Firing Costs and Efficiency in a Frictional Labor Market: Evidence from Brazil*

David Arnold† Joshua Bernstein ‡

September 2, 2018

Abstract

We study how tenure-dependent employment protection legislation (EPL) can be used to identify the equilibrium welfare impacts of firing costs. Our framework incorporates tenure-dependent regulations into the search and matching framework of Moscarini (2005). Using administrative data from Brazil, we estimate the key structural parameters of the the model by matching the empirical pattern of the job termination hazard. Using our estimated model, we find that the costs of labor market regulation are small (0.1% of GDP), and that these costs stem from a violation of the Hosios (1990) efficiency condition.

*We thank Leah Boustan, Will Dobbie, Henry Farber, Mike Golosov, Alex Mas, David Schoenherr and numerous seminar participants for helpful comments and suggestions.
†Princeton University. Email: dharnold@princeton.edu
‡Princeton University. Email: jmsb@princeton.edu

I. Introduction

An extensive literature studies the impact of employment protection legislation (EPL) on unemployment. While some (Lazear 1990, Di Tella and MacCulloch 2005) find a positive correlation between strictness of employment protection legislation (EPL) and unemployment, others find a negative correlation (Garibaldi and Violante 2005) or no correlation (Kugler and Pica 2008). However, these mixed results, together with their reduced-form nature, make it difficult to infer the equilibrium welfare impacts of EPL. For example, it is unclear whether EPL corrects for or exacerbates the inefficiencies created by search frictions in standard models of frictional labor markets.

The goal of this paper to is to estimate the equilibrium impact of labor market regulations on both the labor market and the aggregate economy. Adopting a structural approach, and using a rich dataset on job tenure lengths in Brazil, we find that the imposition of labor market regulation has a small negative impact on the economy, causing GDP to be lower by 0.1%, welfare to fall by 0.04%, and the unemployment rate to increase by 0.1 percentage points. The negative effect stems from 2 features of our estimated structural model. First, we find that the Hosios (1990) efficiency condition is violated such that the equilibrium rate of job separation is lower than the rate chosen by a social planner. Second, the labor market regulations that we analyze lower the rate of job separation even more, and thus exacerbate the initial inefficiency, rather than mitigate it.

We begin by setting up our structural model. We adapt the seminal paper of Moscarini (2005), which analyzes a labor market with 2 key frictions. First, vacant firms and unemployed workers face search frictions as captured by a matching function in the style of Mortensen and Pissarides (1994), which results in positive involuntary unemployment in the steady state of the economy. Second, when firms and workers meet, they do not know how productive the match will turn out to be. Instead, the firm and worker observe output flows produced by the match over time, and use this data to infer the probability that the match is of high or low quality. When the firm and worker are confident enough that the match is of low quality, they can agree to terminate the match. The economy therefore features endogenous match terminations, and hence a hazard rate of match termination that depends on job tenure in a non-trivial manner.

Our main modeling innovation is the inclusion of labor market regulation, which we model as a fixed cost that firms must pay to terminate a match. Importantly, the regulation only applies to jobs that have lasted at least 3 months, which matches the tenure at which regulations becomes active in our Brazilian data. We show that the inclusion of such regulation results in a spike in the theoretical hazard rate at 3 months of tenure. This spike is also a key feature of the empirical hazard rate schedule, and therefore guides our estimation strategy.

We estimate key parameters of our model using the empirical hazard rate schedule in the data. In particular, we argue that salient features of the empirical hazard rate schedule (average level, slope, long run level) allow us to identify structural parameters governing the bargaining share of workers, the speed at which employers and employees learn about the true quality of the match, and the exogenous rate of match termination. In this way, we allow the data to tell us the values of parameters that would otherwise be difficult to directly measure.

In order to infer and estimate the real cost of labor market regulations, we exploit the tenure-dependence to argue that the observed spike in the empirical hazard rate that occurs at a tenure of 3 months is the direct result of the regulations becoming active. Therefore, we can identify the fixed cost parameter in the model from the size of this hazard rate spike in the data. In this way we adapt the methodology of Garicano et al. (2016), and let the observed termination decisions of firms inform our estimate of the fixed cost, rather than try to directly measure the fixed cost from the wide range of labor market regulations that apply in Brazil. This approach also allows us to analyze labor market regulations as a whole, rather than focusing on any 1 regulation in particular.

We use the empirical job termination rate to validate key assumptions of our model by leveraging the richness of the administrative data. In our model, the spike in the job termination hazard is driven by employer learning. However, separations may also be driven by other factors such as demand volatility and short-term contracts for temporary workers. We restrict analysis to permanent contracts and provide a number of tests to show that other considerations such as demand volatility are not driving the shape of the job termination hazard. Finally, we consider alternative calibrations and find that our main results are robust to these changes.

Related Literature This paper relates to two distinct literatures. First there is an extensive literature which explores the impact of labor market regulation on labor market and aggregate outcomes (Daruich, Addario, and Saggio 2018, Di Tella and MacCulloch 2005, Garibaldi and Violante 2005, Kugler and Pica 2008, Lazear 1990). Existing papers generally rely on two identification strategies. One literature utilizes cross-country variation in employment protection legislation while the other uses within-country variation, for example, exploiting reforms or size-contingent laws. Our paper takes a more structural approach and uses the job termination hazard rate to estimate key parameters of the model. Our strategy is therefore similar to Garicano et al. (2013) who use the firm size distribution to infer the costs of EPL as well as Best et al. (2018) who combine a structural model with a bunching approach to estimate the intertemporal elasticity of substitution.

Our paper also relates to a theoretical literature which explores the effects of employment protection through the lens of search models (Garibaldi and violante 2005, Ljungqvist 2001). Prior research often models EPL as creating a fixed cost for hiring permanent workers. However, as Cahuc et. al (2016) points out, in most countries, including our setting Brazil, there is a probationary period during which it is costless to fire workers permanent workers. While this feature of employment protection legislation is generally ignored, it is central to our model and estimation strategy.

The paper is structured as follows: section II describes the structural model and analyzes its equilibrium properties. Section III describes our estimation and identification strategy, while section IV discusses the institutional setting and data. Section V contains the results of our estimation and counterfactual exercise, while section VI contains the robustness exercises. Finally, section VII concludes.

II. Model

The model is based on Moscarini (2005). As such, our exposition closely follows his except at points where our addition of firing costs plays a role.

A final good is produced in continuous time by pairwise firm-worker matches. The match-specific productivity of a match, μ, is ex-ante uncertain, and evolves stochastically over the life of the match. Upon forming the match, both firm and worker share a common prior on μ that is independent of their histories. The prior puts mass on two points $\{\mu^L, \mu^H\}$ where $\mu^L < \mu^H$, and μ^L denotes a bad match, μ^H a good match. Let $p_0 = \Pr\left(\mu = \mu^H\right)$ be the initial prior that the match is good.

Final good production is linear in match productivity, so that in a small interval dt, production of the good X is given by

$$dX_t = \mu dt + \sigma dZ_t$$

where dZ_t is a standard Brownian motion. In other words, output is a noisy indicator of true match quality, with the noise being scaled by $\sigma > 0$. The presence of noise creates an inference problem that firms and workers solve by using the information provided by the history of output, denoted by the filtration $\mathcal{F}_t^X$, to update their prior belief in a Bayesian manner,

$$p_t = \Pr\left(\mu = \mu^H | \mathcal{F}_t^X\right)$$

When the belief is low enough, firms and workers will optimally choose to terminate the match. However, upon termination, firms must pay a firing cost $\kappa(t)$ that depends on the tenure t of the match in the following way:

$$\kappa(t) = \begin{cases} 0 & \text{if} \quad t < T_1 \\ \\ \kappa & \text{if} \quad t \geq T_1 \end{cases}$$

where $\kappa > 0$ is a constant, and T_1 is a tenure after which the firing cost becomes active. In addition to this endogenous separation, matches are also subject to exogenous termination at rate $\delta > 0$.

There is a large mass of ex-ante homogeneous firms, who can post vacancies at flow cost $c > 0$ when unmatched (the large mass ensures free-entry in equilibrium). We normalize the mass of workers to unity, and assume that unemployed workers receive a flow value of leisure b. All agents are risk-neutral, and discount future payoffs at rate $r > 0$. In order to split the surplus generated by a match, we assume that the firm and worker use a generalized Nash bargaining rule, and denote the bargaining weight of workers by $\beta \in (0, 1)$.

Finally, given a mass of vacancies v, and unemployment u, new matches are created according to the matching function

$$m = zu^{\eta}v^{1-\eta}$$

where z is a matching efficiency parameter. From this matching function we can define labor market tightness, $\theta = \frac{v}{u}$, the job finding rate for unemployed workers, $\lambda = z\theta^{1-\eta}$, and the job filling rate for vacant firms, $q = z\theta^{-\eta}$.

We study the steady state equilibrium of this economy. As such, aggregate variables do not have a time subscript, and we use $t \geq 0$ to unambiguously denote tenure for individual level variables from now on.

A. Belief Dynamics

The solution to the continuous time inference problem has a well known solution in the form of the stochastic differential equation

$$dp_t = p_t\left(1 - p_t\right) s d\bar{Z}_t$$

where $s = \frac{\mu^H - \mu^L}{\sigma}$ is the signal-to-noise ratio and

$$d\bar{Z}_t = \frac{1}{\sigma}\left(dX_t - \left(p_t\mu^H + \left(1 - p_t\right)\mu^L\right)dt\right)$$

is a standard Brownian motion with respect to the filtration generated by X_t, $\mathcal{F}_t^X$.[1] Intuitively, beliefs move faster the more uncertain is the current belief ($p\left(1-p\right)$ has a maximum at $p = \frac{1}{2}$).

An advantage of the Moscarini (2005) model is that the current belief p is a state variable for the firm and worker problems, thus rendering them tractable analytically and computationally. As such, it is useful to define

$$\Sigma\left(p\right) = \frac{1}{2}s^2p^2\left(1 - p\right)^2$$

which is interpreted as half the variance of the change in beliefs over a small change in tenure.

B. Matched Firm Value Functions

In order to analyze the firm problem in the presence of firing costs, it is useful to divide the problem into 2 stages of tenure: $t \geq T_1$, and $t \in [0, T_1)$. Let the value functions and match termination threshold functions be denoted by J^2 and J^1, and $\underline{p}^{2,J}$ and $\underline{p}^{1,J}$ respectively. Similarly, let the wage functions be denoted w^2, and w^1. Finally, let $\bar{\mu}\left(p\right) = p\mu^H + \left(1 - p\right)\mu^L$ be expected productivity when the belief is p.

In the following analysis, superscripts are labels, while subscripts denote derivatives. E.g. J^i_{xx} is the second derivative of the function J^i with respect to the argument x.

B.1 $t \geq T_1$

For matches of tenure greater than or equal to T_1, the value function only depends on the belief p since the firing cost is fixed. As a result, the termination threshold will also not depend on tenure.

[1] For a discussion of this result, see Moscarini (2005) and the references therein.

Therefore, J^2 and $\underline{p}^{2,J}$ must jointly satisfy the Hamilton-Jacobi Bellman (HJB) equation

$$rJ^2(p) = \bar{\mu}(p) - w^2(p) + \Sigma(p) J^2_{pp}(p) - \delta J^2(p)$$

and boundary conditions

$$J^2(p) < \infty$$

$$J^2\left(\underline{p}^{2,J}\right) = V - \kappa$$

$$J^2_p\left(\underline{p}^{2,J}\right) = 0$$

The HJB equation has the usual interpretation. The value of the match to the firm equals the flow profits (production minus the wage) plus capital gains stemming from the change in beliefs and the possibility of exogenous separation. The first boundary condition ensures the value function is finite, while the second states that, given any termination threshold $\underline{p}$, the value of the match at the threshold is equal to the value to the firm from termination, which is equal to the value of entering the vacancy posting stage minus the firing cost (the so called "value matching" condition).[2] Solving the HJB equation with the first two boundary conditions yields a function $\tilde{J}^2\left(p; \underline{p}\right)$ that is indexed by the arbitrary belief threshold.

The firm chooses $\underline{p}^{2,J}$ so that the implied value function $J^2(p) = \tilde{J}^2\left(p; \underline{p}^{2,J}\right)$ maximizes the value of a match at any belief p. The optimal threshold is pinned down by the final boundary condition, which states that, at the termination threshold, the slope of the value function must be zero. Intuitively, this condition ensures that the firm is indifferent between terminating the match as soon as the belief hits the threshold, and waiting for a small amount of time to see what happens. Without this so called "smooth pasting" condition, the termination threshold would not be optimal.

In order to save space, we assume that all value functions are finite from now on. This means that we can drop the first boundary condition, and focus only on the "value matching" and "smooth pasting" conditions.

