ON THE SOLUTIONS OF NP-COMPLETE PROBLEMS BY MEANS OF JNEP RUN ON COMPUTERS

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Abstract: We have used jNEP (a JAVA simulator of a natural computing device named Networks of Evolutionary

Processors) to solve some cases of well-known NP-complete problems. We have followed the most relevant papers in the literature. In this paper, we describe the difficulties found in this process and some conclusions

about the design, the simulation and some useful tools for NEPs.

1 INTRODUCTION

1.1 Bio inspired computational devices

The so-called natural computing devices (such as multiagent systems, P systems, cellular automata, L systems and NEPs) are formal complex systems that are able to compute and could, therefore, be used as computers. All of them share two main characteristics: their inspiration in the way in which Nature efficiently solves complex tasks and an intrinsic parallelism that makes it possible to develop algorithms which improve the temporal performance of classic von Neumann architectures. This paper is specifically devoted to NEPs. As they could be considered as an alternative to the von Neumann architecture, a great research effort is currently being made to study the necessary tools to program them. We tackle this goal in two forms: studying the techniques to design NEPs which solve given problems, and developing and using a real hardware/software platform to run these NEPs.

1.2 NP-complete problems

In this section we informally introduce this topic. A formal description could be found in any manual

(Garey and Johnson, 1979) on complexity and is out of the scope of this paper.

NP may be informally defined as the set of decision problems that can be solved in polynomial time on a no deterministic Turing machine.

An NP problem is also *complete* if and only if every other problem in NP can be easily (in polynomial time) transformed into it.

Polynomial performance on a non-deterministic Turing machine frequently corresponds to at least exponential performance on a deterministic Turing machine. Classical von Neumann computers can be considered the closest implementation of deterministic Turing machines.

Even more informally, the reader can consider a non-deterministic Turing machine as a set of as many Turing machines as needed, searching in parallel for a solution of the problem. Such a device will stop as soon as the first solution is found. Each Turing machine is expected to check its solution in polynomial time. In the previous statement, "as many Turing machines as needed" usually means "an exponential number of machines".

The reader can easily understand that if the same work has to be done by a single Turing machine, it has to check each of the possible solutions (an exponential amount of them) in a polynomial time, which results in a final exponential performance.

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1.3 NEPs

NEP (Castellanos 2001 y Castellanos 2003) stands for *Network of Evolutionary Processors*. NEPs are an abstract model of distributed/parallel symbolic processing inspired by biological cells. They distribute a set of simple string processors in the nodes of a fixed graph. Processors contain strings of symbols, change them in a predefined way and filter them to communicate some of these words to the other processors of the graph.

Despite the simplicity of each processor, the entire net can efficiently carry out very complex tasks. Many different works demonstrate the computational completeness of NEPs (Csuhaj-Varju 2005 and Manea 2007) and their ability to solve NP problems with linear or polynomial resources (Manea 2006 and Castellanos 2001). The emergence of such a computational power from very simple units acting in parallel is one of the main interests of NEPs

A NEP is built from the following elements:

- a) a set of symbols which constitutes the alphabet of the words which are manipulated by the processors,
- b) a set of processors,
- an underlying graph where each vertex represents a processor and the edges determine which processors are connected so they can exchange words,
- d) an initial configuration defining which words are in each processor at the beginning of the computation and
- e) one or more stopping rules to halt the NEP.

An evolutionary processor has three main components:

- a) a set of evolutionary rules to modify its words,
- b) some input filters that specify which words can be received from other processors and
- some output filters that delimit which words can leave the processor to be sent to others.

The variants of NEPs mainly differ in their evolutionary rules and filters. They perform very simple operations, like altering the words by replacing all the occurrences of a symbol by another, or filtering those words whose alphabet is included in a given set of words.

NEP's computation alternates evolutionary and communication steps: an evolutionary step is always followed by a communication step and vice versa.

Computation follows the scheme below: when the computation starts, every processor has a set of initial words.

At first, an evolutionary step is performed: the rules in each processor modify the words in the same processor. Next, a communication step forces some words to leave their processors and also forces the processors to receive words from the net.

The communication step depends on the constraints imposed by the connections and the output and input filters

The model assumes that an arbitrary number of copies of each word exists in the processors, therefore all the rules applicable to a word are actually applied, resulting in a new word for each rule.

The NEP stops when one of the stopping conditions is met, for example, when the set of words in a specific processor (the ouput node of the net) is not empty. A detailed formal description of NEPs can be found in (Castellanos 2003, Csuhaj-Varju 2005 or Manea 2007).

