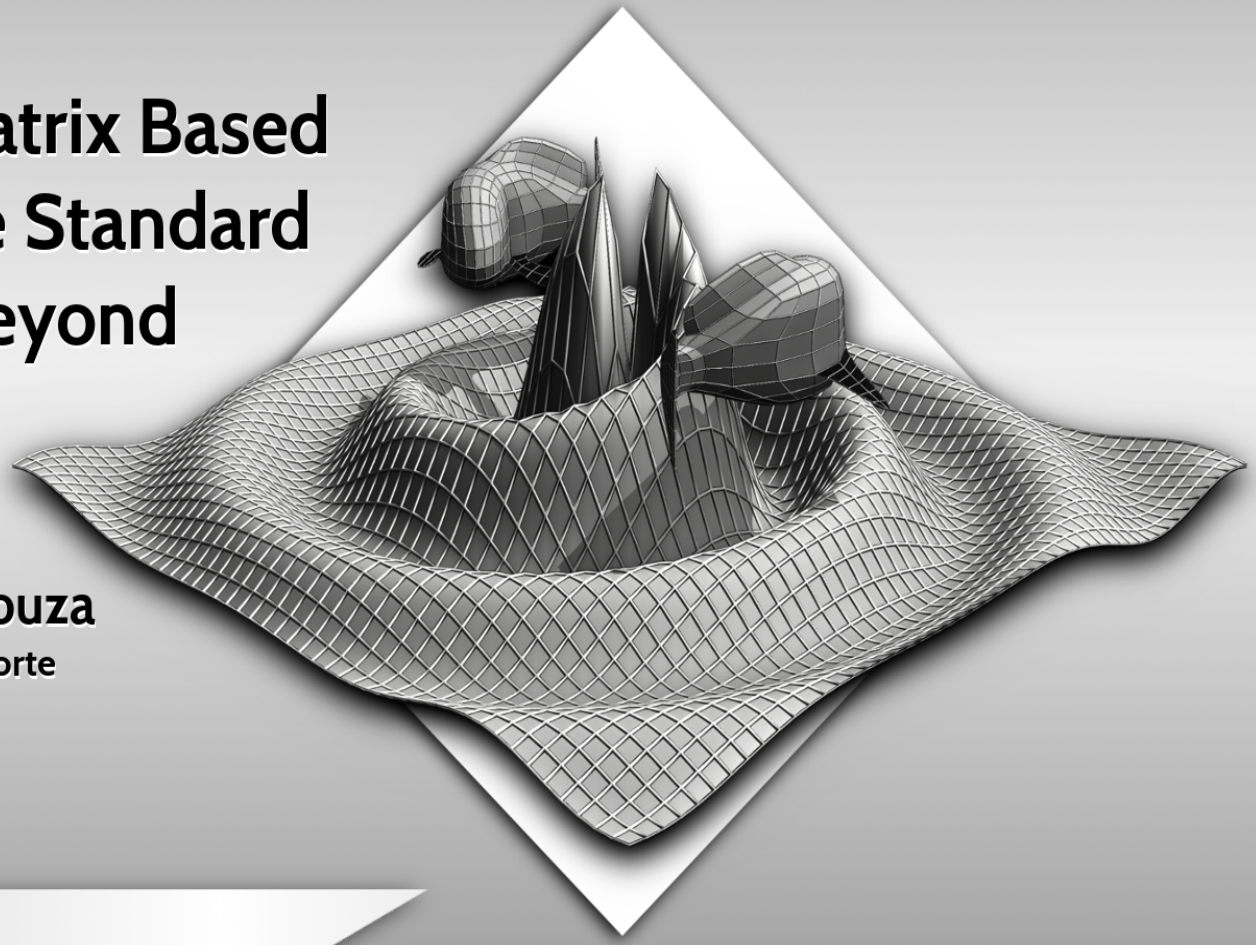
GWDALI: A Fisher Matrix Based Software to Estimate Standard Sirens Parameters Beyond Gaussianity-Level