B.2 $t \in [0, T_1)$

For matches of tenure $t \in [0, T_1)$, the value function depends on both the belief p and tenure t since the time until firing costs become non-zero changes with tenure. Therefore, J^1 and $\underline{p}^{1,J}$ must jointly satisfy the HJB equation

$$rJ^1(p,t) = \bar{\mu}(p) - w^1(p,t) + J^1_t(p,t) + \Sigma(p) J^1_{pp}(p,t) - \delta J^1(p,t)$$

and boundary conditions

$$J^1\left(\underline{p}^{1,J}(t), t\right) = V$$

[2]Technically, the value function must be bounded above by the surplus function (defined later), which itself must be finite.

$$J_p^1\left(\underline{p}^{1,J}(t), t\right) = 0$$

$$J^1(p, T_1) = J^{*,2}(p)\ \forall p \geq \underline{p}^{1,J}(T_1)$$

The final boundary condition says that at tenure $t = T_1$, the value function J^1 is pinned down by a function $J^{*,2}$ which will be determined in equilibrium. Note that in general $J^{*,2} \neq J^2$ since the firm has the option to terminate the match an instant before $t = T_1$ as zero cost, while waiting until $t = T_1$ would incur a positive firing cost when $\kappa > 0$. We will later show that this jump is firing costs at $t = T_1$ results in a discontinuity in the wage function and hence a discontinuity in the value function for the firm.

The remaining boundary conditions have the same interpretations as before, except now they also depend on tenure, and the firing cost is zero.

C. Worker Value Functions

Similarly to the firm analysis, it is useful to divide the employed worker problem into 2 stages of tenure: $t \geq T_1$, and $t \in [0, T_1)$. Let the value functions and termination threshold functions be denoted by W^2 and W^1, and $\underline{p}^{2,W}$ and $\underline{p}^{1,W}$ respectively. Also, let U be the value function of unemployment, which satisfies the HJB equation

$$rU = b + \lambda\left(W^1(p_0, 0) - U\right)$$

The value of being unemployed equals the flow value from leisure plus the gain from becoming employed times the probability of entering a match.

C.1 $t \geq T_1$

For matches of tenure greater than T_1, the value function only depends on the belief p since the firing cost is fixed. Therefore, W^2 and $\underline{p}^{2,W}$ must jointly satisfy the Hamilton-Jacobi Bellman (HJB) equation

$$rW^2(p) = w^2(p) + \Sigma(p)\, W_{pp}^2(p) - \delta\left(W^2(p) - U\right)$$

and boundary conditions

$$W^2\left(\underline{p}^{2,W}\right) = U$$

$$W_p^2\left(\underline{p}^{2,W}\right) = 0$$

The economics is analogous to the firm case. The value of the match to the worker equals the flow benefit (the wage) plus capital gains stemming from the change in beliefs and the possibility of exogenous separation. The “value matching" condition states that value of the match at the threshold is equal to the value to the worker from termination, which is equal to the value of unemployment. The “smooth pasting" condition states that, at the termination threshold, the slope of the value function must be zero so that the worker is indifferent between terminating the match as soon as the belief hits the threshold, and waiting for a small amount of time to see what happens.

C.2 $t \in [0, T_1)$

For matches of tenure $t \in [0, T_1)$, the value function depends on both the belief p and tenure t since the time until firing costs become non-zero changes with tenure. Therefore, W^1 and $\underline{p}^{1,W}$ must jointly satisfy the HJB equation

$$rW^1(p,t) = w^1(p,t) + W_t^1(p,t) + \Sigma(p) W_{pp}^1(p,t) - \delta\left(W^1(p,t) - U\right)$$

and boundary conditions

$$W^1\left(\underline{p}^{1,W}(t), t\right) = U$$

$$W_p^1\left(\underline{p}^{1,W}(t), t\right) = 0$$

$$W^1(p, T_1) = W^{*,2}(p)\ \forall p \geq \underline{p}^{1,W}(T_1)$$

The final boundary condition says that at tenure $t = T_1$, the value function W^1 is pinned down by a function $W^{*,2}$ which will be determined in equilibrium. Note that in general $W^{*,2} \neq W^2$ for the same reasons as in the firm case. The remaining boundary conditions have the same interpretations as before, except now they also depend on tenure.

D. Wage Determination via Nash Bargaining

With a slight abuse of notation, we can define the surplus of a match with current belief p and tenure t as the value of the match to the firm and worker minus their respective outside options,

$$S^i(p,t) = J^i(p,t) - V + \kappa(t) + W^i(p,t) - U$$

where $i \in \{1, 2\}$ is value function label that corresponds to the tenure t. The Nash bargaining solution is hence given by

$$w^i(p,t) = \arg\max_w \left(W^i(p,t) - U\right)^\beta \left(J^i(p,t) - V + \kappa(t)\right)^{1-\beta}$$

which has the FOC

$$\beta\left(J^i(p,t) - V + \kappa(t)\right) = (1-\beta)\left(W^i(p,t) - U\right)$$

Note that this condition together with the "value matching" boundary conditions of the firm and worker HJB equations imply that

$$\underline{p}^{i,J}(t) = \underline{p}^{i,W}(t) = \underline{p}^i(t)$$

so that optimal firm and worker belief thresholds coincide at all tenures. Using the expressions for the value functions to solve for the wage function yields

$$w^i(p,t) = (1-\beta)\,b + \beta\left(\bar{\mu}(p) + \lambda J^1(p_0,0) - (r+\delta+\lambda)\,V + (r+\delta)\,\kappa(t)\right)$$

This expression shows how the worker's wage is a weighted average of her outside option (leisure during unemployment) and her inside option (the expected surplus flow plus the continuation value from a match). Importantly, we see that the firing cost enters positively. Intuitively, the worker can exploit the fact that the firm must pay a firing cost today in order to terminate the match, to increase her share of the surplus.

D.1 $t \uparrow T_1$

Recall that we have postulated a discontinuity in the value functions at $t = T_1$, where the firing costs jump from zero to a positive number. This discontinuity stems from the change in wages as tenure approaches T_1 from below. For $t < T_1$, the wage is given by

$$w^1(p,t) = (1-\beta)\,b + \beta\left(\bar{\mu}(p) + \lambda J^1(p_0,0) - (r+\delta+\lambda)\,V\right)$$

while for $t \geq T_1$, the wage is given by

$$w^2(p,t) = (1-\beta)\,b + \beta\left(\bar{\mu}(p) + \lambda J^1(p_0,0) - (r+\delta+\lambda)\,V + (r+\delta)\,\kappa\right)$$

Since $\kappa(t)$ jumps at $t = T_1$, the wage will also jump. In particular, holding the belief fixed, as tenure crosses T_1, the wage jumps by an amount

$$\omega^1 = \beta\,(r+\delta)\,\kappa > 0$$

E. Equilibrium Matched Firm Value Functions

We can now substitute the wage functions into the corresponding firm value functions, where we maintain the division of tenure for clarity.

E.1 $t \geq T_1$

J^2 and $\underline{p}^2$ must satisfy the HJB equation

$$(r+\delta)\,J^2(p) = (1-\beta)\,(\bar{\mu}(p) - b) + \Sigma(p)\,J^2_{pp}(p,t) - \beta\lambda J^1(p_0,0) + \beta\,(r+\delta+\lambda)\,V - \beta\,(r+\delta)\,\kappa$$

and boundary conditions

$$J^2\left(\underline{p}^2\right) = V - \kappa$$

$$J^2_p\left(\underline{p}^2\right) = 0$$

E.2 $t \in [0, T_1)$

Noting that $\kappa(t) = 0$ for $t \in [0, T_1)$, J^1 and $\underline{p}^1$ must satisfy the HJB equation

$$(r+\delta) J^1(p,t) = (1-\beta)(\bar{\mu}(p) - b) + J_t^1(p,t) + \Sigma(p) J_{pp}^1(p,t) - \beta\lambda J^1(p_0, 0) + \beta(r+\delta+\lambda) V$$

and boundary conditions

$$J^1\left(\underline{p}^1(t), t\right) = V$$

$$J_p^1\left(\underline{p}^1(t), t\right) = 0$$

$$J^1(p, T_1) = J^{*,2}(p) \ \forall p \geq \underline{p}^1(T_1)$$

where we can now derive the function $J^{*,2}$ as the value of the match with belief p at tenure T_1 were the wage not to jump by ω^1 due to the firing cost,

$$J^{*,2}(p) = J^2(p) + \beta\kappa$$

F. Equilibrium Employed Worker Value Functions

We can also substitute the wage functions into the corresponding worker value functions, where we maintain the division of tenure for clarity.

F.1 $t \geq T_1$

W^2 and $\underline{p}^2$ must satisfy the HJB equation

$$rW^2(p) = (1-\beta) b + \beta\left(\bar{\mu}(p) + \lambda J^1(p_0, 0) - (r+\delta+\lambda) V + (r+\delta)\kappa\right) + \Sigma(p) W_{pp}^2(p) - \delta\left(W^2(p) - U\right)$$

subject to the boundary conditions

$$W^2\left(\underline{p}^2\right) = U$$

$$W_p^2\left(\underline{p}^2\right) = 0$$

F.2 $t \in [0, T_1)$

Noting that $\kappa(t) = 0$ for $t \in [0, T_1)$, W^1 and $\underline{p}^1$ must satisfy the HJB equation

$$rW^1(p,t) = (1-\beta) b + \beta\left(\bar{\mu}(p) + \lambda J^1(p_0, 0) - (r+\delta+\lambda) V\right) + W_t^1(p,t) + \Sigma(p) W_{pp}^1(p,t) - \delta\left(W^1(p,t) - U\right)$$

subject to the boundary conditions

$$W^1\left(\underline{p}^1(t), t\right) = U$$

$$W_p^1\left(\underline{p}^1(t), t\right) = 0$$

$$W^1(p, T_1) = W^{*,2}(p) \ \forall p \geq \underline{p}^1(T_1)$$

where we can now derive the function $W^{*,2}$ as the value of a match with belief p at tenure T_2 were the firing cost function to remain flat so that wages did not jump by ω^1,

$$W^{*,2}(p) = W^2(p, T_1) - \beta\kappa$$

G. Ergodic Distribution of Beliefs

Recall that the posterior belief that a match is of high quality evolves according to the process

$$dp_t = p_t(1-p_t)\, s d\bar{Z}_t$$

where $s = \frac{\mu^H - \mu^L}{\sigma}$ is the signal/noise ratio and

$$d\bar{Z}_t = \frac{1}{\sigma}\left(dX_t - \left(p_t\mu^H + (1-p_t)\mu^L\right) dt\right)$$

is a standard Brownian motion with respect to the filtration generated by X_t, $\mathcal{F}_t^X$. We would like to characterize the steady-state distribution of beliefs induced by this stochastic process. We again divide to derivation into 2 stages, according to match tenure, and define 2 belief distribution functions f^1, and f^2. In what follows, the function $f^i(p,t)$ should be interpreted as the distribution over the unit interval of beliefs for a cohort of matches that began production at the same moment in time, and have reached tenure t.[3]

G.1 $t \leq T_1$

We can characterize the evolution of the distribution of beliefs using the Kolmogorov Forward Equation (KFE), which states that f^1 evolves according to the equation

$$\frac{\partial}{\partial t} f^1(p,t) = \frac{\partial^2}{\partial p^2}\left[\Sigma(p) f^1(p,t)\right] - \delta f^1(p,t)$$

for $p \in (\underline{p}^1(t), 1]$, with boundary conditions

$$f^1\left(\underline{p}^1(t), t\right) = 0$$

$$\int_{\underline{p}^1(0)}^{1} f^1(p,0)\, dp = \lambda u$$

$$f^1(p,0) = \Delta(p - p_0)$$

where we recall that λ is the finding rate and u is the mass of unemployed workers, and we define Δ as the Dirac delta function that places all mass at the initial prior p_0.

The KFE states that the change in density at a belief p for a small change in tenure is the sum of

[3] The f functions are not strictly distributions since I do not require them to sum to 1. Instead, their total mass will be total employment, and hence 1 minus total unemployment since we have normalized the mass of workers to unity.

two components. First, beliefs move around in the distribution according the evolution equation. The change in density caused by these movements is captured by the first term on the right hand side. Second, at any belief, a fraction δ of matches end exogenously, causing a negative change to the density captured by the second term.

The first boundary condition ensures that there is always zero mass at the termination threshold at tenure t, since when a match belief reaches the threshold, it is immediately terminated. The second imposes that the size of a new cohort is equal to the flow out of unemployment. Finally, the third condition imposes that all initial matches start with prior belief p_0.

G.2 $t \geq T_1$

The KFE states that f^2 evolves according to

$$\frac{\partial}{\partial t} f^2(p,t) = \frac{\partial^2}{\partial p^2}\left[\Sigma(p) f^2(p,t)\right] - \delta f^2(p,t)$$

for $p \in (\underline{p}^2, 1]$, with boundary conditions

$$f^2\left(\underline{p}^2, t\right) = 0$$

$$\int_{\underline{p}^2(T_1)}^{1} f^2(p,T_1)\,dp = \int_{\underline{p}^2(T_1)}^{1} f^1(p,T_1)\,dp$$

$$f^2(p,T_1) = f^1(p,T_1)\,\mathbf{1}\left\{p \in \left[\bar{p}^{1,J}(T_1), 1\right]\right\}$$

The interpretations of the KFE and the first boundary condition are the same as above. The second condition ensures consistency of total mass as match tenure crosses T_1. The final condition initializes f^2 as the truncation of the final f^1 distribution for beliefs above the threshold $\bar{p}^{1,J}$. This condition captures the effect of the spike termination rate that occurs at tenure T_1.