1.4 Clusters of computers running Java

Java is currently one of the most popular object oriented programming languages. Java may be slower than other programming languages for computation-intensive problems. Nevertheless it is possible to run Java programs on a cluster of computers by means of a special Distributed Java Virtual Machine (DJVM), which supports parallel execution of Java threads. In this way, a multithreaded Java application runs on a cluster just as if it were running on a single machine, but with the same performance as a big multi-processor machine.

DJVMs are not included in the Sun's standard Java distributions. There are several different kinds of DJVM, for example: Java-Enabled Single-System-Image Computing Architecture 2 (JESSICA2), the cluster virtual machine for Java developed by IBM (IBM cJVM), Proactive PDC (Proactive), DO! (Launay 97), JavaParty (JavaParty), Jcluster (Jcluster), MPJ Express (MPJ), and Terracota (Terracota 2008).

The simulator used in this paper has been developed with both, standard JVM and JavaParty.

2 JNEP

In a previous work (Rosal 2008) we have explained the reasons to develop a NEP simulator to be run in a cluster of computers.

jNEP offers an implementation of NEPs as general, flexible and rigorous as possible. This is not an obvious goal, because we have observed that different authors understand the model definition in slightly different ways. These subtle differences imply, nevertheless, hard to overcome problems in the development of a computer application that implements all of them.

As shown in figure 1, the design of the NEP class mimics the NEP model definition. In jNEP, a NEP is composed of evolutionary processors and an underlying graph (attribute *edges*) to define the net topology and the allowed inter processor interactions. The *NEP* class coordinates the main dynamic of the computation and rules the processors (instances of the *EvolutionaryProcessor* class), forcing them to perform alternate evolutionary and communication steps. It also stops the computation when needed.

The core of the model includes these two classes, together with the *Word* class, which handles the manipulation of words and their symbols.

jNEP is kept as general and rigorous as possible by means of the following mechanisms: *Java interfaces* and the development of *different versions* to widely exploit the parallelism available in the hardware platform. Further details can be found in (Rosal 2008) and at (http://jnep.e-delrosal.net)

jNEP currently has two lists of choices (concurrency approach and platform) to select the parallel/distributed platform on which it runs (jNEP versions for any possible combination of them are available in http://jnep.e-delrosal.net).

Concurrency is implemented with two different Java approaches: *Threads* and *Processes*.

The supported platforms are standard JVM and clusters of computers (by means of JavaParty).

jNEP reads the definition of the NEP that is being simulated from a configuration file that follows XML conventions. Roughly speaking the

configuration file contains special tags for any relevant component in the NEP (alphabet, stopping conditions, the complete graph, each edge, each evolutionary processor with its rules, filters and initial contents).

Although some fragments of these files will be shown in these pages, all the configuration files mentioned in this paper can be found at (http://jnep.e-delrosal.net). Despite the complexity of these XML files, the interested reader can see that these tags and their attributes have self-explaining names and values.

3 SOLVING NP-COMPLETE PROBLEMS WITH JNEP

In our previous work (Rosal 2008) we showed, as an example, how to solve the propositional logic SAT problem for three variables by means of a NEP with a kind of special rules (splicing) taken from (Manea 2007).

In this work we show how jNEP has been used to solve some instances of other two well-known NP-complete problems: the Hamiltonian path problem and the 3-coloring problem.

3.1 Hamiltonian path problem

This well-known NP-complete problem searches an undirected graph for a Hamiltonian path, that is, one that visits each vertex exactly once.

In his work (Adleman 1994) Adleman proposed a way to solve this problem with polynomial resources by means of DNA manipulations in the laboratory. Figure 2 shows the graph used by Adleman. The solution is in this case obvious (path 0-1-2-3-4-5-6) Despite its simplicity, Adleman described a general algorithm applicable to almost any graph with the same performance.

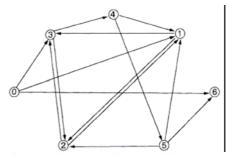


Figure 2. Graph studied by Adleman

Adleman's algorithm can be summarized as follows:

- 1. Generating randomly all the possible paths.
- 2. Selecting those paths that begin and end in the proper nodes.
- 3. Selecting only the paths that contain exactly the total number of nodes.
- 4. Removing those paths that contain some node more than once.
- 5. The remaining paths are solutions for the problem.

The present work follows a similar approach.