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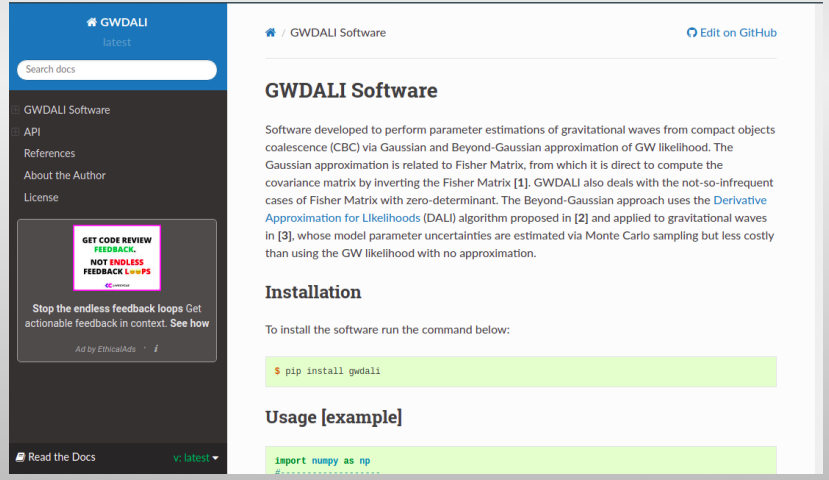
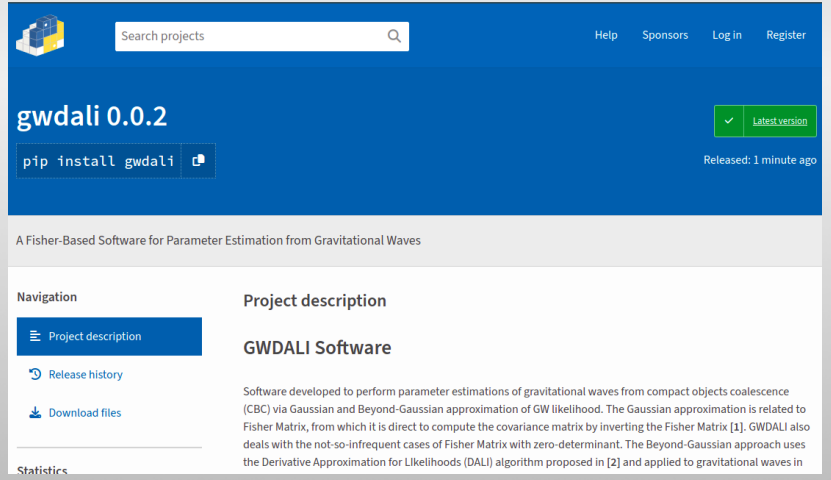
Installation and Documentation

$ pip install gwdali

https://gwdali.readthedocs.io/



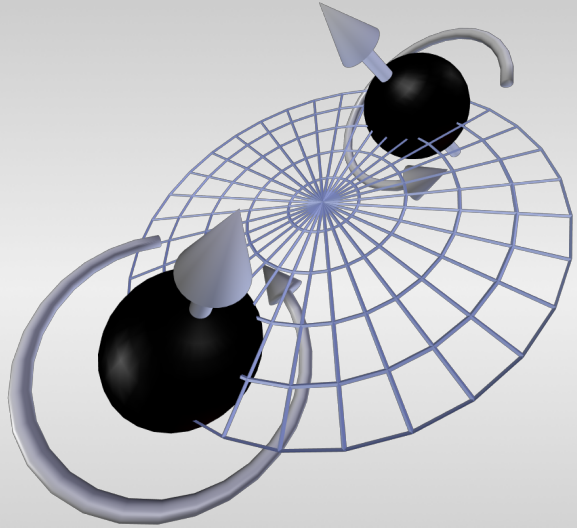
https://pypi.org/project/gwdali/



Choose Source Parameters (15)

