Cosmology With Gravitational Wave Simulations

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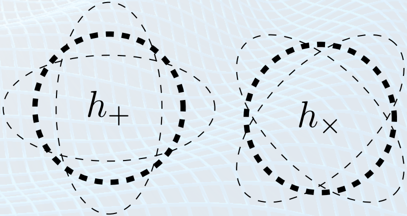
Background

- (1915) Publication of the *Einstein Field Equations* of gravity;
- (1916) Prediction of the *Gravitational Waves*(GW);
- **(1986) B.Shutz reports how to measure H_o with GW's;**
- (1992) Proposal for construction of LIGO;
- (1999) Conclusion of LIGO construction;
- (2002) Start operation of LIGO;
- (2015) First detection of a GW from a BHS (signal GW150914);
- (2017) Nobel Prize due GW detections;

$$g_{\mu\nu}(x^\alpha) = \eta_{\mu\nu} + h_{\mu\nu}(x^\alpha)$$

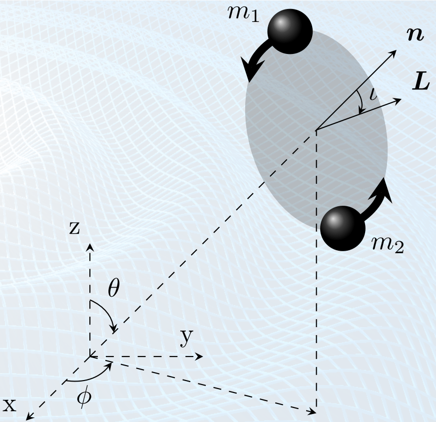
$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 0$$

$$\boxed{\Box h_{\mu\nu}(x^\alpha) = 0}$$



Gravitational Waves as Standard Sirens

$$h_{ij}^{TT} = \begin{pmatrix} h_+ & h_\times & 0 \\ h_\times & -h_+ & 0 \\ 0 & 0 & 0 \end{pmatrix}$$



$$h_+ = \frac{4}{d_L}(G\mathcal{M}_c)^{5/3}(\pi f_{gw})^{2/3}\left(\frac{1+cos^2\iota}{2}\right)cos(\Phi(t))$$

$$h_\times = \frac{4}{d_L}(G\mathcal{M}_c)^{5/3}(\pi f_{gw})^{2/3}cos\iota\ sin(\Phi(t))$$

$$h(t) = F_+h_+(t) + F_\times h_\times(t)$$

$$\boxed{h \propto \frac{1}{d_L}}$$

Output GW Detectors

GW Signal and Output Detector:

$$h(t) = F_+ h_+(t) + F_\times h_\times(t)$$

$$O(t) = h(t) + n(t)$$

Power Spectral Density ($\boldsymbol{S_n}$):

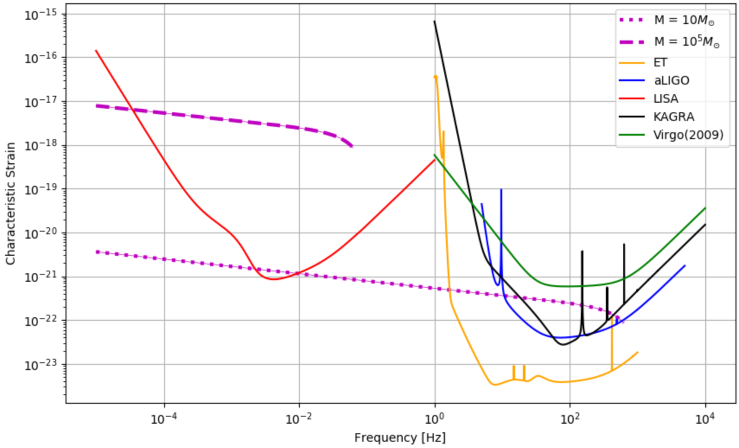
$$\langle \tilde{n}(f)\tilde{n}^*(f')\rangle = \frac{1}{2}\boldsymbol{S_n}(\boldsymbol{f})\delta(f - f')$$

Signal to Noise Ratio:

$$SNR = 2\left[\int_0^\infty \frac{df}{S_n(f)}|\tilde{h}(f)|^2\right]^{1/2}$$

Figure: From **https://www.virgo-gw.eu/**.