- The NEP graph is very similar to the one studied above: an extra node is added to ease the definition of the stopping condition.
- The set {i,0,1,2,3,4,5,6} is used as the alphabet. Symbol i is the initial content of the initial vertex (v₀)
- Each node (except the final one) adds its number to the string received from the network.
- Input and output filters are defined to allow the communication of all the possible words without any special constraint.
- The input filter of the final node excludes any string which is not a solution. It is easy to imaging a regular expression for the set of solutions (those words with the proper length, the proper initial and final node and where each node appears only once). The NEP basic model allows defining filters by means of regular expressions.
- It is also easy to devise a set of additional nodes that performs the previous filter following Adleman's checks (proper beginning and end, proper length, and number of occurrences of each node). For the shake of simplicity we have used explicitly the solution word (i_0_1_2_3_4_5_6) instead of a more complex regular expression or a greater NEP.

The reader will find at http://jnep.e-delrosal.net the complete XML file for this problem (Adleman.xml). Some of their sections are explained below:

• The XML file for this example defines the alphabet with this tag

```
<aLPHABET symbols="i_0_1_2_3_4_5_6" ^{\prime}
```

• the initial content of node 0 in the following way

```
<NODE initCond="i">
```

• The rules for adding the number of the node to its string are defined as follows (here for node 2)

```
<RULE ruleType = "insertion"
    actionType = "RIGHT"
    symbol = "2" newSymbol="" />
```

• There are several ways of defining filters for the desired behavior (to allow the communication of all the possible words without any special constraint). We have used only the permitted input and output filters. A string can enter a node if it contains any of the symbols of the alphabet and no string is forbidden.

The behavior of the NEP is sketched as follows:

- 1. In the initial step the only non empty node is 0 and contains the string i
- 2. After the first step, 0 is added to this string and thus, node 0 contains i_0
- 3. This string is moved to the nodes connected with node 0. In the next steps only nodes 1, 3 and 6 contain i_0.
- 4. These nodes add their number to the received string. In the next step their contents are respectively i_0_1, i_0_3 and i_0_6
- 5. This process is repeated as many times as needed to produce a string that meets the conditions of the solution. In this final step the solution string (i_0_1_2_3_4_5_6 is sent to node 7 and the NEP stops.

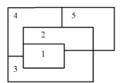
The definition of filters in NEP model poses some difficulties to the design of NEPs and, thus, to the development of a simulator.

These filters are defined (Castellanos 2001 and Castellanos 2003) by means of two couples of filters (forbidden and allowed) to each operation (input and output). There exist, in addition, different ways of combining and apply the filters to translate them into a set of strings. This mechanism contains obvious redundancies that make it difficult to design NEPs. It could be advisable a more general agreement of the researchers to ease and simplify the development of NEPs simulators.

3.2 3-coloring problems

This problem introduces a map whose regions have to be colored with only three colors, and with a different one for each pair of adjacent regions.

We have used the NEP defined in (Castellanos 2003). The map is translated into an undirected graph whose nodes stand for the regions and whose edges represent the adjacency relationship between regions. Figure 3 shows one of the examples studied in this paper. It is ease to prove that there is no solution to this map.



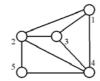


Figure 3. Example of a map and its adjacency graph. In this case, there is no solution for the 3-colorability problem

The NEP has a complete graph with two special nodes (for the initial and final steps) and a set of seven nodes associated to each edge of the adjacency graph. These nodes perform the tasks outlined below. Next paragraphs describe them with more detail:

- The initial (final) node is responsible of starting (stopping) the computation.
- The seven nodes associated with an edge of the map are grouped in three couples (one for each color). There is, in addition, a special node to communicate with the set of nodes of the next edge.
- Each couple is responsible of the main operation in the NEP: to check that a coloring constraint is not violated for the current edge. It performs this task in the following way:
 - Let us suppose that the color red is the one associated with the pair of nodes.
 - The first node in the NEP associates the color red to the first node of the edge in the map.
 - The second node in the NEP simultaneously keeps all the allowed coloring (two, in this

- case) for the second node of the edge: (blue and green)
- It is clear that the only acceptable colorings for this edge are red-blue and red-green.

The behavior of the complete NEP could be sketched as follows:

- 1. The initial node generates all the possible assignment of colors to all the regions in the map and adds a symbol to identify the first edge to be checked. These strings are communicated to all the nodes of the graph.
- 2. The set of nodes associated to each edge accepts only the strings marked with the symbol of the edge. These nodes remove all the strings that violate the coloring constraint for the regions of the edge. One special node in the set replaces the edge mark with that which corresponds to the next edge. In this way, the process continues.
- 3. The final node of the NEP collects the strings that satisfy the constraints of all the edges. It is ease to see that these strings are the solutions.

