

`Python` introduction

NumPy Basics: Fitting and testing

Mauricio Sevilla

email= `jsevilla@credicorpcapital.com`

27.06.19

Our last example was based on how to manipulate data qualitatively, now we are going to start doing some statistics so that later there will be easier for us, do some analysis.

As we showed, pandas is a great alternative when doing data analysis. But sometimes there are some important libraries that uses different kind of data more *simple*.

So during this session we will use a numpy approach using scipy

The interesting part here, is that pandas is based on numpy and scipy

Our main objective today, is getting some insides on numpy , data transformation and fitting.

By the moment, we will just need numpy and matplotlib ,

```
In [1]: import numpy as np  
import matplotlib.pyplot as plt
```

NumPy brought many advantages to Python in terms of velocity of big calculations (One of the biggest problems of Python), we will see some examples and learn the usage meanwhile.



More than 50% is done on C !!

What makes NumPy special?

- arrays!

Let us take two built-in lists,

```
In [2]: x=[1,2,3]  
        y=[3,2,1]
```

For example we would like to sum them, component by component as vectors do

$$\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 4 \\ 4 \\ 4 \end{pmatrix}$$

In [3]: `z=x+y`

In [4]: `print(z)`

[1, 2, 3, 3, 2, 1]

List s do not work like that!, so we have to construct the sum from scratch.

```
In [5]: z=[]  
        for i in range(len(x)):  
            z.append(x[i]+y[i])
```

```
In [6]: print(z)
```

```
[4, 4, 4]
```

There is a different way to do this but with some more structure.

```
In [7]: z=[x[i]+y[i] for i in range(len(x))]
```

```
In [8]: z
```

```
Out[8]: [4, 4, 4]
```

What if we want to use bigger Lists, for example, using the range function

```
In [9]: x=list(range(10))  
        print(x)
```

```
[0, 1, 2, 3, 4, 5, 6, 7, 8, 9]
```

But it has other useful parameters,

```
In [10]: x=list(range(1,10))  
print(x)
```

```
[1, 2, 3, 4, 5, 6, 7, 8, 9]
```

```
In [11]: x=list(range(1,10,3))  
print(x)
```

```
[1, 4, 7]
```

Let us create a bigger list

```
In [12]: N=10000000  
x=list(range(N))  
y=list(range(N))
```

And let us calculate the sum, but!, measuring the time it takes

```
In [13]: %%time  
z=[]  
for i in range(len(x)):  
    z.append(x[i]+y[i])
```

Wall time: 2.41 s

```
In [14]: %%time  
z=[x[i]+y[i] for i in range(len(x))]
```

Wall time: 1.5 s
Compiler : 202 ms

These are called **magic** cells, done for the jupyter notebooks!, for more information use

```
%magic
```

on a code cell.

%magic

Now, using arrays!

As we have been talking before, everytime we call a function from a given library (in this case NumPy), we use first the name (or alias.)

```
In [15]: X=np.array(x)  
         Y=np.array(y)
```

There you can see that we *converted* from lists to arrays!!.

In [16]: %%time
Z=X+Y

Wall time: 25.7 ms

In []:

The array s are created from lists or tuples, not numbers!!

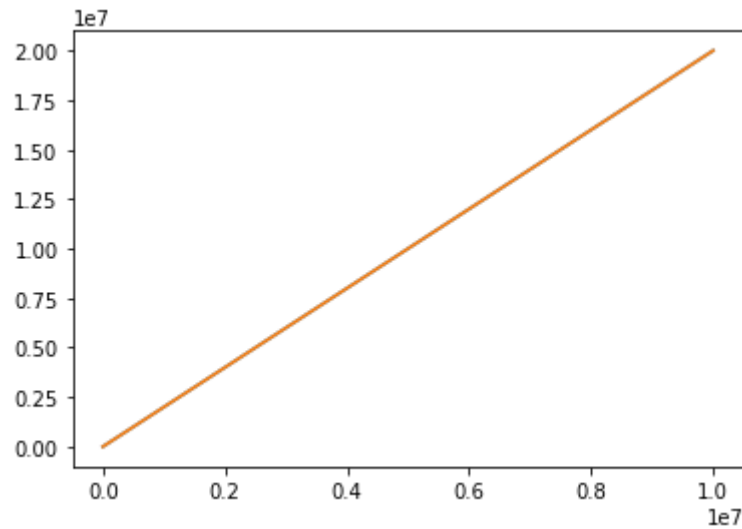
In []:

How can we know if there is any difference between the results from `lists` and `arrays`?