G.3 Ergodic Distribution of Beliefs

Combining f^1 and f^2 yields the ergodic distribution of beliefs,

$$g(p) = \int_0^{T_1} f^1(p,t)\,\mathbf{1}\left\{p \geq \underline{p}^1(t)\right\} dt + \int_{T_1}^{\infty} f^2(p,t)\,\mathbf{1}\left\{p \geq \underline{p}^2\right\} dt$$

Hence the mass of employed workers is given by

$$e = \int_0^1 g(p)\,dp = 1 - u$$

H. The Matching Function

Let the number of matches be given by

$$m = zu^{\eta}v^{1-\eta}$$

where z is matching efficiency, and u and v are the masses of unemployed workers and vacancies posted by firms respectively. This implies the job finding rate

$$\lambda = z\theta^{1-\eta}$$

and job filling rate

$$q = z\theta^{-\eta}$$

where

$$\theta = \frac{v}{u}$$

is labor market tightness.

I. Vacancy Posting

A vacant firm has a value function V that satisfies

$$(r+q)\,V = qJ^{1}\left(p_0, 0\right) - c$$

where q is the job filling rate, and c is the per-period cost of maintaining a vacancy. Free entry into the market for vacancies implies that, in equilibrium, $V = 0$.

J. Equilibrium Analysis

We can now define a stationary equilibrium for our economy.

Definition 1. A stationary equilibrium is a set of scalars

$$\left\{\lambda, q, \theta, u, v, \underline{p}^{2}\right\}$$

and functions

$$\left\{J^{1}, J^{2}, W^{1}, W^{2}, U, \underline{p}^{1}, w^{1}, w^{2}, f^{1}, f^{2}\right\}$$

such that:

1. $\left\{J^{1}, J^{2}, W^{1}, W^{2}, U, \underline{p}^{1}, \underline{p}^{2}\right\}$ satisfy their HJB equations and boundary conditions.
2. $\left\{w^{1}, w^{2}\right\}$ satisfy the Nash bargaining condition.
3. $\left\{f^{1}, f^{2}\right\}$ satisfy their KFEs, and boundary and initial conditions.

4. $\{\lambda, q, \theta\}$ satisfy their definitions.

5. v ensures free entry in the vacancy posting market.

6. u is consistent with total employment implied by $\{f^1, f^2\}$.

J.1 The Effect of Firing Costs

A key feature of our model is that firms must pay a firing cost in order to terminate a match that has tenure of at least T_1. The following logic formalizes the effect that this firing cost has on the hazard rate of match termination.

Define the firm-specific thresholds $\underline{p}^{1,J}(T_1)$ and $\bar{p}^{1,J}(T_1)$, that satisfy the conditions

$$J^1\left(\underline{p}^{1,J}(T_1), T_1\right) = V$$

$$J^2\left(\bar{p}^{1,J}(T_1)\right) = J^1\left(\bar{p}^{1,J}(T_1), T_1\right) - \beta\kappa = V$$

where $\bar{p}^{1,J}(T_1) > \underline{p}^{1,J}(T_1)$ since firm value functions are increasing in the current belief. Then, at tenure T_1, any match with belief $p \in \left[\underline{p}^{1,J}(T_1), \bar{p}^{1,J}(T_1)\right]$ will be immediately terminated since the value of such a match will instantaneously drop to $J^2(p, T_1) < V$. This extra firing at will create a spike in the hazard rate at T_1.

Intuitively, when tenure reaches T_1, there is an interval of beliefs such that a match with a belief in that interval would not be terminated were it not for the presence of a firing cost. While such matches are reasonably productive, they are not productive enough to warrant the firm continuing the match and paying the firing cost if productivity deteriorates later. All matches with beliefs in this interval are therefore terminated at tenure T_1, thus creating a spike in the hazard rate at tenure T_1. We will find strong support for such behavior in the data.

Analogously, we can define the worker-specific thresholds $\bar{p}^{1,W}(T_1)$ and $\underline{p}^{1,W}(T_1)$ that satisfy the conditions

$$W^1\left(\bar{p}^{1,W}(T_1), T_1\right) = U$$

$$W^1\left(\underline{p}^{1,W}(T_1), T_1\right) = U - \beta\kappa$$

where $\bar{p}^{1,W}(T_1) > \underline{p}^{1,W}(T_1)$ since worker value functions are increasing in the current belief. Then, at tenure T_1, any match with belief $p \in \left[\underline{p}^{1,W}(T_1), \bar{p}^{1,W}(T_1)\right]$ will not be terminated since the value of such a match will instantaneously jump to $W^2(p, T_1) \geq U$.

However, applying the FOC from the Nash bargaining protocol to beliefs as $t \uparrow T_1$ yields

$$\bar{p}^{1,W}(T_1) = \underline{p}^{1,J}(T_1)$$

so that the interval of no termination for workers lies strictly in a range of beliefs that would have resulted in match termination at an earlier tenure. Therefore, we can safely ignore the effect of firing costs on worker termination incentives and focus solely on firms.

J.2 Aggregate Output and Social Welfare

Since production is linear in productivity beliefs, we can define aggregate output as

$$Y = \mu^H \int_0^1 pg(p)\,dp + \mu^L \int_0^1 (1-p)\,g(p)\,dp$$

which has the simple interpretation of the number of good matches times the productivity of a good match plus the number of bad matches times the productivity of a bad match.

Exploiting the linearity of preferences, and recalling that $u = 1 - \int_0^1 g(p)\,dp$, we can define a simple measure of steady state welfare as

$$\mathcal{W} = b\left(1 - \int_0^1 g(p)\,dp\right) + \mu^H \int_0^1 pg(p)\,dp + \mu^L \int_0^1 (1-p)\,g(p)\,dp$$

which is the flow value of leisure times the mass of unemployed workers plus the flow value of production, given by aggregate output. This expression also indicates that firing costs affect welfare by altering the shape of the ergodic distribution of beliefs (and hence productivities) g.

J.3 Efficiency

Although our model contains both endogenous job creation and job destruction, the Hosios (1990) condition still applies to the economy without firing costs (see Rogerson et al. (2005) for a simple example). In particular, the equilibrium coincides with the planning solution when the worker's bargaining share is equal to the elasticity of matches with respect to the unemployment rate, i.e. when $\beta = \eta$.

When $\beta > \eta$, the inefficiency of the equilibrium is as follows: the relatively high value of the worker's bargaining share implies that firms do not create enough vacancies in equilibrium since their share of the associated surplus is relatively low. The lower vacancy rate implies a lower job finding rate, which in turn lowers the worker's outside option that enters the Nash bargaining protocol, and hence raises the firm's surplus share and value of being in a match. The increase in firm value leads firms to optimally lower their belief threshold at which matches are terminated. A lower threshold results in fewer endogenous job separations. Therefore, when $\beta > \eta$, both the vacancy creation rate and job separation rate are too low. The opposite logic applies to the case in which $\beta < \eta$, and both the vacancy creation rate and job separation rate are too high relative to the planner's solution.

When the economy without firing costs is inefficient, the imposition of regulation may be welfare-improving. In particular, since the imposition of regulation will lead to a lower match termination threshold, regulation will be welfare-improving if the original threshold was too high, i.e. when $\beta < \eta$. Whether this condition is met depends on the value of the parameters estimated from the data.

III. Estimation Procedure

The model features 13 parameters: $T_1, \mu^L, \mu^H, r, \eta, z, b, p_0, c, \delta, \sigma, \beta, \kappa$. Since our key data variation is the hazard rate schedule, we use these data to estimate the exogenous separation rate, δ, the learning parameter σ, the bargaining share β, and the firing cost parameter κ. We normalize and or calibrate the remaining parameters.

A. Calibration

We set T_1 = 3 months to reflect the tenure at which firing costs and other labor market regulations become active. Since μ^L and μ^H simple set the location and scale of production in the economy, we normalize them to $\mu^L = 0$ and $\mu^H = 1$ respectively. We set the annual discount rate $r = 7.5\%$ which is in line with Brazilian interest rates. The weight on unemployment in the matching function, η, is set to 0.25, in line with the estimates in Hoek (2007) who estimates a Cobb-Douglas matching function for Brazil. The matching efficiency parameter z is another free normalization, and so is set so that market tightness $\theta = 1$, and the job finding rate $\lambda = 0.08$ in our baseline economy. These choices imply a steady state unemployment rate of around 11%, which is consistent with recent Brazilian data. These choices are consistent with a steady state unemployment rate of 10%, which is in line with Brazilian data. The value of leisure is set at $b = 0.6$ in order to reflect 2 considerations. First, b includes unemployment benefits, which are valued at between 15% and 80% of a worker's previous wage. Second, as argued by Hagedorn and Manovskii (2008), the value of leisure parameter should be such that workers are indifferent between working and not working on the margin, so that b is equal to average labor productivity. Our choice of $b = 0.6$ strikes a balance between each of theses forces, and results in a value of leisure equal to 75% of the mean wage. We explore alternative calibrations in the robustness exercises. The vacancy posting cost parameter is set at $c = 0.25$ so that the total cost of hiring a new worker is approximately equal to paying average wages for 2 months, which is in line with Mortensen and Pissarides (1999). Finally, the initial prior belief parameter, p_0, is pinned down endogenously by the free entry condition for vacancy posting (given all other parameters),

$$c = z\theta^{-\eta} J^1 (p_0, 0)$$

B. Estimation

We estimate δ, σ, β, and κ via the method of simulated moments. Formally, letting $\Xi = (\delta, \sigma, \beta, \kappa)$ be the vector of parameters, define the vector of model-generated hazard rates for the first 4 years of tenure as

$$H^{mod}(\Xi) = \left(h_1^{mod}(\Xi), h_2^{mod}(\Xi), ..., h_{96}^{mod}(\Xi)\right)$$

where the vector length of 96 reflects the 15 day spacing of hazard rates to match the data. We use 4 years of hazard rate data to cleanly measure the hazard rate after the effects of the learning process have died out so that we can identify δ. Similarly, define the vector of hazard rates in the data as

$$H = (h_1, h_2, ..., h_{96})$$

Then, we choose parameters Ξ to solve

$$\min_{\Xi} \left(H^{mod}(\Xi) - H \right) W \left(H^{mod}(\Xi) - H \right)'$$

where W is a weighting matrix. We set W to the identity matrix in our baseline estimation. Finally, we obtain standard errors using a bootstrap procedure with 100 replications.

C. Identification

Parameters δ, σ, and β determine the shape of the hazard rate schedule in the absence of firing costs. As just mentioned, the exogenous separation rate δ determines the level to which the hazard rate converges as tenure increases and the learning process becomes less prevalent. The speed of this convergence, and hence the slope of the hazard rate schedule at medium to longer term tenures, is governed by σ since this noise parameter determines the "speed" at which beliefs move around the interval $[0, 1]$: the higher is sigma, the noisier is the belief updating process, and the slower beliefs move. Finally, the bargaining share parameter β controls the average level of hazard rates, especially at shorter term tenures. Intuitively, when β is higher, workers receive a larger fraction of the match surplus, so that firms have weaker incentives to terminate a match since new matches are less valuable to them. Therefore, hazard rates are lower.

In the absence of firing costs, the hazard rate schedule has the usual hump shape. Therefore, the impact of labor market regulation is identified by the distortions to the hazard rate schedule away from this hump shape. In particular, κ is identified from the size of the spike in the hazard rate at 3 months of tenure.

IV. Institutions and Data

A. Institutions

Formal sector employment in Brazil is governed by the *Consolidação das Leis do Trabalho* (CLT) laws. The CLT laws guarantee all workers a set of benefits which include severance pay if dismissed without cause, a bonus salary equivalent to one month's salary paid every December, 30 days notice for any separation, and, if given notice of separation, 2 hours per day to search for a new job. Importantly, these benefits only apply after a 3 month probationary period.

In addition to guaranteed benefits, the firm must also place 8.5 percent of the worker's monthly salary into a savings account (FGTS) which the worker cannot access. In the event of a separation without cause, the worker gains access to the FGTS savings account. Firing with cause only applies to situations in which the worker fails to perform their duties or breaks the law. Importantly, firing a worker due to economic distress is not considered a separation with just cause. If the firm does fire a worker without cause, the firm must pay a firing penalty which amounts to 50 percent of the gross amount accrued in the FGTS account during the employment spell. 80 percent of this penalty goes directly to the worker, while 20 percent goes to the government. Similar to the guaranteed benefit provisions, the firing costs only take effect once tenure has reached 3 months. Lastly, if a job ends after a year of tenure, an examination is made by the Labor Ministry, which investigates whether the firm complied with labor regulations for the entire duration of the job.

The wide-ranging scope of these labor market regulations make it very difficult to directly analyze their effects on firm decisions, especially those related to firing workers. In light of this, we follow Garicano et al. (2016) and use our theoretical framework to recover the costs of regulation using the observed choices made by firms. Crucially, the fact that such regulations do not take effect until job tenure has reached 3 months gives us a clean way to identify the effects that these regulations have on the decision to keep or fire a worker.

B. Data and Descriptive Statistics

Our analysis utilizes administrative data from the *Relação Anual de Informações Sociais* (RAIS), years 2002-2007. The RAIS data contains linked employer-employee records from a mandatory survey administered by the Brazilian Ministry of Labor and Employment (MTE). Fines are levied on firms which provide inaccurate or incomplete information on the survey.