Characteristic Strains: $h_c \equiv 2f|\tilde{h}(f)| \quad vs \quad h_n \equiv \sqrt{f\ S_n(f)}$



Bayesian Parameter Estimation

Bayes Theorem:

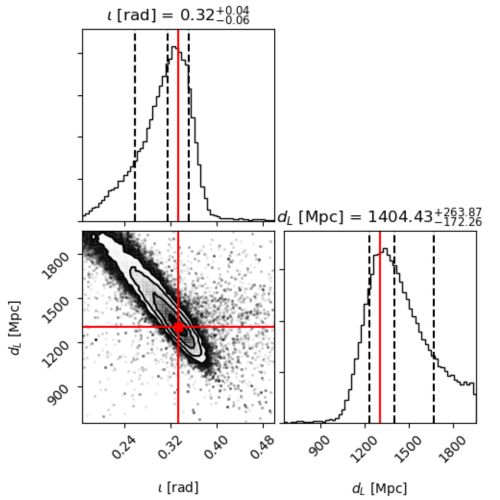
$$P(\theta|d,M) = \frac{\boldsymbol{P(d|\theta,M)}P(\theta|M)}{P(d)}$$

Likelihood:

$$\mathcal{L} \equiv \boldsymbol{P(d|\theta,M)} \Rightarrow \boxed{log\mathcal{L} = -\langle s-h, s-h\rangle}$$

Inner Product Definition:

$$\langle a,b\rangle \equiv \int_0^\infty \frac{df}{S_n}[\tilde{a}^*(f)\tilde{b}(f) + \tilde{a}(f)\tilde{b}^*(f)]$$



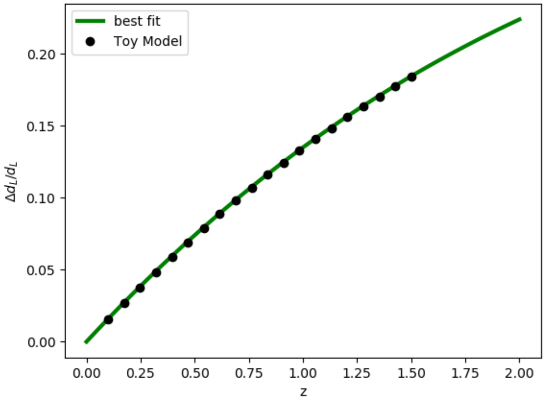
Estimation of Luminosity Distance Uncertainty

Toy Model:
Suposing that the measurements uncertainty behaves like the inverse of SNR.

$$\Delta d_L \approx \frac{2d_L}{SNR}$$

Performing the best fit for this model we have:

$$\boxed{\frac{\Delta d_L}{d_L} = 0.1618z - 0.0289z^2 + 0.002z^3}$$



Events Distribution

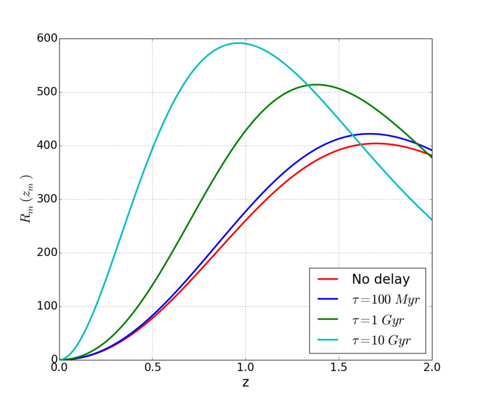
$$\frac{dN}{dt_o dz} = \left[\frac{1}{1+z}\frac{dV_c}{dz}\right] \cdot \left(\frac{dN}{dt_m dV_c}\right)$$

$$\frac{dN}{dt_m dV_c} = \kappa \int_{z_m}^{\infty} dz_f \frac{dt_f}{dz_f} \left(\boldsymbol{\frac{dN}{dt_f dV_c}}\right) P(t_m|t_f,\tau)$$