A: Plotting

```
In [17]: plt.plot(Z)  
plt.plot(z)
```

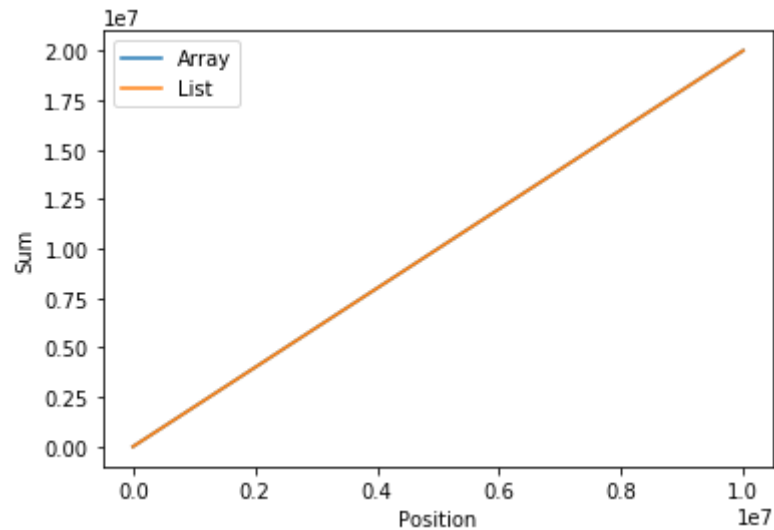
```
Out[17]: [<matplotlib.lines.Line2D at 0x22c07755f98>]
```



That is not a presentable plot!

```
In [18]: plt.plot(Z,label='Array')  
plt.plot(z,label='List')  
plt.xlabel("Position")  
plt.ylabel("Sum")  
plt.legend()
```

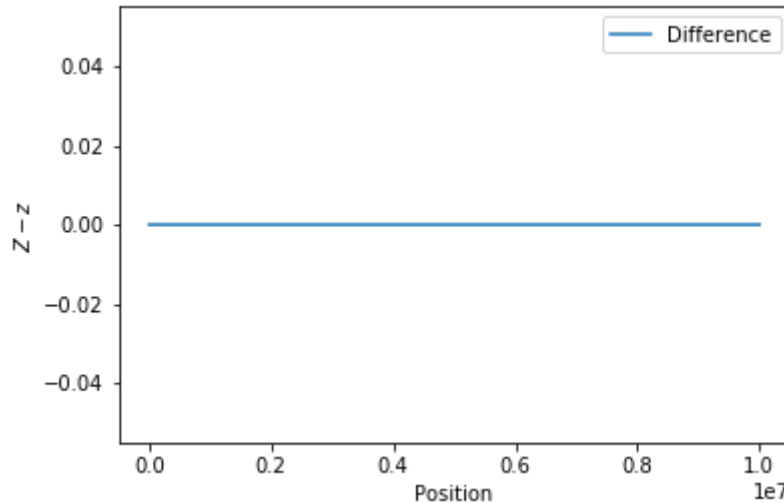
Out[18]: <matplotlib.legend.Legend at 0x22c1c99a470>



Or the difference!

```
In [19]: plt.plot(Z-z,label='Difference')  
plt.xlabel("Position")  
plt.ylabel("$Z-z$")  
plt.legend()
```

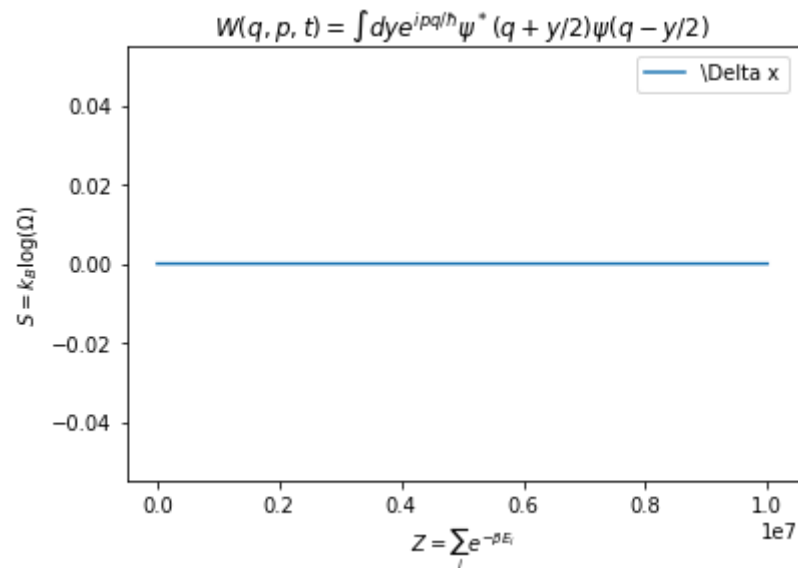
```
Out[19]: <matplotlib.legend.Legend at 0x22c55599f28>
```