Each entry in the RAIS dataset is a employee-employer match. Each individual, firm and establishment are assigned unique administrative identifiers which do not change over time. The data provides demographic data on the individual, such as education, gender, ethnicity, and occupation. It also includes information about the job, such as occupation, tenure, wage, hours, type of labor contract, whether the job has ended, and why the job has ended. Lastly, the data includes information on firms, including region, sector, and legal classification (i.e. private enterprise vs. public company). Most importantly for our study, the RAIS contains data on job tenure. We bin tenure into 15 day intervals due to "heaping" in the distribution of tenure (i.e. it is much more likely to observe a 30 days job spell than a 29 day job spell). For more information about the dataset and the definition of variables, see section C in the appendix.

B.1 Temporary Contracts

An important institutional detail for our study is the existence and regulation of temporary contracts. In Brazil, such contracts must be approved by the Ministry of Labor (MTE). Temporary contracts are only approved to meet a temporary need, such as seasonal variation in demand. While such fixed term contracts may be extended up to a maximum of two years, in our data, we find many fixed term contracts are specified for three months, which confounds the interpretation of the spike in the job termination hazard as the result of labor regulations becoming active. In order to avoid this issue, we drop all temporary contracts from the main analysis.

C. Sample Selection

As just mentioned, we exclude all individuals employed under temporary contracts (about 5 percent of jobs), since including these jobs would confound the interpretation of the hazard rate spike at three months. We further restrict attention to workers aged 18-65, and working in full-time jobs (at least 35 hours per week). We exclude individuals with invalid identifiers (less than one percent of the data). Column 1 of Table 1 presents summary statistics for the population of 18-65 year olds. Column 2 presents summary statistics for jobs which last less than or equal to three months. Short-duration workers are slightly younger (30.4 vs. 31.5), less likely to be a college graduate (3.8 percent vs. 7.2 percent), are paid lower monthly wages (670.90 Real vs 819.21 Real), and are less likely to be in public administration jobs (7.4 percent vs. 2.2 percent) and more likely to be in agricultural jobs (14.0 percent vs. 9.3 percent). In total, there are 92,023,307 jobs corresponding to 29,438,306 unique workers. 24,427,409 jobs last three months or less (i.e. 26.5 percent of all jobs).

V. Results

A. Qualitative Features of the Data

The hazard rate schedule plays a key role in our model estimation strategy. Therefore, we begin by analyzing the pattern of hazard rates in the data, as shown in figure 2. There is a sharp spike in the job termination hazard at exactly three months of tenure. The hazard rate is 0.021 at 2.5 months, 0.047 at 3 months, and then 0.018 at 3.5 months. This pattern of hazard rates is consistent with a key theoretical prediction of our model: a spike in the hazard rate at the tenure at which labor market regulations become active.

Our structural model posits that endogenous match separations are driven by employers and employees learning about the quality of the match over time, and agreeing to separate when the belief that the match is of high quality is sufficiently low. An additional prediction of such a model is that the relationship between wages and tenure should be increasing but concave. Intuitively, conditional on a match reaching a given tenure, the belief that the match is of high quality, and hence the wage, must have risen on average. At long enough tenures, the belief gets closer and closer to 1, so that the wage flattens out as tenure continues to increase. We test this prediction by estimating the relationship between wage and tenure, and find that wages initially increase quickly, and then flatten out at longer tenures, just as predicted by the model. Details of this procedure are in section B in the appendix. This finding also rules out models in which match productivity is subject to idiosyncratic shocks that follow mean-reverting or random-walk processes, as in Prat (2003).

In addition to our learning mechanism, there are at least 2 other possible explanations for the spike in the hazard rate at a tenure of 3 months. First, the spike could be driven by demand volatility. For example, suppose service workers are hired in October to handle excess demand during holiday months, and then laid off January 1^{st}. This would cause a spike in job separations at three months, but the source would not be the learning mechanism. To test this explanation, in appendix section B, we estimate bunching (i.e. the excess mass in job separations at three months) across a range of sectors and look for heterogeneity in the bunching masses. Intuitively, if demand volatility is driving the bunching then we would expect sectors with more demand volatility to exhibit more bunching. However, we find that bunching occurs across almost all sectors and is not significantly correlated with measures of employment (and hence demand) volatility.

Second, firms could be rotating through workers to avoid firing costs. In this case, we would expect to see relatively more bunching in low-skill occupations where it may be relatively easy to replace workers, since in high-skill occupations it seems unreasonable to constantly rotate through workers given the presence of larger training and search costs. However, when we split the sample by the skill of the occupation, we find similar levels of bunching across occupations, indicating that rotating through workers is not driving bunching. Details are in appendix section B.

B. Econometric Results

Table 2 shows the estimation results in our baseline structural model. Standard errors are computed using a bootstrap procedure. Note that the large sample size of our data set ensures that our parameters are very precisely estimated.

The interpretation of δ is simply the hazard rate for jobs of sufficiently long tenure such that there is very little left for the firm and worker to learn about the quality of the match. The estimate of σ implies that, in the absence of firing costs, the hazard rate peaks at a job tenure of around 5 months with a probability of being fired of about 2.5%. The value of β implies that workers receive 36% of the surplus generated by a match. Note that this is larger than the elasticity of unemployment in the matching function, $\eta = 0.25$. Finally, the value of the firing cost parameter κ implies that the cost to a firm of maintaining a job beyond a tenure of 3 months is approximately 1.3% of the mean annual wage in the economy. Recall that this parameter incorporates the wide variety of labor market regulations in place in Brazil. Its estimated value is therefore a monetary summary of how all of this regulation impacts firms.

Hazard Rates Figure 3 plots the hazard rate as a function of job tenure both in the data and in the estimated model. Our sparse parameterization does a reasonable job of capturing key features of the hazard rate schedule shape. In particular, the presence of a fixed firing cost at tenures beyond 3 months results in a spike in the model hazard rate at 3 months, just as in the data.

C. The Effect of Firing Costs

Figure **??** plots the hazard rate in the data, estimation, and in the structural model with the firing costs removed ($\kappa = 0$). In the absence of firing costs, the hazard rate function resembles the standard hump shape. In particular, the hazard rate without firing costs does not spike at 3 months, and is uniformly higher than the hazard with firing costs at tenures greater than 3 months, but lower than the hazard with firing costs at tenures less than 3 months. In other words, the presence of firing costs causes firms to terminate matches earlier than they otherwise would, just as the theory predicted.

Table 3 summarizes the key aggregate implications of this change in hazard rate pattern, where the change is measured from the structural model with firing costs to the model without firing costs. Removing firing costs, holding all other structural parameters fixed, leads to an increase in output of 0.11%, and a decrease in the unemployment rate of 0.1 percentage points. As a result, social welfare also increases by 0.04%. Therefore, the presence of firing costs has a small negative effect on the aggregate economy. The final row also indicates that removing firing costs raises the long-run productivity threshold in the economy by 0.2 percentage points.

C.1 Explaining the effect of firing costs

The results indicate that labor market regulations have a small negative impact on aggregate outcomes in the economy. The negative impact of regulation stems from the fact that $\beta > \eta$ in the estimated structural model, which violates the Hosios (1990) condition for efficiency. In particular, it implies the rate of endogenous match separation is inefficiently low. Intuitively, when $\beta > \eta$, the worker's share of the match surplus is too high, which implies that firms do not create enough vacancies in equilibrium. The lower vacancy rate implies a lower job finding rate, which lowers a worker's outside option in the Nash bargaining protocol, thus raising the firm's value of being in a match. The increase in firm value implies that the optimal choice of belief threshold must fall in equilibrium in order to satisfy the "value matching" condition. A lower threshold then results in fewer endogenous separations.

The fact that the endogenous separation rate is too low implies that any policy that lowers it further cannot be welfare improving. Unfortunately, this is exactly what the labor market regulations analyzed in this paper achieve. The imposition of a fixed cost of termination causes firms to lower their threshold even more, thus moving the economy further away from the efficient point. Fortunately, the estimation results imply that the regulations do not impose a large real cost on firms, so that the decrease in welfare and output is small in magnitude.

VI. Robustness

I this section we consider the robustness of our results to changes in the value of leisure parameter b. We consider 2 additional values of this parameter, $b \in \{0.2, 0.4\}$, which imply values of leisure equal to 55% and 30% of the mean wage respectively. In each case, we re-estimate the model, and re-calibrate the vacancy posting cost parameter c to make each case comparable. We then compute the results of our counterfactual exercise in which we set the firing cost parameter to zero.

A. Results

Table 4 shows the values of the estimated and re-calibrated parameters in each robustness exercise, where we have reprinted the baseline for ease of comparison. Table 5 shows the corresponding results of the counterfactual exercise for each value of b.

The estimation results show that the violation of the Hosios (1990) condition becomes worse as b falls. However, while the implied cost of labor market regulation grows as b declines, it remains quantitatively small. Even when the value of leisure is just 30% of the mean wage, the cost of regulation is 0.3% of GDP. Therefore, we conclude that our baseline quantitative results are not driven by our choices of calibrated parameters.

VII. Conclusion

We use a general equilibrium model of a frictional labor market as a lens to understand the aggregate impacts of labor market regulation. We exploit tenure-dependence of the regulations, and use a rich employer-employee matched dataset in order to estimate key structural parameters of our model, including the bargaining share of workers, and the real cost of the regulations themselves. We use our estimated model to compute the impact of regulation by comparing outcomes to a counterfactual economy in which the regulations are absent, and find that the cost of regulations are approximately 0.1% of GDP. These costs stem from a violation of the Hosios (1990) condition that we estimate, and that the regulations only serve to exacerbate rather than correct for.

References

[1] Di Tella, R. and MacCulloch, R., 2005. The consequences of labor market flexibility: Panel evidence based on survey data. European Economic Review, 49(5), pp.1225-1259.

[2] Boeri, T., 2011. Institutional reforms and dualism in European labor markets. Handbook of labor economics, 4(Part B), pp.1173-1236.

[3] Bosch, M. and Maloney, W., 2007. Comparative analysis of labor market dynamics using markov processes: An application to informality.

[4] Cahuc, P., Charlot, O. and Malherbet, F., 2016. Explaining the spread of temporary jobs and its impact on labor turnover. International Economic Review, 57(2), pp.533-572.

[5] Cahuc, P. and Postel-Vinay, F., 2002. Temporary jobs, employment protection and labor market performance. Labour economics, 9(1), pp.63-91.

[6] Chetty, R., Friedman, J.N., Olsen, T. and Pistaferri, L., 2011. Adjustment costs, firm responses, and micro vs. macro labor supply elasticities: Evidence from Danish tax records. The Quarterly Journal of Economics, 126(2), pp.749-804.

[7] Cosar, A.K., 2010. Adjusting to trade liberalization: Reallocation and labor market policies. University of Chicago Booth School of Business, unpublished working paper.

[8] Daruich, D., Di Addario, S., and Saggio, R., 2018. The Effects of Partial Employment Protection Reforms: Evidence from Italy", unpublished working paper.

[9] Farber, H.S., 1999. Mobility and stability: The dynamics of job change in labor markets. Handbook of labor economics, 3, pp.2439-2483.

[10] Farber, H.S. and Gibbons, R., 1996. Learning and wage dynamics. The Quarterly Journal of Economics, 111(4), pp.1007-1047.

[11] Garibaldi, P. and Violante, G.L., 2005. The employment effects of severance payments with wage rigidities. The Economic Journal, 115(506), pp.799-832.

[12] Gonzaga, G., Maloney, W.F. and Mizala, A., 2003. Labor turnover and labor legislation in Brazil. Economia, 4(1), pp.165-222.

[13] Hoek, J., 2007. Labor Flows in Formal and Informal Labor Markets in Brazil. Unpublished manuscript.

[14] Kahn, L.B., 2013. Asymmetric information between employers. American Economic Journal: Applied Economics, 5(4), pp.165-205.

[15] Kugler, A. and Pica, G., 2008. Effects of employment protection on worker and job flows: Evidence from the 1990 Italian reform. Labour Economics, 15(1), pp.78-95.

[16] Menezes-Filho, N.A. and Muendler, M.A., 2011. Labor reallocation in response to trade reform (No. w17372). National Bureau of Economic Research.

[17] Garibaldi, P. and Violante, G.L., 2005. The employment effects of severance payments with wage rigidities. The Economic Journal, 115(506), pp.799-832.

[18] Lazear, E.P., 1990. Job security provisions and employment. The Quarterly Journal of Economics, 105(3), pp.699-726.

[19] Ljungqvist, L., 2002. How do lay-off costs affect employment? The Economic Journal, 112(482), pp. 829-853.

[20] Moscarini, G., 2005. Job matching and the wage distribution. Econometrica, 73(2), pp.481-516.

[21] Rogerson, R., Shimer, R., and Wright, R. 2005. Search-Theoretic Models of the Labor Market: A Survey. Journal of Economic Literature, 43 (4): 959-988.

[22] Van Doornik, B., Schoenherr, D. and Skrastins, J., 2017. Unemployment Insurance, Strategic Unemployment, and Firm-Worker Collusion. Working Paper.