$$P(t_m|t_f,\tau) = \frac{1}{\tau} exp\left[-\frac{(t_f - t_m)}{\tau}\right]$$

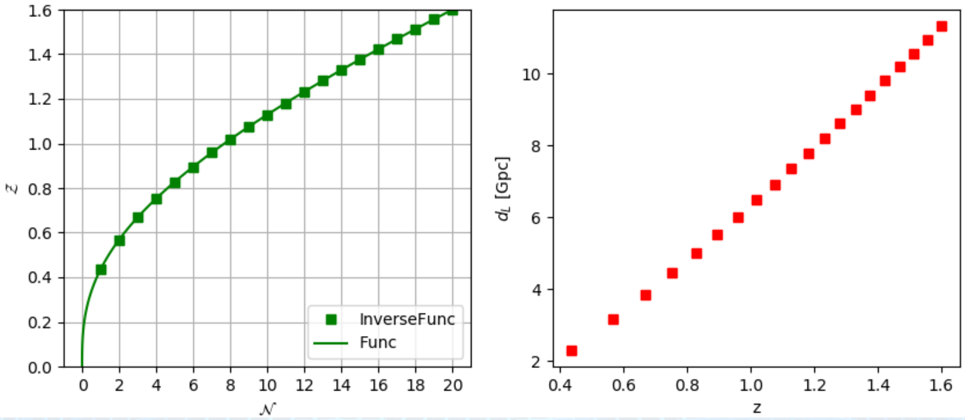
where κ is choosen such that
$N_{tot} = \int_0^{z_{max}} \left(\frac{dN}{dzdt_0}\right) dz.$

$$\Rightarrow \boxed{\mathcal{N}(z) \equiv \int_0^z \left(\frac{dN}{dzdt_0}\right) dz}$$

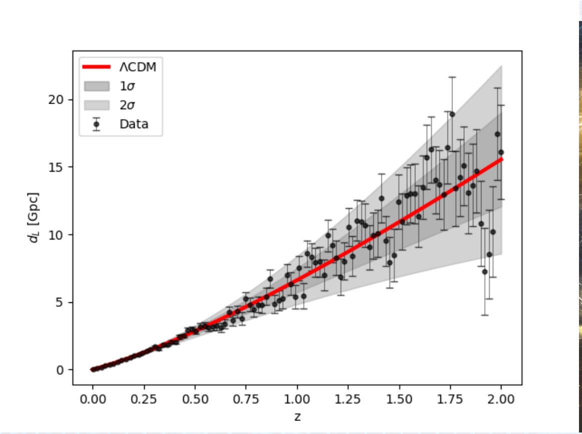


Madau, P. & Dickinson, M.(arXiv:1403.0007)
Vitale, S. et al.(arXiv:1808.00901)

Cosmology With GW Simulations



Cosmology With GW Simulations



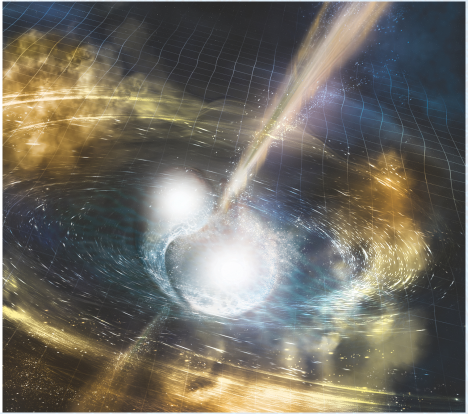


Figure: (Left) Data simulation of events like GW170817. (Right) Artisit representation of a merging BNS (from https://www.ligo.caltech.edu/).

Bayesian Model Selection

Bayes Theorem:

$$P(\theta|d,M) = \frac{P(d|\theta,M)P(\theta|M)}{\boldsymbol{P(d)}}$$

Likelihood:

$$\mathcal{L} = exp\left[-\frac{1}{2}\sum_{i=1}^{N}\frac{|d_i - f(\theta,z_i)|^2}{\sigma_i^2}\right]$$

Bayesian Evidence:

$$\boldsymbol{Z} \equiv \boldsymbol{P(d)}$$

Bayes Factor:

$$\boxed{\mathcal{B}_{0,1} = \frac{Z_0}{Z_1}}$$

Nested Models:

$$\boldsymbol{\Lambda CDM}\text{: } \omega_{DE} = -1$$

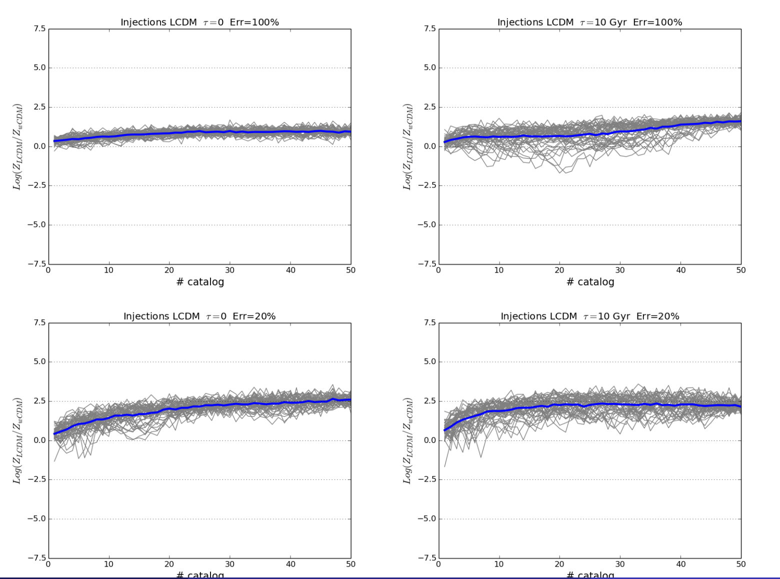
$$\boldsymbol{\omega_o CDM}\text{: } \omega_{DE} = \omega_0 \neq -1$$