Look at the label on y . The font looks different. Matplotlib supports *L^AT_EX*!!!. Let us make the previous plot, but with hypothetical mathematical labels

```
In [20]: plt.plot(Z-z,label='\Delta x')
plt.xlabel("$Z=\sum_i e^{\beta E_i}$")
plt.ylabel("$S=k_B\log(\Omega)$")
plt.title("$W(q,p,t)=\int dy e^{ipq/\hbar}\psi^*(q+y/2)\psi(q-y/2)$")
plt.legend()
```

Out[20]: <matplotlib.legend.Legend at 0x22c1f096a20>



It can get even worse. Matrix sum.

First of all, let us see how can we construct a *matrix* using lists.

```
In [21]: %%time  
N=4  
x=[list(range(i*N,N+i*N)) for i in range(N)]
```

Wall time: 167 ms

This is basically a list done of lists.

In [22]: `print(len(x),len(x[2]))`

4 4

In [23]: `print(x)`

[[0, 1, 2, 3], [4, 5, 6, 7], [8, 9, 10, 11], [12, 13, 14, 15]]

Now, let us use a bigger matrices

```
In [24]: %%time  
N=10000  
x=[list(range(i*N,N+i*N)) for i in range(N)]  
y=[list(range(i*N,N+i*N)) for i in range(N)]
```

Wall time: 8.09 s

And summing them.

```
In [25]: %%time
z=[]
for i in range(N):
    aux=[]
    for j in range(N):
        aux.append(x[i][j]+y[i][j])
    z.append(aux)
```

Wall time: 28.3 s

and just in the same way we did it for the case of *vectors*, we will convert the `list` s to `array` s

```
In [26]: %%time  
X=np.array(x)  
Y=np.array(y)
```

Wall time: 20.5 s

And summing them,

```
In [27]: %%time  
Z=X+Y
```

Wall time: 952 ms

Look that in this case, we didn't have to define a new *sum*

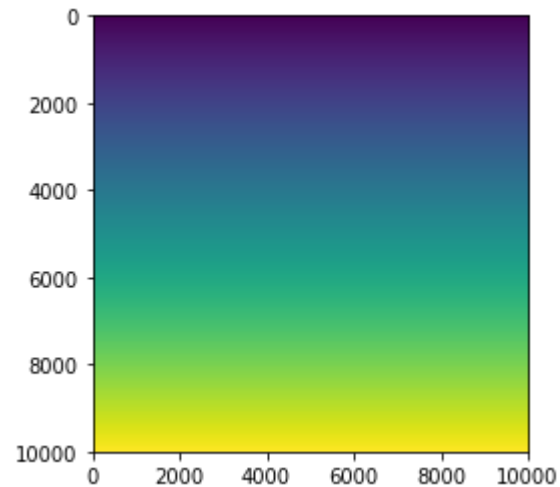
How is the difference on times?

HUGE!

In the same sense than before, we can plot the matrices and their difference

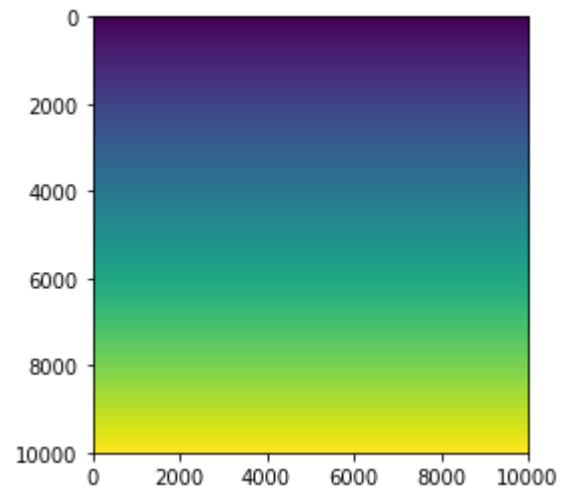
```
In [28]: plt.imshow(Z)
```

```
Out[28]: <matplotlib.image.AxesImage at 0x22fd2801eb8>
```