Table 1: Descriptive Statistics for Estimation Sample, 2002-2007

	All Jobs	Short Duration Jobs
Panel A: Demographics	(1)	(2)
Age	31.535	30.405
High School Graduate	0.336	0.305
College Graduate	0.072	0.038
Male	0.658	0.693
Panel B: Job Characteristics		
Monthly Wage	819.212	670.909
Tenure	13.332	1.681
Hours	43.108	43.444
Panel C: Firm Characteristics		
Manufacturing	0.199	0.185
Agriculture	0.093	0.140
Public Administration	0.074	0.022
Health and Education	0.039	0.023
All Other Sectors	0.595	0.631
Unique Workers	29,438,306	13,312,346
Number of Jobs	92,023,307	24,427,409

Note: Column 1 reports descriptive statistics for jobs between age 18-65 who began a job after after 1986, excluding workers on temporary contracts. Column 2 reports descriptive statistics for jobs which last less than three months. Tenure is measured in months. Wages are denominated in Brazilian Real.

Table 2: Parameter Estimation

Parameter	Estimate	Standard Error
δ	0.0040	4.2e-05
σ	12.14	0.01
β	0.36	0.005
κ	0.27	0.004

Note: This table reports the estimated model parameters: δ is the rate of exogenous job destruction, σ controls the speed of employer learning, β is the worker's bargaining share and κ is the firing costs. The model is estimated using a method of simulated moments procedure which chooses parameters to match the empirical job termination hazard. Standard errors are computed using a bootstrap procedure which re-samples at the worker level, estimates the job termination hazard, and re-estimates the model parameters. The standard errors are equal to the standard deviation of the estimated model parameters across 500 bootstrap replications.

Table 3: Aggregate Results

	Δ
Output	0.11%
Unemployment Rate	-0.1 p.p
Welfare	0.04%
$\underline{p}^2$	0.2 p.p

Note: This table reports the results from a counterfactual simulation in which firing costs are set to zero. $\underline{p}^2$ is the long-run productivity threshold below which workers are endogenously terminated.

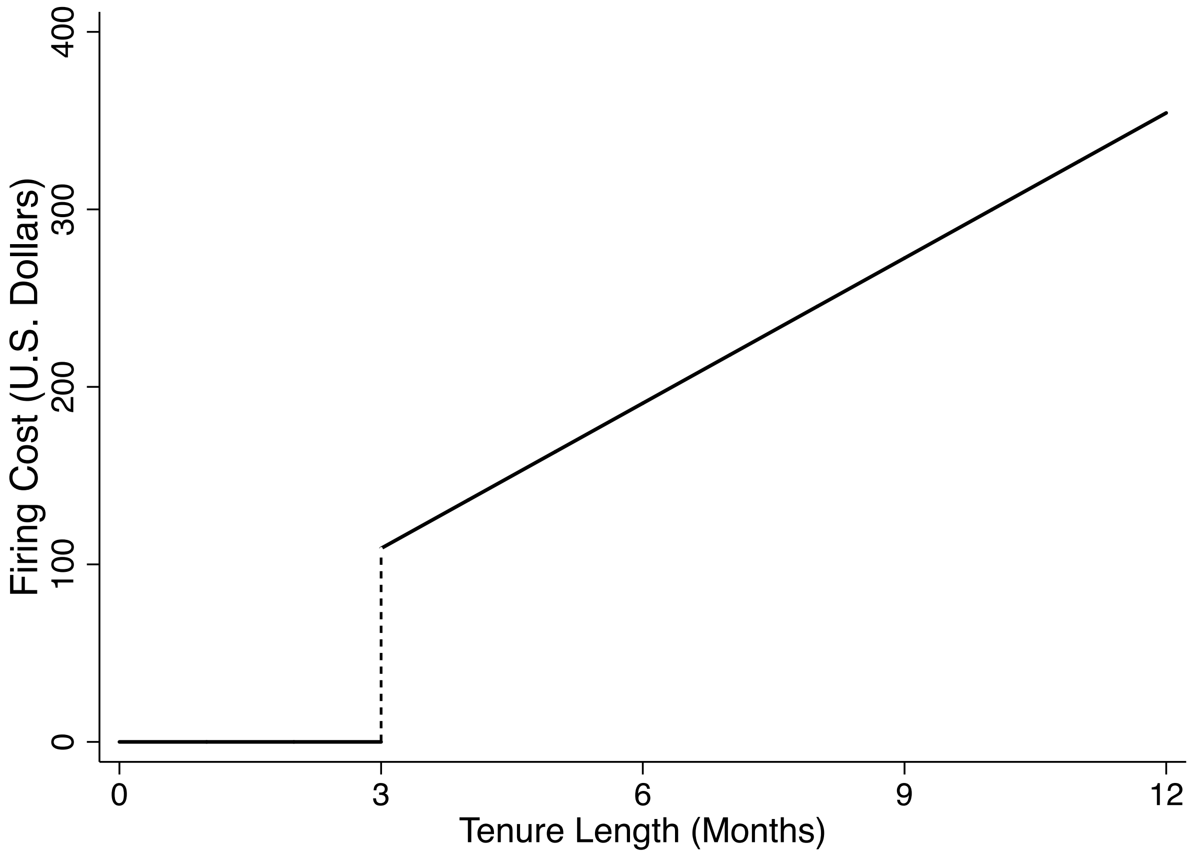
Table 4: Robustness Checks: Estimated Parameters

b	0.2	0.4	0.6
δ	0.003	0.0035	0.004
σ	13.86	14.65	12.14
β	0.63	0.55	0.36
κ	0.24	0.33	0.27
c	0.2	0.2	0.25

Table 5: Robustness Checks: Aggregate Results

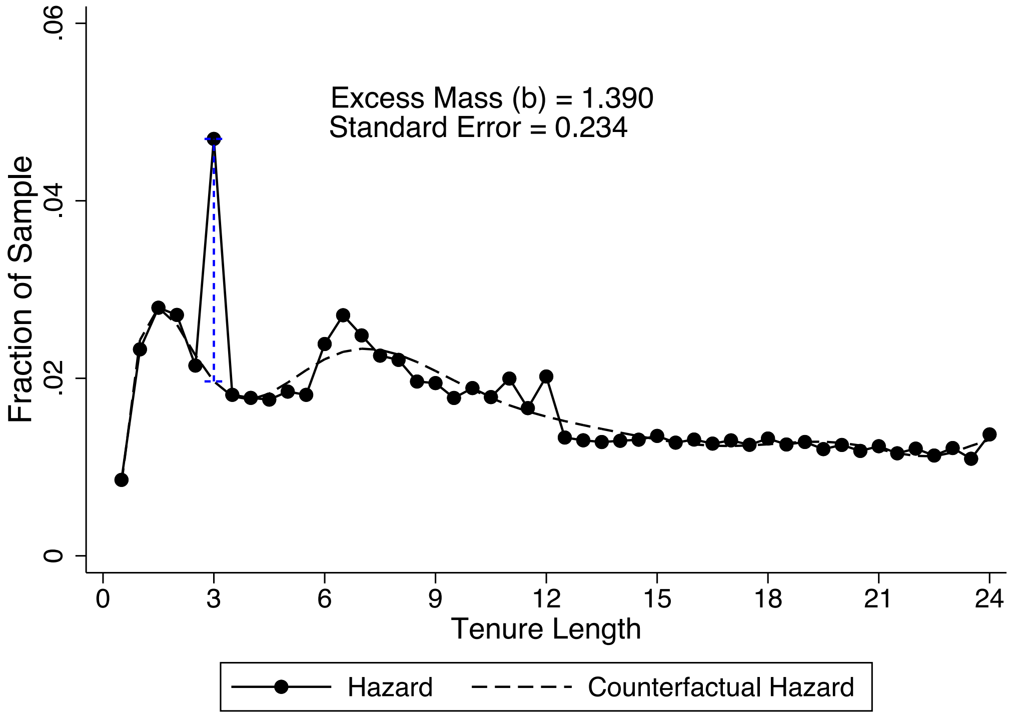
b	0.2	0.4	0.6
ΔY	0.31%	0.27%	0.11%
ΔW	0.21%	0.13%	0.04%
Δu	-0.3 p.p	-0.2 p.p	-0.1 p.p
$\Delta \underline{p}^2$	0.1 p.p	0.2 p.p	0.2 p.p

Figure 1: Firing Cost Schedule



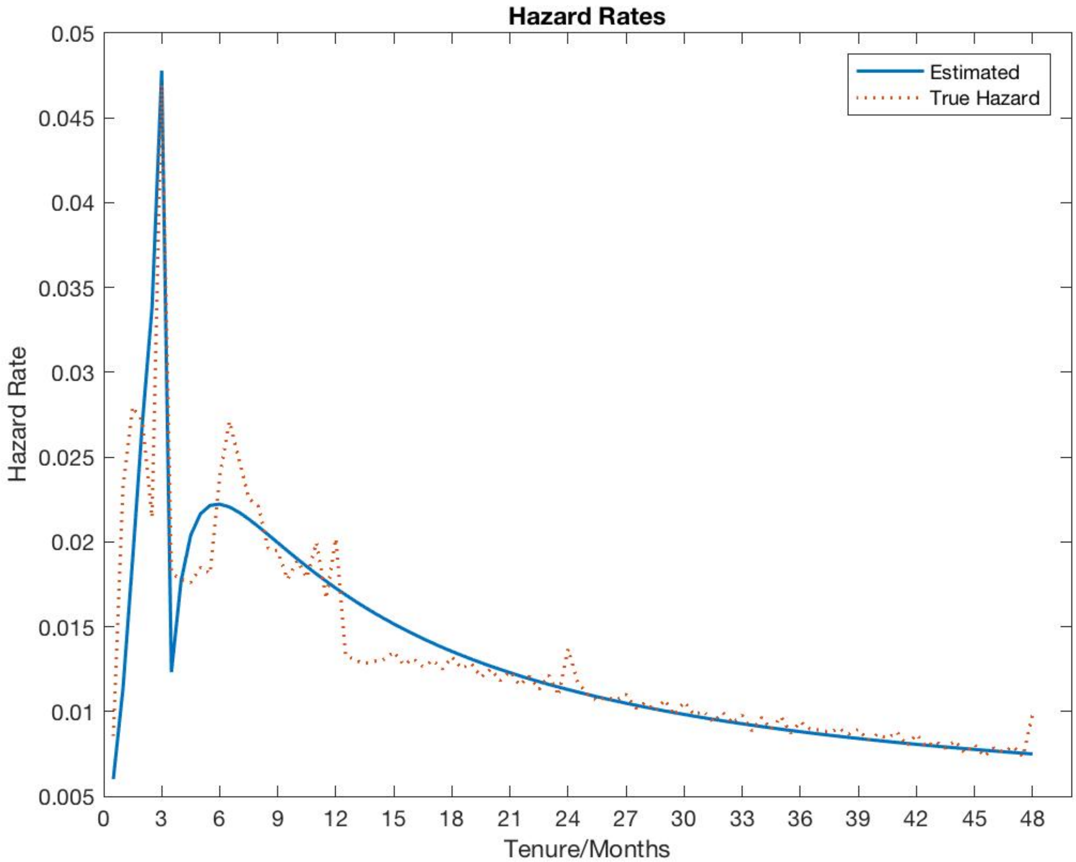
Note: This figures depicts the firing cost schedule for a worker with the average monthly wage in Brazil in 2016 (2115 Real=641.33 USD). 80 percent of the penalty is paid to the worker, while 20 percent is paid to the government

Figure 2: Hazard Rates around Firing Cost Notch



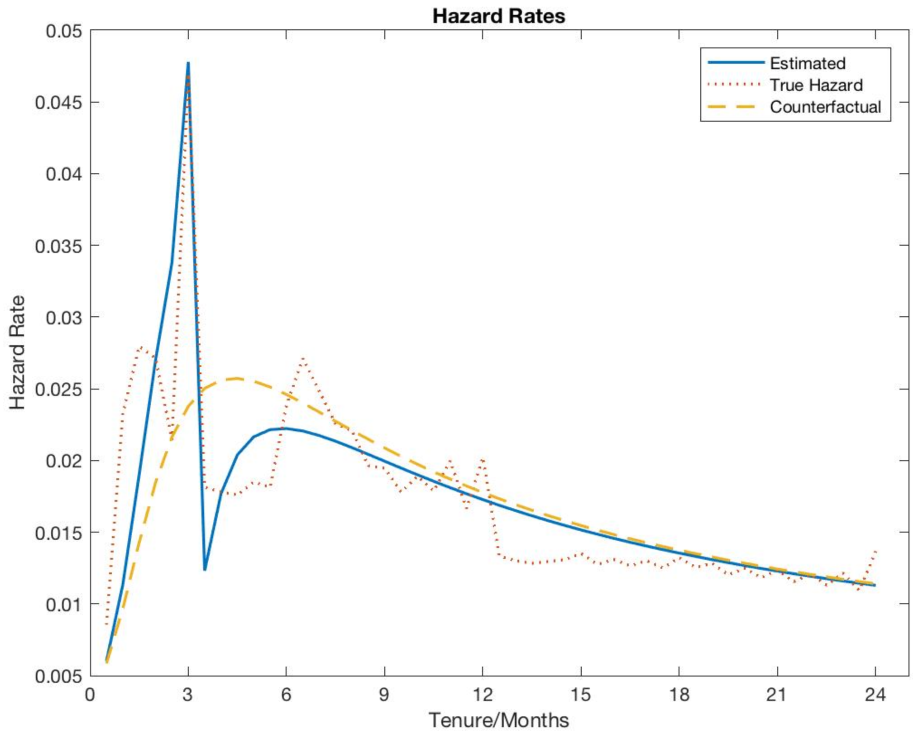
Note: This figure plots the layoff hazard rate. Tenure duration is binned into 15 day intervals. The dashed line is a tenth-degree polynomial fitted to the empirical hazard rate, excluding points 15 days away from the notch, as in Equation (1). The vertical dotted line displays the excess mass B, while the normalized excess mass b and standard error is reported in the figure. The standard error is computed using a residual bootstrap procedure.