Some fragments of the XML file for this example (3Coloring.xml) are shown below to describe the above behavior with more detail:

The alphabet of the NEP is defined as follows:

<ALPHABET

symbols="b1_r1_g1_b2_r2_g2_b3_r3_g3_b4_
r4_g4_b5_r5_g5_B1_R1_G1_B2_R2_G2_B3_R3_
G3_B4_R4_G4_B5_R5_G5_a1_a2_a3_a4_a5_X1_
X2_X3_X4_X5_X6_X8_X9"/>

- This alphabet contains the following subsets of symbols:
 - o {a1,...,a5} represents the initial situation of the regions (uncolored).
 - o {b1, r1, g1,..., b5, r5, g5} represents the assignment of the colors to the regions.
 - o {B1, R1, G1,... B5, R5, G5} is a copy of the previous set to be used while checking the constraint associated with a couple of adjacent regions.

 The string contained in the initial node at the beginning represents the complete map uncolored and the number of the first edge to be tackled (X1)

```
<NODE initCond="a1_a2_a3_a4_a5_X1">
```

 The rules of the initial node assign all the possible colors to all the regions. The following rules refer to the second region:

```
<RULE ruleType = "substitution"
    actionType = "ANY"
    symbol="a2" newSymbol="b2"/>
<RULE ruleType="substitution"
    actionType="ANY"
    symbol="a2" newSymbol="r2"/>
<RULE ruleType="substitution"
    actionType="ANY"
    symbol="a2" newSymbol="g2"/>
```

• The node in the NEP that assigns a color (red, in this case) to the first region (1 in the example) of an edge in the map uses the following rule:

```
<RULE ruleType="substitution"
   actionType="ANY" symbol="r1"
   newSymbol="R1"/>
```

 The other node ensures that the adjacent region (2 in this case) has a different color by means of these rules:

```
<RULE ruleType="substitution"
    actionType="ANY"
    symbol="b2"
    newSymbol="B2"/>
<RULE ruleType="substitution"
    actionType="ANY" symbol="g2"
    newSymbol="G2"/>
```

 The node used for starting the process in the next edge removes any special (capitalized) color symbol and sets the edge marking to the next one. The following rules correspond to the first edge

```
<RULE ruleType="substitution"
    actionType="ANY" symbol="R1"
    newSymbol="r1"/>
<RULE ruleType="substitution"
    actionType="ANY" symbol="B1"
    newSymbol="b1"/>
<RULE ruleType="substitution"
    actionType="ANY" symbol="G1"
    newSymbol="g1"/>
```

```
<RULE ruleType="substitution"
    actionType="ANY" symbol="R2"
    newSymbol="r2"/>
<RULE ruleType="substitution"
    actionType="ANY" symbol="B2"
    newSymbol="b2"/>
<RULE ruleType="substitution"
    actionType="ANY" symbol="G2"
    newSymbol="g2"/>
<RULE ruleType="substitution"
    actionType="ANY" symbol="G2"
    newSymbol="g2"/>
```

- We have found difficulties when applying the input and output filters as they are in (Castellanos 2003). We have previously explained our opinion on the advisability of a greater standardization to minimize this situations.
- Notice that the nodes associated with the last edge (in this case with the number 8) mark its strings with the following number that does not correspond with any edge in the graph (9 in our example). This is important for the design of the final node.
- A special node of the NEP checks the stopping condition (Non Empty Node Stopping Condition). This final node only accepts strings with the corresponding mark (one that does not correspond to any edge in the adjacency graph).

Figure 4 shows other map that could be colored with 3 colors. Splitting region 3 and 4 in figure 3 generates this map. Figure 4 also summarizes the sequence of steps for one of the possible solutions. It is worth noticing that all the solutions are simultaneously kept in the configurations of the NEP.