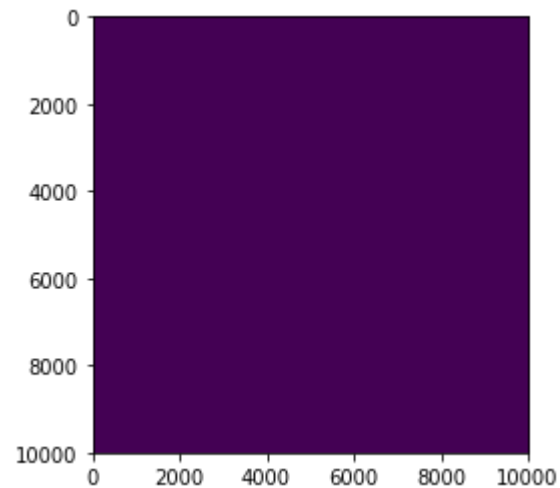
In [29]: `plt.imshow(z)`

Out[29]: `<matplotlib.image.AxesImage at 0x22c1f1a8b70>`



In [30]: `plt.imshow(Z-z)`

Out[30]: `<matplotlib.image.AxesImage at 0x22c077bdb00>`



But, what if we want to do a more rigorous study of the time certain process take?

We will use the library time

```
In [31]: import time
```

The available methods are,

```
In [32]: print(dir(time))
```

```
['_STRUCT_TM_ITEMS', '__doc__', '__loader__', '__name__', '__package__', '__spec__',  
'altzone', 'asctime', 'clock', 'ctime', 'daylight', 'get_clock_info', 'gmtime', 'localtime', 'mktime', 'monotonic', 'monotonic_ns', 'perf_counter', 'perf_counter_ns', 'process_time', 'process_time_ns', 'sleep', 'strftime', 'strptime', 'struct_time', 'thread_time', 'thread_time_ns', 'time', 'time_ns', 'timezone', 'tzname']
```

For instance,

```
In [33]: time.localtime()
```

```
Out[33]: time.struct_time(tm_year=2019, tm_mon=6, tm_mday=27, tm_hour=12, tm_min=54, tm_sec=2,  
tm_wday=3, tm_yday=178, tm_isdst=0)
```

We will concentrate on the function `time.time()`

This function counts how many seconds have passed since 1 Jan. 1970.

```
In [34]: time.time()
```

```
Out[34]: 1561658042.7488396
```

Without being rigorous, one can more or less test that,

```
In [35]: time.time()/60/60/24/365
```

```
Out[35]: 49.5198516864145
```

```
In [36]: 2019-1970
```

```
Out[36]: 49
```


So, we will measure the time elapsen on a given command as,

```
In [37]: a=time.time()  
         time.sleep(2)  
         b=time.time()
```

So that the difference is now the amount of seconds,

```
In [38]: b-a
```

```
Out[38]: 2.0002174377441406
```

For instance, on our example of the time it takes to sum two built in list s

```
In [39]: a=time.time()
N=10000000
x=list(range(N))
y=list(range(N))
z=[]
b=time.time()
for i in range(len(x)):
    z.append(x[i]+y[i])
c=time.time()
print(c-b,"sec",c-a,"sec")
```

2.331831455230713 sec 12.639823913574219 sec

We would like to have *something* that, given the size returns the time it takes, so that we can plot the time consumed by the computer for a sum of a given size pair of vectors.

Functions

A function is a piece of code, that can be *called* without typing the commands again, the structure goes as,

```
In [40]: def test1(a,b):  
         return a*b
```

In [41]: test1(2,3)

Out[41]: 6

In [42]: test1(2.0,3.0)

Out[42]: 6.0

In [43]: test1([1,2],3)

Out[43]: [1, 2, 1, 2, 1, 2]

In [44]: test1("a",10)

Out[44]: 'aaaaaaaaaa'

Our function should

- Receive the size N
- Generate the vectors \mathbf{x} and \mathbf{y}
- Create the Output vector \mathbf{z}
- Sum $\mathbf{z} = \mathbf{x} + \mathbf{y}$

```
In [45]: def list_sum(N):  
         x=list(range(N))  
         y=list(range(N))  
         z=[]  
         b=time.time()  
         for i in range(len(x)):  
             z.append(x[i]+y[i])  
         c=time.time()  
         return c-b
```