Figure 3: Estimated and Empirical Hazard Rates



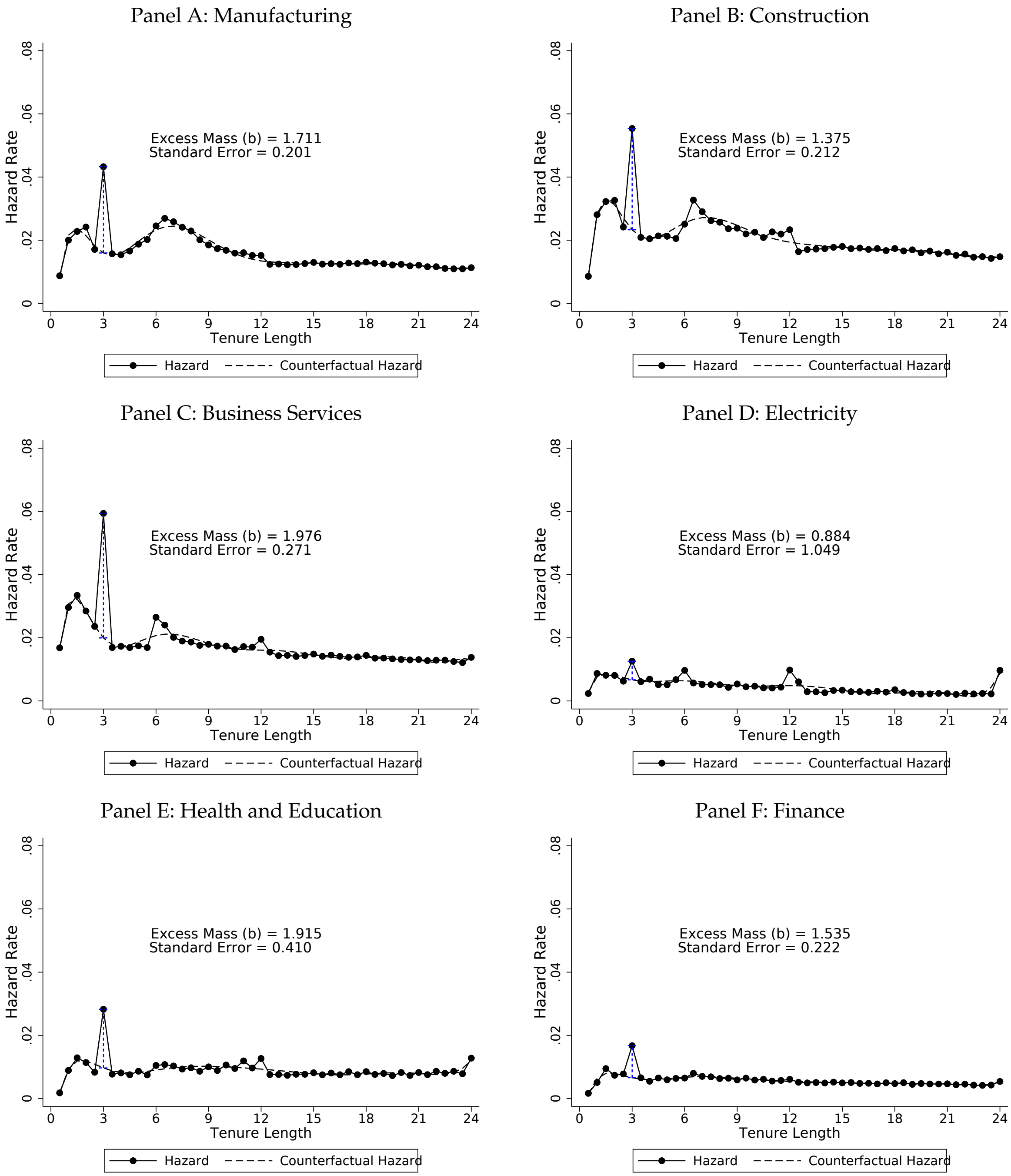
Note: This figure plots the empirical layoff hazard rate (dotted) as well as the hazard rate from the estimated model (solid). Tenure duration is binned into 15 day intervals to estimate the empirical layoff hazard.

Figure 4: Estimated, Empirical, and Counterfactual Hazard Rates



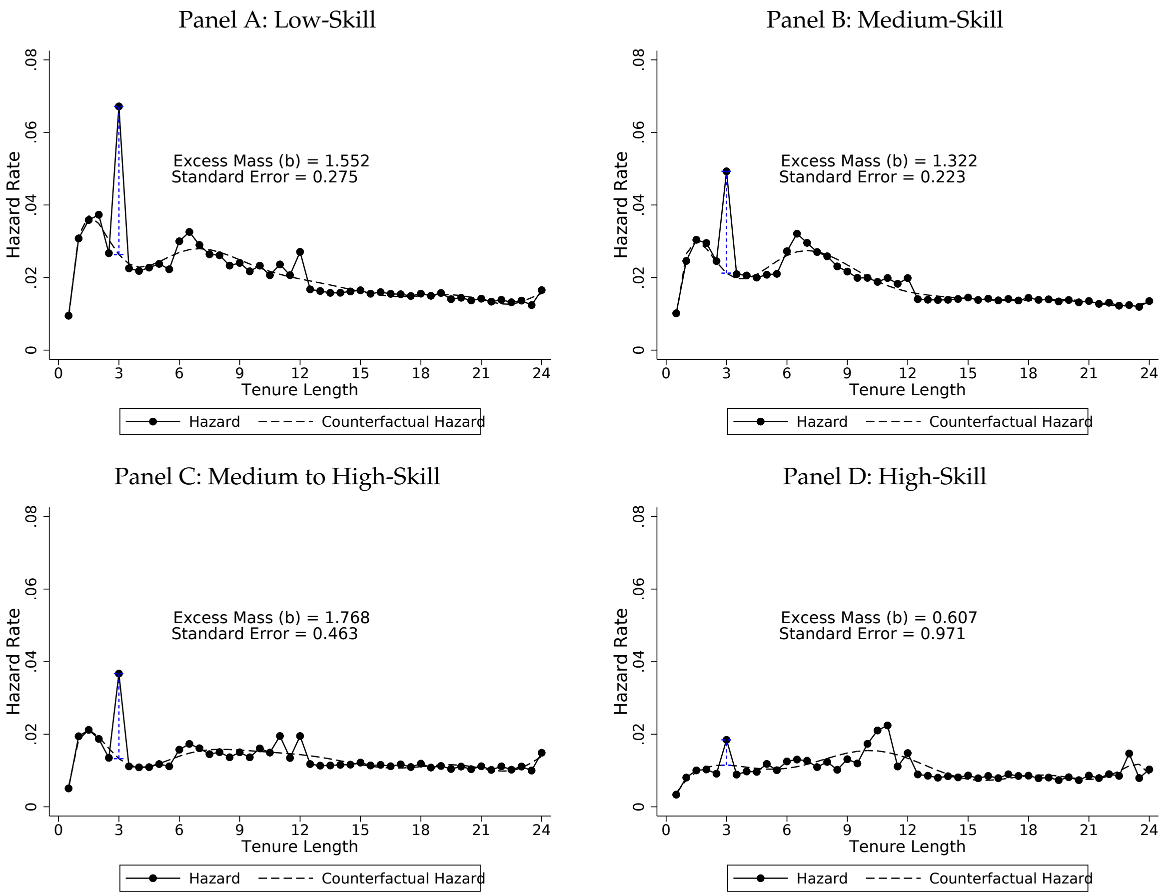
Note: This figure plots the empirical layoff hazard rate (dotted), the estimated hazard rate generated by the model (solid), and a counterfactual hazard rate (dashed) when firing costs are set to zero. Tenure duration is binned into 15 day intervals to estimate the empirical layoff hazard.

Appendix Figure 1: Heterogeneity in Bunching by Industry



Note: This figure plots the layoff hazard rate by different industries. Tenure duration is binned into 15 day intervals. The dashed line is a tenth-degree polynomial fitted to the empirical hazard rate, excluding points 15 days away from the notch, as in Equation (1). The vertical dotted line displays the excess mass B, while the normalized excess mass b and standard error is reported in the figure. The standard error is computed using a residual bootstrap procedure.

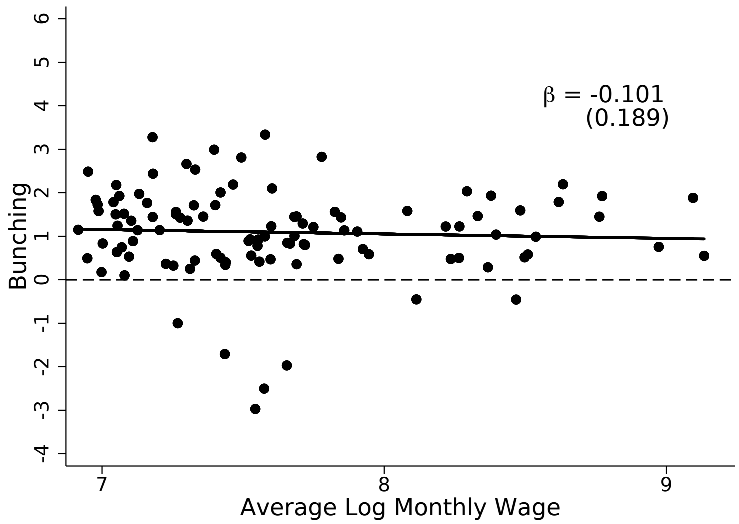
Appendix Figure 2: Heterogeneity in Bunching by Occupation and Skill



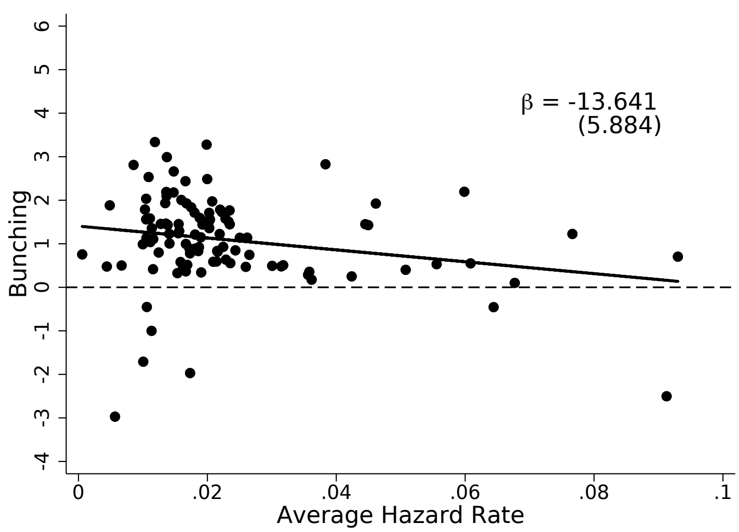
Note: This figure plots the layoff hazard rate by different occupation skill levels. Skill level is defined by the International Standard Classification of Occupations (ISCO). Low skill occupations are characterized by the performance of simple and routine physical tasks, and includes occupations such as cleaners and construction laborers. Medium-skill jobs involve performing more complex tasks, such as operating machinery, and includes occupations such as office clerks and skilled craftsman. Medium to High-skill jobs require workers to perform complex tasks and requires significant practical knowledge, and is composed primarily of technicians. High-skill occupations require complex problem solving and includes occupations such as managers and scientific professionals. Tenure duration is binned into 15 day intervals. The dashed line is a tenth-degree polynomial fitted to the empirical hazard rate, excluding points 15 days away from the notch, as in Equation (1). The vertical dotted line displays the excess mass B, while the normalized excess mass b and standard error is reported in the figure. The standard error is computed using a residual bootstrap procedure.

Appendix Figure 3: Is Bunching Consistent Across Occupations?

