

- 1 Under what circumstances might it be more appropriate to select an asymptotically “slower” sorting algorithm? Be as specific as possible. (1)
- 2 Which is “asymptotically greater”,  $n!$  or  $n^n$ ? Explain your reasoning. (1)
- 3 Explain why the following program segment runs in  $O(\lg n)$  time. (2)

```
while (n > 0) {
    System.out.print(n % 2);
    n /= 2;
}
```

- 4 Explain why it is important to understand when a particular problem is algorithmically “unsolvable.” What can be done to tackle these types of problems? (2)
- 5 Consider *Euclid’s Algorithm* given below. (4)

Algorithm:

**Input:**  $a, b$ : any two, positive integers.

Example:

**Input:** 12, 18

```
while a ≠ b
    if a > b
        a ← a − b
    otherwise
        b ← b − a
return a
```

a	b
12	18
12	6
6	6

**Output:** 6

- (a) Follow *Euclid’s Algorithm* for each of the following pairs of numbers.
  - i. 40, 16
  - ii. 75, 105
  - iii. 156, 286
- (b) How is the result of *Euclid’s Algorithm* “mathematically significant”? That is, what does this algorithm actually produce as a result?
- (c) Implement *Euclid’s Algorithm* in Java.
- 6 The *Sieve of Eratosthenes* is an ancient algorithm designed for finding prime numbers below a given maximum value. It accomplishes this by marking as non-prime all multiples of each prime, starting with 2. The following steps summarize how the sieve works. (6)

1. Create a list of consecutive integers from 2 through  $n$ :  $[2, 3, 4, 5, \dots, n]$ .
2. Initialize  $p$  to 2, the smallest prime number.
3. Starting with  $2p$ , mark each multiple of  $p$  less than or equal to  $n$  as non-prime.
4. Set  $p$  to the next number greater than  $p$  that is not marked as non-prime. If no such number exists, stop.
5. When the algorithm stops, all numbers not marked non-prime are all the prime numbers below  $n$ .

Create the method, `Eratosthenes()`, that will take  $n$  as a parameter and use the *Sieve of Eratosthenes* to find all prime numbers less than or equal to  $n$ . Your methods should return an array containing only these prime numbers.