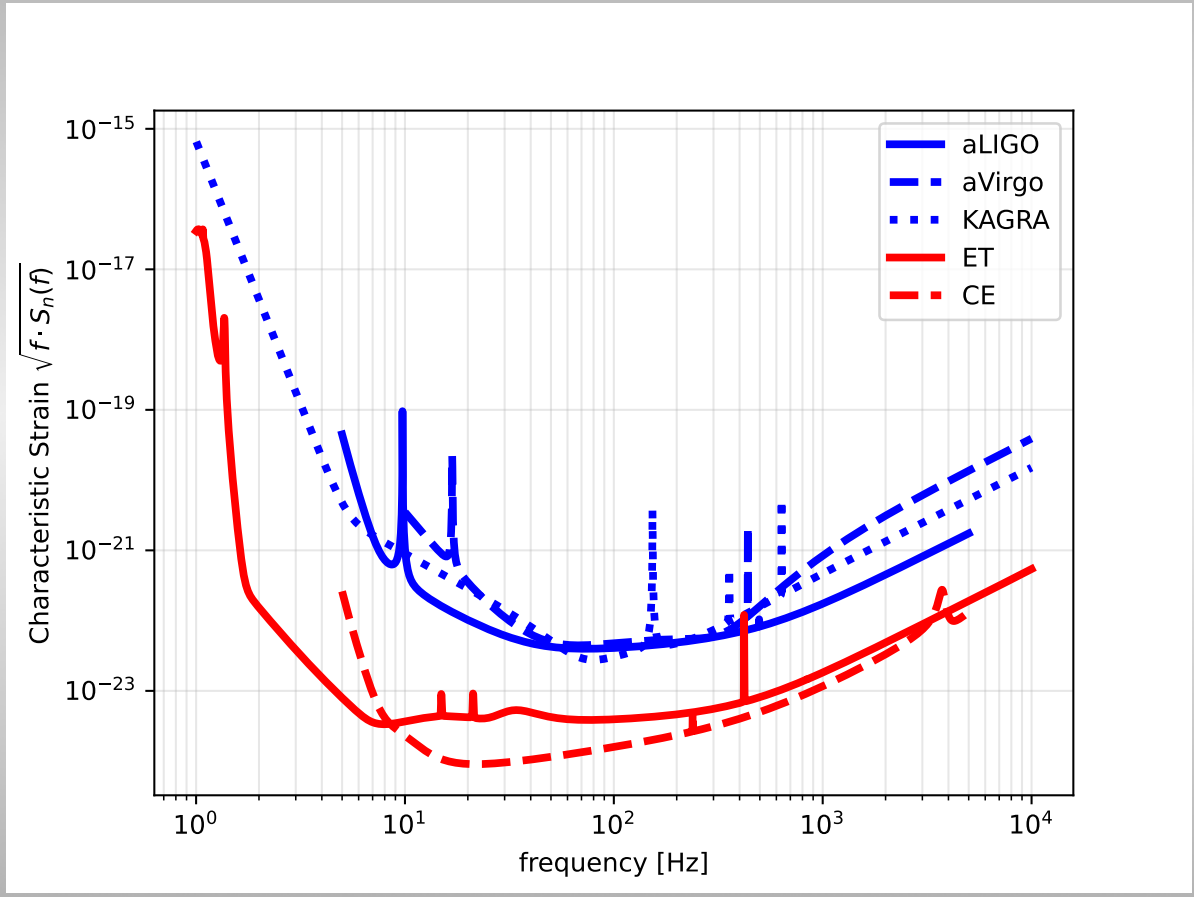
Parameters:

- “m1”
- “m2”
- “sx1”
- “sy1”
- “sz1”
- “sx2”
- “sy2”
- “sz2”
- “RA”
- “Dec”
- “DL”
- “iota”
- “psi”
- “t_coal”
- “phi_coal”