$$\boldsymbol{\omega_o \omega_a CDM}\text{: } \omega_{DE} = \omega_0 + (1-a)\omega_a$$

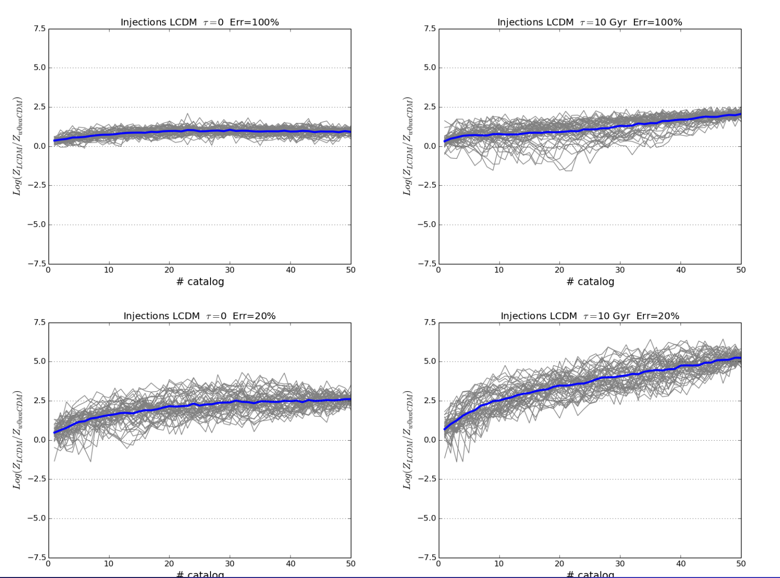
Parameters to be estimated:

$$\boxed{H_0, \Omega_M, \omega_0, \omega_a}$$

$\omega_o CDM$ vs ΛCDM: (arXiv:1905.03848)