Panel A: Correlation between Bunching and Average Log Monthly Wage

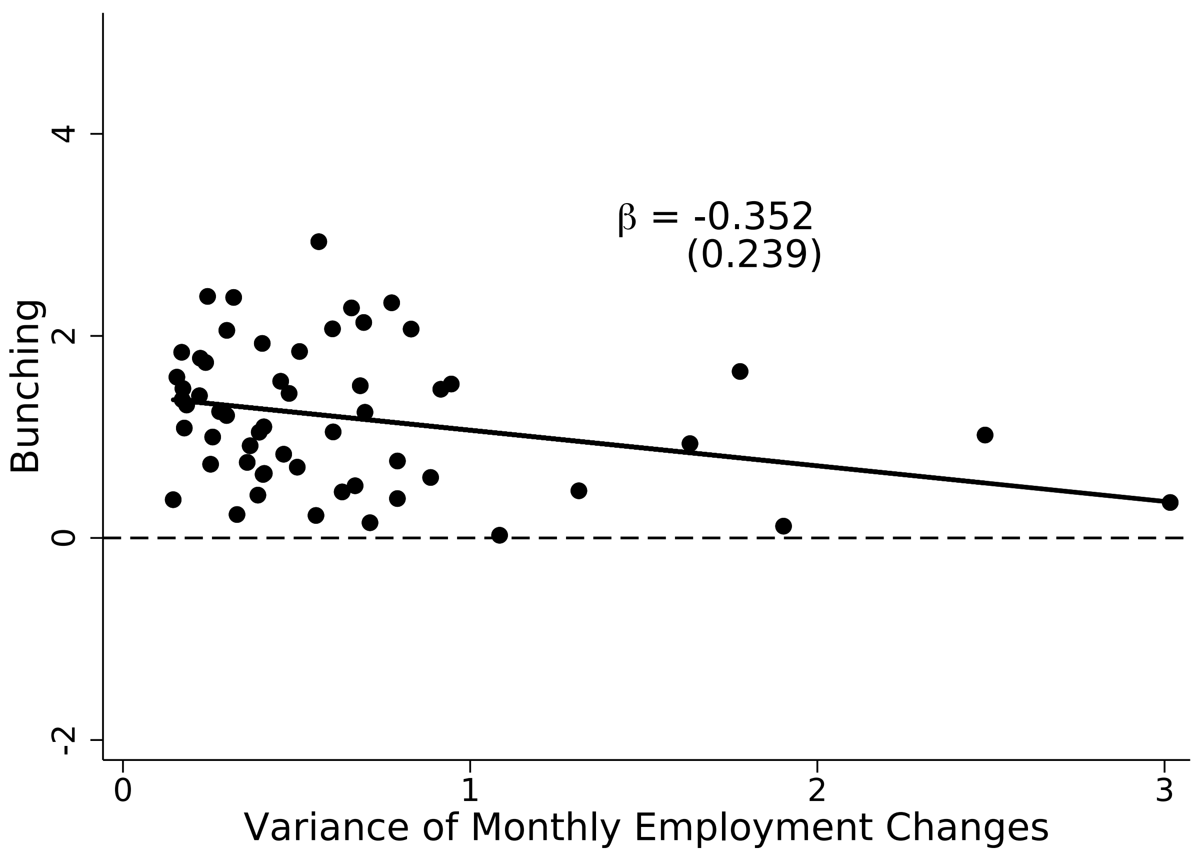


Panel B: Correlation between Bunching and Average Hazard Rate



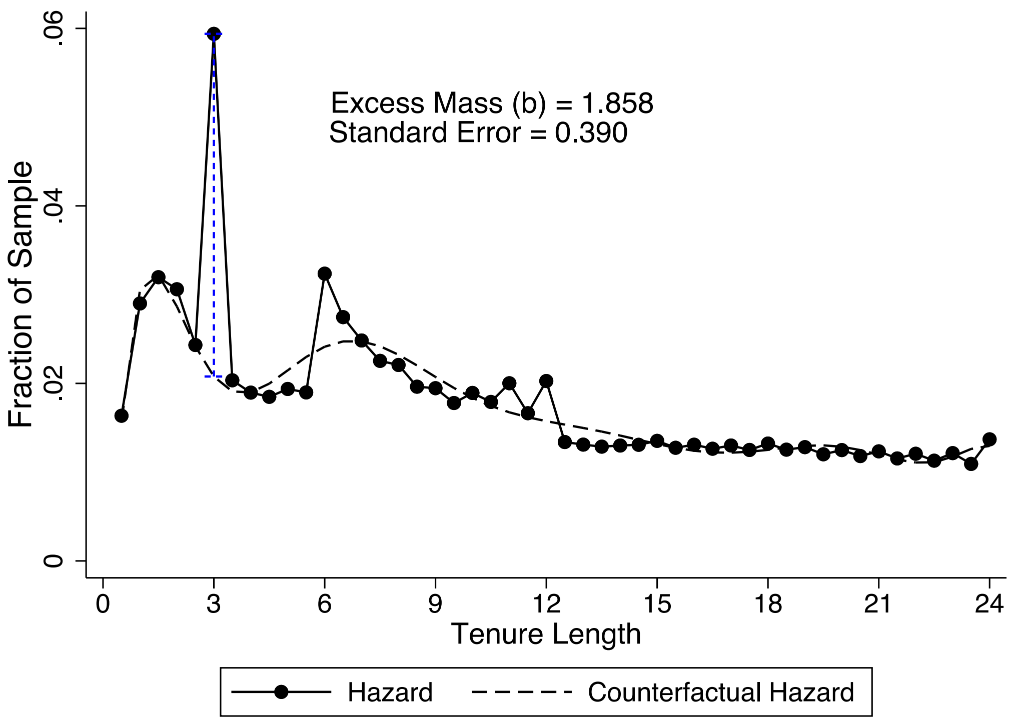
Note: This figure plots the bunching estimate by occupation (defined by three-digit ISCO identifier) and correlates the bunching to average wages (Panel A) and the average occupation-specific hazard rate (Panel B). The hazard rate for panel B is estimated as in Figure 2. The coefficient displayed is the estimate of the coefficient of a regression of the bunching estimate on the variable on the x-axis.

Appendix Figure 4: Bunching and Demand Volatility



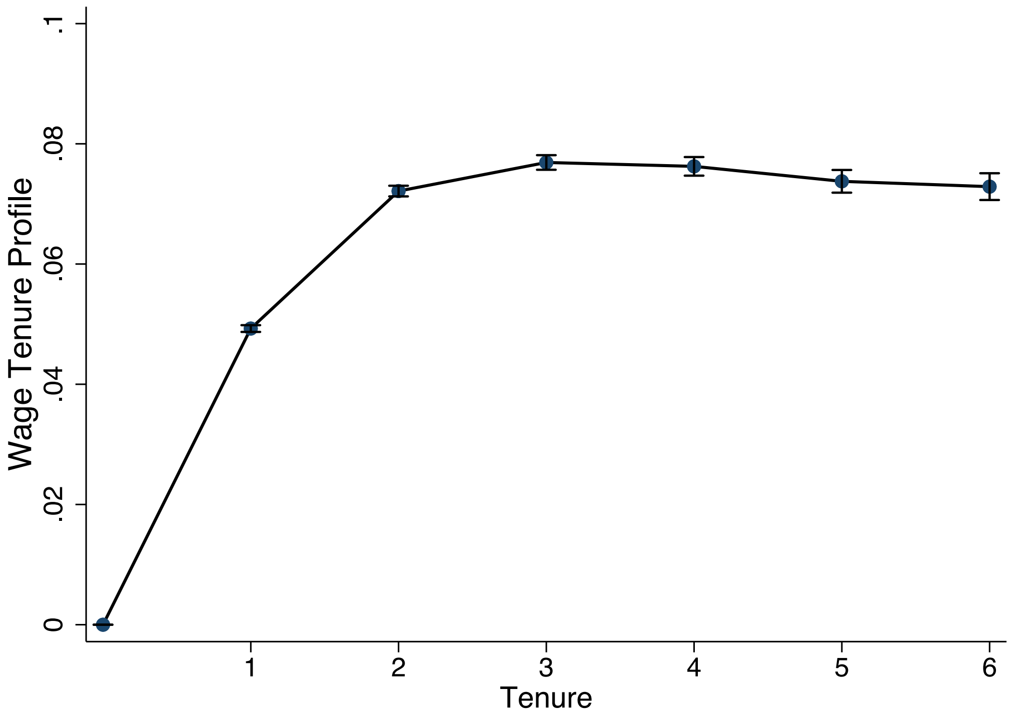
Note: This figure plots the bunching estimate by sector (defined by three-digit CNAE classification) and correlates the bunching to volatility in employment. The volatility is calculated as the standard deviation of month-to-month employment changes over the course of a year.

Appendix Figure 5: Bunching Including Temporary Contracts



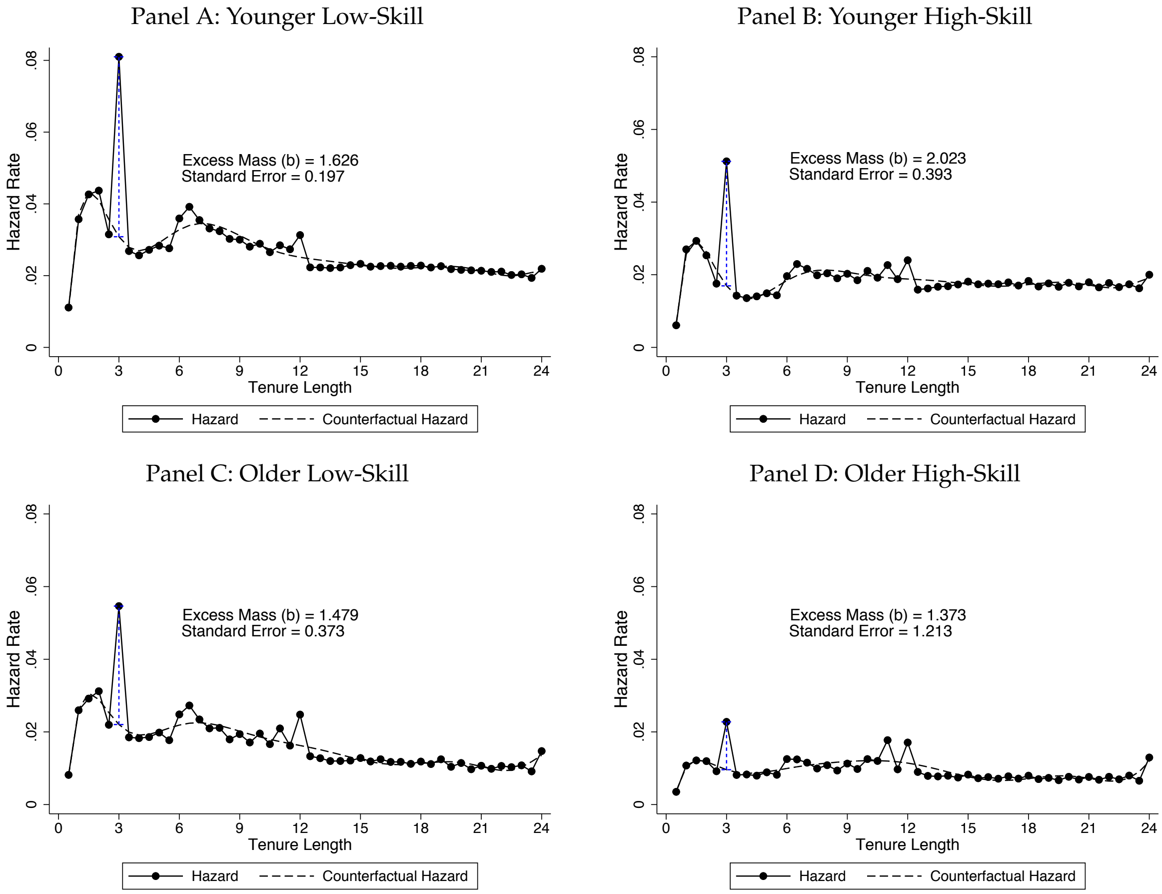
Note: This figure plots the job termination hazard rate which includes temporary contracts.

Appendix Figure 6: Wage-Tenure Relationship



Note: This figure plots the return to tenure within a job spell. Standard errors are clustered at the worker level.

Appendix Figure 7: Relationship between Bunching and Age



Note: This figure plots the layoff hazard rate by different occupation skill levels and age. Skill level is defined by the International Standard Classification of Occupations (ISCO). Younger workers are individuals who are between 20 and 25 years of age at the beginning of the employment spell. Older workers are individuals who are between 40 and 45 years of age at the beginning of the employment spell. Tenure duration is binned into 15 day intervals. The dashed line is a tenth-degree polynomial fitted to the empirical hazard rate, excluding points 15 days away from the notch, as in Equation (1). The vertical dotted line displays the excess mass B, while the normalized excess mass b and standard error is reported in the figure. The standard error is computed using a residual bootstrap procedure.

Appendix A: Numerical Implementation

A. Algorithm to solve the model

Solving the model can be broken into two steps: first solve for the equilibrium firm value functions and belief thresholds, and then solve for the equilibrium unemployment rate.

A.1 Solving for J^i and $\underline{p}^i$

After substituting the wage into the firm value functions, we essentially have to solve for 3 HJB equations together with the optimal belief threshold functions. To do this, we use a finite-difference approximation to the HJB equation, and exploit the fact that the optimal stopping problem characterizing the thresholds can be solved as a linear complementarity problem (see the information and MATLAB codes on Benjamin Moll's website). Specifically, we start by solving for J^3 and $\underline{p}^3$ for $t \geq T_2$ since these objects are stationary. Given these objects, we can then work backwards to solve for J^2 and J^1, and $\underline{p}^2$ and $\underline{p}^1$.

A.2 Solving for f^i

Given the thresholds and a guess of the unemployment rate, we can use a similar finite-difference approximation to solve the KFE forward in time, using the appropriate initial and boundary conditions, to get f^1, f^2, and f^3. Using these distributions to compute an implied unemployment rate then yields a simple iterative scheme to find the equilibrium unemployment rate.

Given these objects, all other equilibrium objects can be computed using the relationships stated in the theoretical exposition.