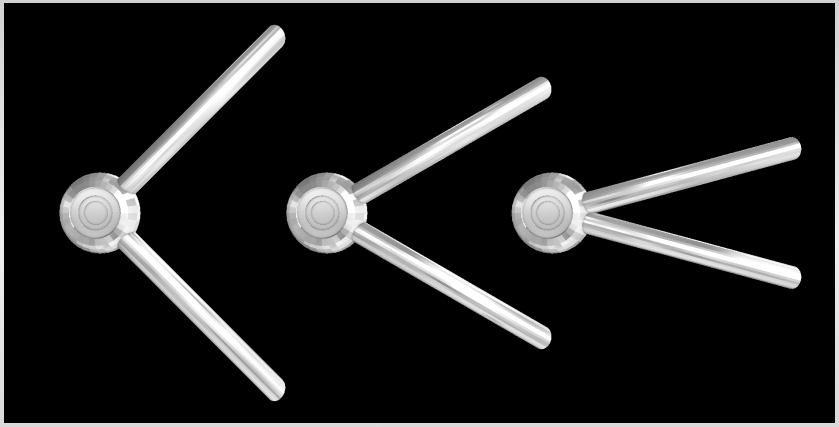


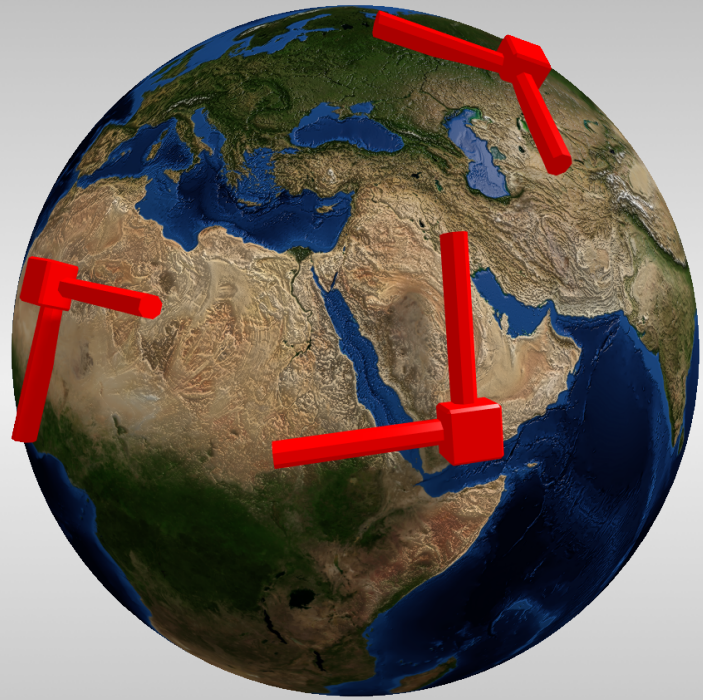
gw_params = { “m1”:1.5 , “m2”:1.5, “DL”: 100 , ...}

Choose The Detector Sensitivity



Choose Any Detector Coordinates/Shape





```
det = {"name":"CE","lon":-119,"lat":46,"rot":45,"shape":90}
```

Parameter Estimation (Standard/Fisher/DALI)

Standard: $$\boxed{log\mathcal{L} = -\frac{1}{2}\sum_{a=1}^{N_{det}}\langle H_a - h_a(\{\theta\})|H_a - h_a(\{\theta\})\rangle}$$

where,

- H_a: Output of the a-th detector (the data)
- $h_a(\{\theta\})$: The GW template in the a-th detector (the model)
- $\{\theta\}$: Set of GW parameters

Scalar Product:

$$\langle X_a|Y_a\rangle \equiv 4\text{Real}\left\{\int_0^\infty df\,\frac{\tilde{X}_a^*(f)\tilde{Y}_a(f)}{S_{n,a}(f)}\right\}$$

N_{det}: Number of detectors.