Let us test it,

```
In [46]: list_sum(1)
```

```
Out[46]: 0.0
```

```
In [47]: list_sum(10)
```

```
Out[47]: 0.0
```


In [48]: `list_sum(100)`

Out[48]: 0.0006272792816162109

In [49]: `list_sum(1000)`

Out[49]: 0.0

In [50]: `list_sum(10000)`

Out[50]: 0.0020236968994140625

In [51]: `list_sum(100000)`

Out[51]: 0.013962507247924805

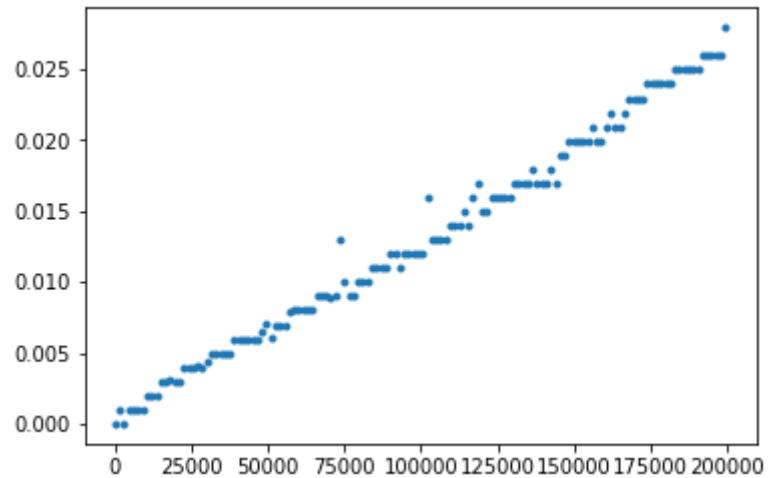
In [52]: `list_sum(10000000)`

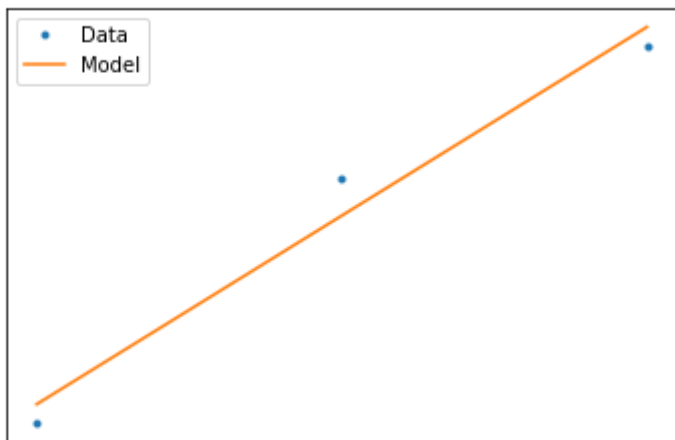
Out[52]: 1.506976842880249

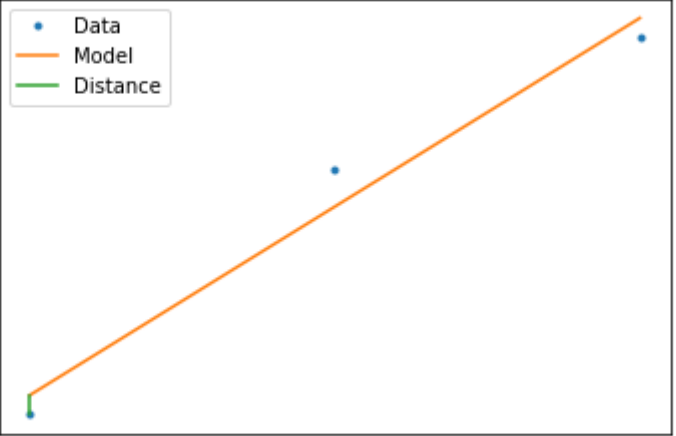
```
In [53]: times=[]  
t=[]  
for i in range(0,200000,1500):  
    t.append(i)  
    times.append(list_sum(i))  
t=np.array(t)
```

