f = 0.0055.$ $\therefore \tilde{\beta} = \Phi^{-1}(1-\tilde{P}_f) = 2.5447.$

(e) from posterior mean and standard deviation derived at (c).

we can get $\mu_\beta \cong \mu_\beta^{FO} = \beta(\underline{M}_\theta) = -\dfrac{\mu_\mu''-300}{\sqrt{30^2+45^2}} \doteq 2.7016$

$$\sigma_\beta^2 \cong (\sigma_\beta^2)^{FO} = \nabla_\theta \beta(\underline{M}_\theta)\cdot \Sigma_{\theta\theta} \nabla_\theta \beta(\underline{M}_\theta)^T = \left(-\frac{1}{\sqrt{30^2+45^2}}\right)\cdot \sigma_\mu^{2\,\prime\prime}\cdot\left(-\frac{1}{\sqrt{30^2+45^2}}\right)$$

$\doteq 0.1272$

$\therefore \beta \sim N(2.7016, 0.3566^2)$

$\therefore \langle\beta\rangle_{70\%} = 2.7016 \pm 1.04 \times 0.3566 = [2.3308, 3.0724]$

$\langle\beta\rangle_{95\%} = 2.7016 \pm 1.96 \times 0.3566 = [2.0027, 3.4005]$

using $\tilde{\beta}$ instead of μ_β^{FO}, we'll get

$\langle \tilde{\beta} \rangle_{90\%} = [2.1738, 2.9155]$ $\langle \tilde{\beta} \rangle_{95\%} = [1.8458, 3.2436]$.