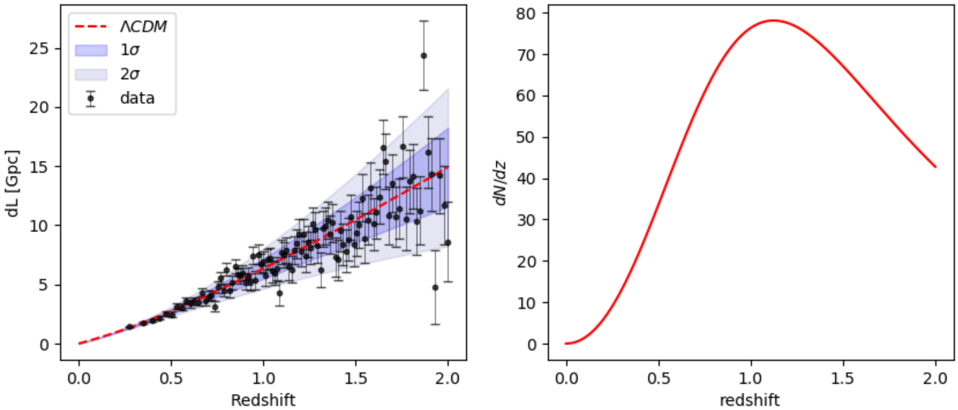


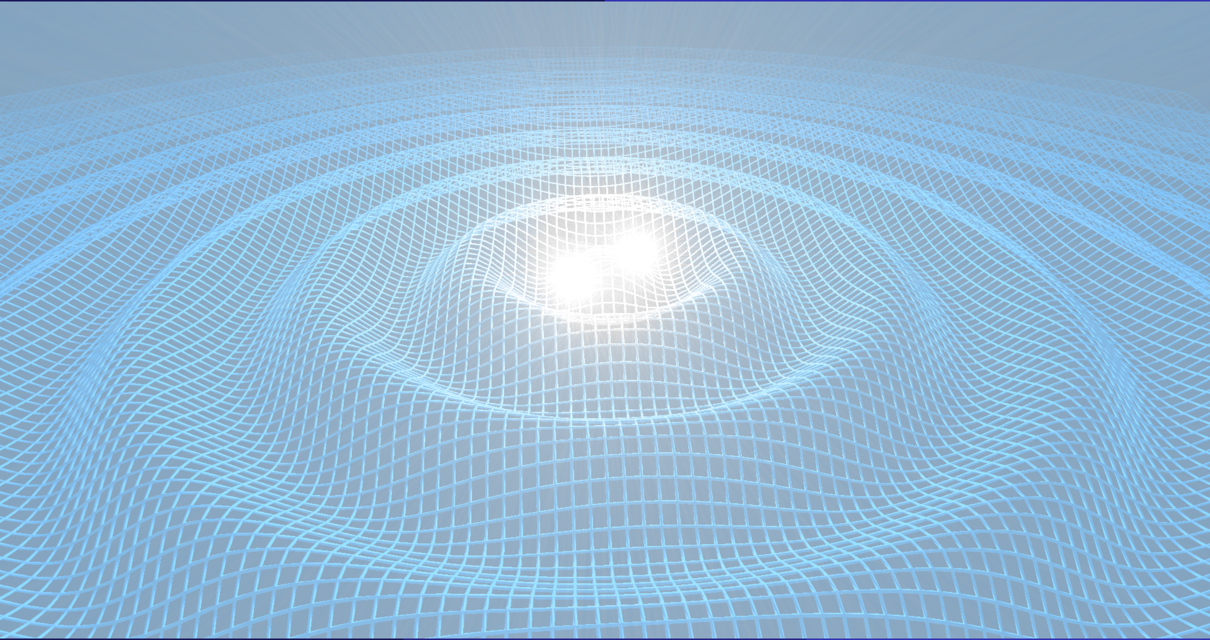
$\omega_o\omega_a CDM$ vs ΛCDM: (arXiv:1905.03848)



Thank You

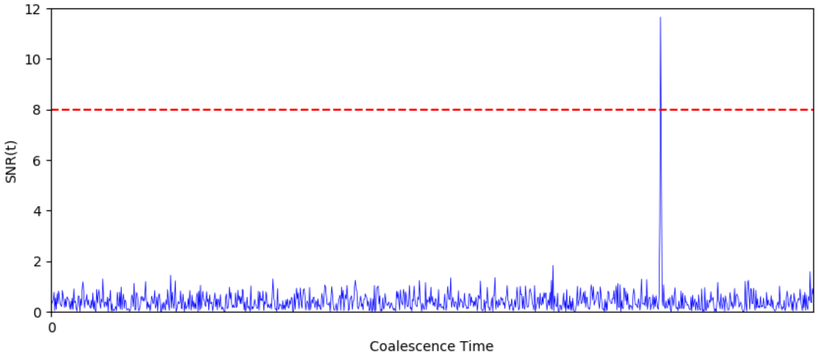
Figure: Python code available in **https://github.com/jmsdsouzaPhD/GW**





Matched Filtering

$$SNR(t) = \left[\int_{-\infty}^{\infty} \frac{df}{S_n(f)} \tilde{O}(f)\tilde{h}^*(f) e^{2\pi i f t}\right] \cdot \left[\frac{1}{2}\int_{-\infty}^{\infty} \frac{df}{S_n(f)} |\tilde{h}(f)|^2\right]^{-1/2}$$



Antenna Pattern Functions

L-shape Detectors (LIGO/Virgo)

$$F_{+} = \left(\frac{1+cos^2\beta}{2}\right) cos(2\alpha)cos(2\psi) - cos\beta sin(2\alpha)sin(2\psi)$$

$$F_{\times} = \left(\frac{1+cos^2\beta}{2}\right) cos(2\alpha)sin(2\psi) + cos\beta sin(2\alpha)cos(2\psi)$$

Δ-Shape Detectors (ET/LISA):

$$F_{+} = \frac{\sqrt{3}}{2}\left[\left(\frac{1+cos^2\beta}{2}\right) cos(2\alpha)cos(2\psi) - cos\beta sin(2\alpha)sin(2\psi)\right]$$

$$F_{\times} = \frac{\sqrt{3}}{2}\left[\left(\frac{1+cos^2\beta}{2}\right) cos(2\alpha)sin(2\psi) + cos\beta sin(2\alpha)cos(2\psi)\right]$$