Appendix B: Empirical Support for Learning Model

In our model, bunching stems from employer learning. However, there are at least three alternative explanations for the spike in the job termination at three months. First, a spike at three months could be driven by demand volatility. Second, firms could be rotating through workers to avoid firing costs. Lastly, rather than employer learning, a model of idiosyncratic productivity movements as in Prat (2003) could generate many of the same predictions as our model.

To address these concerns, we estimate bunching (i.e. the excess mass in job separations at three months) in response to firing costs along a variety of dimensions to discern whether alternative stories can explain the pattern of the job termination hazard rate. To proceed, we bin job tenure into 15 day intervals and then estimate the hazard non-parametrically by computing the probability a job is terminated in a given interval, given the job has lasted until that interval. A job is defined as begin terminated if the employer ends the job. In particular, quits are not counted as a job termination in our estimation of the hazard rate.

Panel A of Figure 2 presents the job termination hazard for the population of 18-65 year olds excluding all temporary contracts. As can be seen in the figure, the hazard rate exhibits a sharp spike at 90 days, just before the firing costs jump. To go from the hazard rate to a bunching moment, we need to estimate the counterfactual hazard rate (i.e. the hazard rate that would prevail if there was no discontinuous jump in firing costs).

To estimate the counterfactual hazard, we follow Chetty et al. (2012) and fit a flexible polynomial to the data, excluding data from around the notch point T. Formally, let $B_j = \{15, 30, ...\}$ define the bins and H_j indicate the hazard rate in bin j (i.e. H_{90} denotes the probability a job ends between 75 and 90 days, given the job has lasted for 75 days). To estimate the counterfactual hazard, we estimate the following regression:

$$H_j = \sum_{i=0}^{q} \beta_i \cdot (B_j)^i + \sum_{i=-R}^{R} \gamma_i \cdot \mathbb{1}[B_j = i] + \varepsilon_j^0 \tag{1}$$

where q is the order of the polynomial and R denotes the width of the excluded region around the notch in firing costs. In practice, we set $q = 10$ and $R = 15$, and therefore exclude any tenure durations which end between 75 and 105 days in the estimation of the counterfactual hazard rate. We use the results from Equation (1) to estimate the counterfactual hazard as:

$$\hat{H}_i = \sum_{i=0}^{q} \hat{\beta}_i (B_j)^i \tag{2}$$

The excess mass is defined as the difference in the true hazard rate and the counterfactual hazard rate at the notch point $T = 90$:

$$B = H_T - \hat{H}_T \tag{3}$$

The normalized excess mass, which we will refer to as bunching b, is defined as the excess mass divided by the counterfactual hazard rate at tenure duration T.

$$b = \frac{B}{\hat{H}_T} \tag{4}$$

To compute standard errors for the excess mass and normalized excess mass, we generate hazards and excess mass by resampling the residuals in Equation (1). The standard error is then equal to standard deviation of the distribution of excess mass estimates generated by bootstrapping.

Panel A of Figure 2 displays the job termination hazard rate. The normalized excess mass is equal to 1.008 (se=0.165), indicating that the true hazard rate is twice as large as the counterfactual hazard rate at 3 months tenure. As can be seen in the figure, there is another spike in the hazard rate around six months. Van Doornik et al. (2017) shows that job terminations increase around six months in Brazil due to "fake" separations. If a worker is fired after six months of tenure, the worker can receive unemployment insurance from the government. This incentivizes firms to fire workers and then split the unemployment insurance.

The first alternative explanation for bunching is that demand fluctuations create natural spikes in the job termination hazard. To alleviate concerns about demand fluctuations creating spikes in the job termination hazard, we eliminate all temporary contracts from the analysis. Unsurprisingly, this restriction reduces the magnitude of the bunching (from 1.86 to 1.39). [4]. However, as stated previously, regulation on temporary contracts requires firms to receive permission to use temporary workers. It could be that firms avoid going through this application process, but just hire workers with the intent to fire them before the probationary period ends, and therefore focusing only on permanent contracts may not completely alleviate this concern. To determine whether demand volatility is driving the bunching we re-estimate the hazard rates separately for different industries. Figure 1 displays the results for six different industries. As can be seen in the figure, bunching occurs consistently across a variety of industries, including industries where demand volatility is unlikely to explain bunching. For example, the bunching in health and education is equal to 1.92, larger than the bunching in construction 1.38. The only industry displayed without statistically significant bunching is the electricity industry, an industry with significant state ownership.[5]

To provide further evidence that demand volatility does not drive bunching once temporary contracts are eliminated, we estimate bunching separately for each three-digit sector and then correlate bunching with the month-to-month variation in employment in the given three-digit-sector. If bunching is driven by demand volatility, we would expect industries with high employment volatility to also display greater magnitudes in bunching. To estimate demand volatility, we compute a normalized measure of monthly employment changes in sector j at time t as:

$$\Delta E_{j,t} = \frac{hires - fires}{hires} \tag{5}$$

We then compute volatility of sector j as $V_j = Var(\Delta E_{j,t} - \Delta E_{j,t-1})$. In words, we create time series of month-to-month net employment changes scaled by the total number of hires. We then take the variance of the first difference as our measure of employment volatility. We then correlate this measure with bunching by running the following regression:

$$\hat{V}_j = \beta_0 + \beta_1 \hat{b}_j \tag{6}$$

Where β_1 capture the correlation between bunching and employment volatility. Figure 4 shows the results of this regression. As can be seen in the figure, demand volatility is negatively correlated with bunching, although the correlation is not significant. This suggests that the bunching estimate is not driven by changes in demand volatility.

The second alternative explanation for bunching is that firms are rotating through workers to avoid paying the firing costs. In our model, all workers are hired with the possibility that they will become permanent workers. Under this alternative explanation, some workers are hired with the expectation that the job will only last three months. Conceptually, we would like to map the bunching to a parameter which is informative about how aggregate employment is affected by firing costs, with larger bunching indicating a greater reduction in aggregate employment.

[4] See Appendix Figure 5 for the job termination hazard which includes temporary contracts

[5] In unreported results, We also estimate hazards separately depending on the hiring month and find consistent bunching across months

However, if firms are substituting towards short-tenure jobs, then aggregate employment may be unaffected, while there may be a strong substitution effect towards short-duration jobs.

Again, eliminating temporary contracts alleviates this concern by allowing us to focus on workers with at least an expectation that the job may become a permanent position. Additionally, in Figure 1 we find bunching occurs across a variety of industries, including industries such as health and education, where, a priori, it would seem unlikely that firms would find it profitable to cycle through workers.

In Figure 2 we also estimate bunching separately by occupational skill level. Skill level is defined by the International Standard Classification of Occupations (ISCO). Low-skill occupations are characterized by the performance of simple and routine physical tasks, and includes occupations such as cleaners and construction laborers. Presumably, it may be easy to cycle through low-skill workers, as the tasks they perform require little training. Medium-skill jobs involve performing more complex tasks, such as operating machinery, and includes occupations such as office clerks and skilled craftsman. Medium to High-skill jobs require workers to perform complex tasks and requires significant practical knowledge. Common occupations in this category include different types of technicians. High-skill occupations include managers and professionals, which requires complex problem solving, and in most cases, some form of advanced education. As can be seen in Figure 2, bunching occurs across all skill levels. For example, bunching in the highest skill category is equal to 1.502 (se=0.768), while bunching in the lowest skill category is 1.207 (se=0.180). It seems unlikely that bunching for high-skill positions is driven by firms finding it profitable to cycle through high-skill workers at three-month intervals.

To provide more evidence that bunching is consistent across occupations, Figure 3 plots the bunching estimate for 105 different occupations. Of the 105 different occupations, we estimate positive bunching for 98 of the occupations. Panel A shows that bunching is not correlated with the average wage in the occupation. If bunching was driven by firms cycling through workers, we would expect more low-skill low-wage jobs to exhibit more bunching.

Lastly, while we follow Moscarini (2005) in modeling jobs as a signal extraction problem, an alternative is to allow match-specific productivity to be subject to idiosyncratic shocks, as in Prat (2003). This model yields many of the same predictions as Moscarini (2005), although the source of turnover is no longer the learning process, but idiosyncratic volatility. As discussed in Prat (2003), one way to distinguish between the two models is to look at wage growth within a job. Prat (2003) predicts wage growth within a job to be approximately constant. To estimate the return to tenure, in our data, we estimate the following regression:

$$ln(w_{mt}) = \alpha_m + Year_t + \sum_{k=2}^{6} \delta^k T_{mt}^k \tag{7}$$

Where m indexes a match between a worker and a firm, t indexes calendar time, α_m are match fixed effects, $Year_t$ are calendar year effects, and T_{mt}^k is equal to one if match m has tenure k at time t. δ^k captures the change in wages from tenure k relative to the base level of tenure, which is one or fewer years. We restrict to only six years of tenure given our focus is on the early portion of job spells. If the rate of return is constant as predicted in Prat (2003), then δ^k should lie on a line. A learning model, however, has no clear prediction about the relation between wages and tenure.

Figure 6 displays the coefficients δ^k. As can be seen in the figure δ^k increases dramatically at the beginning of a job spell, but then flattens out at longer tenure durations. While this wage-tenure relationship is consistent with the learning model illustrated in Section II, it is not consistent with a model based on idiosyncratic drift in match productivity which predicts a roughly constant return to tenure.

To provide further evidence that learning is an important determinant of turnover, we explore whether turnover and bunching is greater for workers where there is more likely to be greater uncertainty on the employer side. If a given worker has significant labor market experience, then the uncertainty in hiring the worker is likely lower. Kahn (2013) calibrates a model of employer learning and finds that an outside firms can reduce their initial expectation error of a worker by roughly a third of what the incumbent firm can reduce theirs by. Therefore, while more labor market experience does not eliminate the uncertainty for future employers, it does reduce the expectation error for a given worker. To provide evidence that learning is an important driver of layoffs and bunching, we split workers into age bins and re-estimate bunching, the idea being that older workers have accumulated more labor market experience and therefore there should be less uncertainty in the hiring process. Figure 7 presents results for high-skill and low-skill occupations for workers between 20 and 25 and workers between 40 and 45. As can be seen in the figure, the job termination hazard rate is shifted down for older workers, and the magnitude of bunching has decreased, consistent with employers facing less uncertainty about older workers. We find this relationship to hold consistently across 5 year age bins from 20 to 55, but only report the 20-25 and 40-45 bins for readability.

Summarizing, while demand volatility and substitution towards short-duration jobs may be important considerations when interpreting the spike in the job termination hazard rate at three months, we believe by eliminating temporary contracts, we have greatly lessened the impact of these alternative explanations. In particular, we find bunching across a wide variety of sectors and occupations, including sectors such as health and education, where demand volatility and substitution towards short-duration jobs are likely to play a small role in explaining the spike in the job termination hazard.

Appendix C: Data Appendix

A. Overview

The *Relação Anual de Informações Sociais* (RAIS) is an employer-employee matched dataset which includes information on all workers and firms in the formal sector of Brazil. The main use of the RAIS is to compute federal wage-supplements (*Abono Salarial*). While not reporting can in theory result in fines, these fines are rarely issued in practice. However, workers and firms are incentivized to provide accurate wage information given the federal public wage-supplement is based on the wage reported in the RAIS.

B. Sample Selection

In the RAIS, workers are identified by an individual-specific PIS (Programa de Integração Social), a unique time-invariant worker identifier similar to a social security number. We follow Menezes-Filho and Muendler (2011) and drop workers with PIS identifiers less than 11 digits, as these are not valid identifiers. Errors in worker identifiers may be caused by (1) bad compliance and book-keeping errors or (2) to allow workers to withdraw from their severance account through fake layoffs and rehires. We eliminate jobs for workers which begin on the same day for the same employer. A single employer may report multiple accounts for one worker so that the workers may access their employer-funded severance payment account, which by law should only be accessed in the case of a firing or for health-related reasons. However, individuals must work at an employer for more than six months in order to access the FGTS account. Therefore, the spike in the job termination hazard cannot be due to employers reporting multiple jobs for the same worker.

C. Variable Definitions

PIS: A PIS is a worker identifier that is unique to a given worker over time.

CNPJ: The CNJP is an establishment-level identifier issued by the Brazilian tax authority which is unique to a given establishment over time. The first eight digits of the CNPJ corresponds to the firm of the establishment, while the last six correspond to the establishment within the firm.

Education: The RAIS records education at eight different categories. I re-code these variables to four categories: (1) Less than High-School (2) High-School Graduate (3) College Graduate. Education for an individual worker is set to the modal value of education for the worker over the sample period.

Occupation: Occupations are defined by the Classificação Brasileira de Ocupações (CBO) into 2355 distinct groups. We map these occupations to .

Sector: Sectors are reported under the *CNAE* four-digit classification (*Classificação Nacional de Atividade Econômica*) for 654 industries. I aggregate sectors to the two-digit level in the paper, unless specified otherwise.

Wage: Wage refers to total payments, including regular salary payments, holiday bonuses, performance-based and commission bonuses, tips, and profit sharing agreements, divided by total months worked during the year for that employer. Payments that are not considered part of the wage include severance payments for layoffs and indemnity pay for maternal leave.
Tenure: The duration the worker has been employed at the establishment. We recode the tenure duration so that it increases in increments of two weeks.