Parameter Estimation (Standard/Fisher/DALI)

GW Fisher Approach [arXiv:gr-qc/9301003]

$$log\mathcal{L} \approx -\frac{1}{2}\sum_{a=1}^{N_{det}}\sum_{i,j}^{N_p}\left\langle \frac{\partial h_a(\{\theta\})}{\partial\theta_i}\middle|\frac{\partial h_a(\{\theta\})}{\partial\theta_j}\right\rangle_0 (\theta_i-\hat{\theta}_i)(\theta_j-\hat{\theta}_j)$$

$\{\hat{\theta}\}$: GW parameters of the injection (best-fit parameters).

$$\text{Fisher Matrix: } \Gamma_{ij} \equiv \sum_{a=1}^{N_{det}}\left\langle \frac{\partial h_a(\{\theta\})}{\partial\theta_i}\middle|\frac{\partial h_a(\{\theta\})}{\partial\theta_j}\right\rangle_0$$

$$\text{Covariance Matrix: } C_{ij} = (\Gamma^{-1})_{ij}$$

N_p: Number of free parameters.

Parameter Estimation (Standard/Fisher/DALI)

DALI (Doublet) [arXiv:1401.6892 & arXiv:2203.02670]

$$log\mathcal{L} \approx -\left[\frac{1}{2}\sum_{i,j}\langle\partial_i h|\partial_j h\rangle_0\,\Delta\theta_i\Delta\theta_j\right]$$
$$-\left[\frac{1}{2}\sum_{i,j,k}\langle\partial_i h|\partial_j\partial_k h\rangle_0\,\Delta\theta_{ijk}+\frac{1}{8}\sum_{i,j,k,l}\langle\partial_i\partial_j h|\partial_k\partial_l h\rangle_0\,\Delta\theta_{ijkl}\right]$$

where,

$$\begin{aligned}\Delta\theta_i &\equiv \theta_i-\hat{\theta}_i\\ \Delta\theta_{ij} &\equiv \Delta\theta_i\Delta\theta_j\\ \Delta\theta_{ij..n} &\equiv \Delta\theta_i\Delta\theta_j\ldots\Delta\theta_n\end{aligned}$$

Parameter Estimation (Standard/Fisher/DALI)

DALI (Triplet) [arXiv:1401.6892 & arXiv:2203.02670]

$$
\begin{aligned}
log\mathcal{L} \approx & -\left[\frac{1}{2}\sum_{i,j}\langle\partial_i h|\partial_j h\rangle_0\,\Delta\theta_i\Delta\theta_j\right] \\
& -\left[\frac{1}{2}\sum_{i,j,k}\langle\partial_i h|\partial_j\partial_k h\rangle_0\,\Delta\theta^{ijk}+\frac{1}{8}\sum_{i,j,k,l}\langle\partial_i\partial_j h|\partial_k\partial_l h\rangle_0\,\Delta\theta_{ijkl}\right] \\
& -\left[\frac{1}{6}\sum_{i,\cdots,l}\langle\partial_i h|\partial_j\partial_k\partial_l h\rangle\,\Delta\theta_{ijkl}+\frac{1}{12}\sum_{i,\cdots,m}\langle\partial_i\partial_j h|\partial_k\partial_l\partial_m h\rangle\,\Delta\theta_{ijklm}\right. \\
& \left.+\frac{1}{72}\sum_{i,\cdots,n}\langle\partial_i\partial_j\partial_k h|\partial_l\partial_m\partial_n h\rangle\,\Delta\theta_{ijklmn}\right]
\end{aligned}
$$

Main Module

GWDALI function

```
GWDALI.GWDALI(Detection_Dict, FreeParams, detectors, approximant='TaylorF2',
dali_method='Fisher_Sampling', sampler_method='nestle', save_fisher=True,
save_cov=True, plot_corner=True, save_samples=True, hide_info=False, index=1,
rcond=1.e-4, npoints=300)
```

method: standard, fisher, fisher_sampling , doublet , triplet.
sampler_method: Any method of *Bilby Package* (e.g. 'nestle', 'dynest', 'emcee')

GWDALI Output: Fisher Matrix, Covariance Matrix, SNR, MCMC samples, Recovered Parameters, Uncertainties (CL=60%)

Thank You