The behavior of the NEP for this map could be summarized as follows:

- The initial content of the initial node is a1_a2_a3_a4_a5_X1.
- This node produces all the possible coloring combinations. In the second step of the computation, for example, it contains the following strings:

```
      b1_a2_a3_a4_a5_X1
      r1_a2_a3_a4_a5_X1

      g1_a2_a3_a4_a5_X1
      a1_b2_a3_a4_a5_X1

      a1_r2_a3_a4_a5_X1
      a1_g2_a3_a4_a5_X1

      a1_a2_b3_a4_a5_X1
      a1_a2_r3_a4_a5_X1

      a1_a2_g3_a4_a5_X1
      a1_a2_a3_b4_a5_X1

      a1_a2_a3_r4_a5_X1
      a1_a2_a3_b4_a5_X1

      a1_a2_a3_g4_a5_X1
      a1_a2_a3_g4_a5_X1

      a1_a2_a3_a4_r5_X1
      a1_a2_a3_a4_r5_X1

      a1_a2_a3_a4_g5_X1
      a1_a2_a3_a4_r5_X1
```

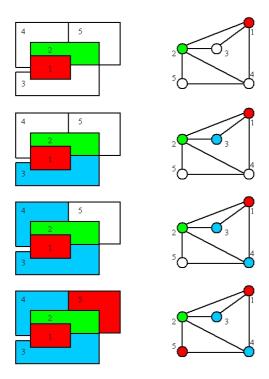


Figure 4. Sequence of steps in the solution of a 3-coloring problem by jNEP

- The NEP still needs a few more steps to get all the combinations.
- After that, the coloring constraints are applied simultaneously to all the possible solutions and those assignments that violate some constraint are removed. We describe below a sequence of strings generated by the NEP that corresponds to the solution graphically shown in figure 4:
 - o r1_g2_b3_b4_r5_X1 is generated in the initial steps.
 - o After checking the 1st edge (regions 1 and 2) the NEP contains these two strings

R1_g2_b3_b4_r5_X1 and R1_G2_b3_b4_r5_X1

- O After checking the 2nd edge (regions 1 and 3) R1_g2_B3_b4_r5_X2
- And after checking the edges 3, 4, 5, 6 and 8 (remember that edge 7 was removed to make the map colorable) associated respectively with the pairs of regions 1-4, 2-3, 2-4, 2-5 and 4-5, the following strings are in the NEP:

o Finally, the complete solution is found

r1_g2_b3_B4_R5_X9 and r1_g2_b3_b4_r5_X9

 This NEP processes all the solutions at the same time. It removes all the coloring combinations that violate any constraint. The final node contains in the last step all the solutions found.

(Castellanos 2003) describes one of the kinds of NEPs (simple NEPs) that is simulated by jNEPs. As we have briefly mentioned before, we have observed that the authors have used slightly different filters for the 3-coloring problem. We could not use these filters and we had to change some of them (most of the output filters) in order to get a proper behavior of the NEP. The complete XML file is available at http://jnep.e-delrosal.net

4 CONCLUSIONS AND FURTHER RESEARCH LINES

We have tackled the solution of several NPcomplete problems found in the literature by means of jNEP. We have observed that there exist different ways of implementing the same formal model, mainly with respect to input and output filters. These open aspects have to be defined when the model is implemented to solve given problems. We conclude that simulation needs both: a formal definition and also some standardization in the way in which different authors particularize these open aspects in the implementation of their own NEPs. These differences make it very difficult to fully understand the behavior of the proposed NEPs as well as their simulation. Although we have not found any significant mistake in the simulation of the formal model, we had to modify and improve jNEP in several subtle details in order to ease the handling of the NEPs described in the literature.

We have also identified some common techniques to these different NP problems. They suggest us some tools that could be added to jNEP to increase the comfort of the NEPs designer. In the future we plan to develop a more abstract input format. For example, most of the NEPs defined to solve NP problems uses complete graphs. The current XML configuration file explicitly defines each edge, which implies a big amount of tedious and mechanical work. It will be very useful some automatic mechanism to do this task.

It could be also very useful adding some diagnose tool to check the correctness of the NEPs.

It is worth noticing that jNEP is just a block that will be used to build more complex applications. One of them is a full graphic simulation environment for NEPs that ease their design to solve given problems. Our research group is also interested in some evolutionary techniques to automatic design NEPs. jNEP will be used as a part of the fitness function that this kind of algorithms needs.

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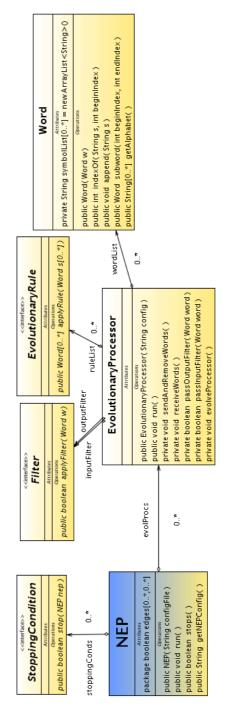


Figure 1: simplified class diagram for jNEP