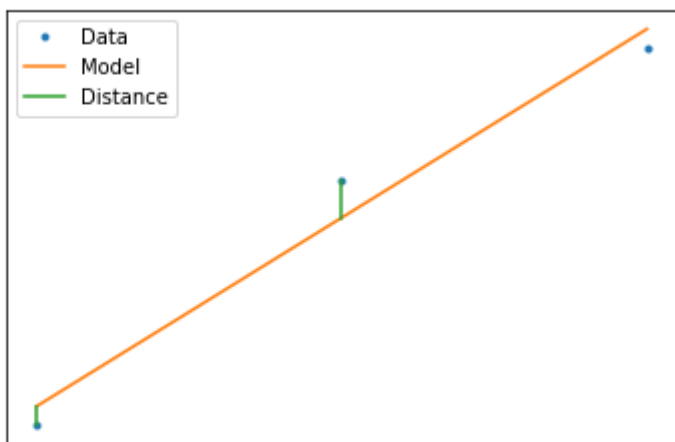
```
In [54]: plt.plot(t,times, '.')
```

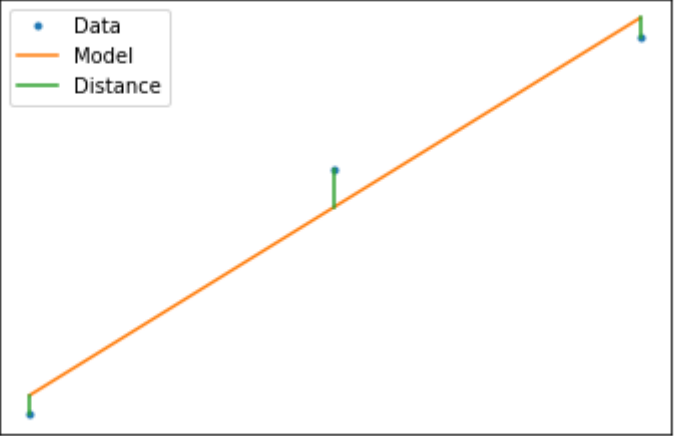
```
Out[54]: [<matplotlib.lines.Line2D at 0x22c30aa4630>]
```







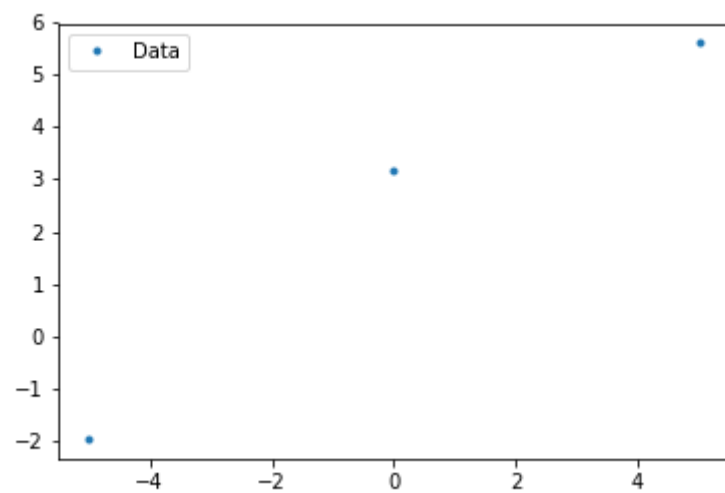


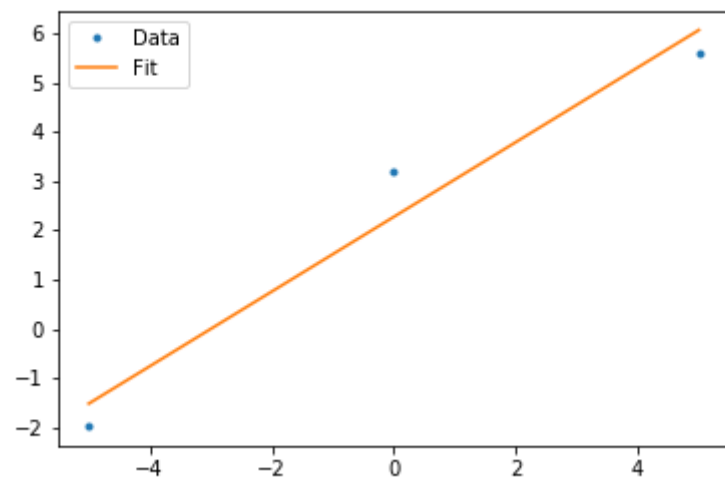


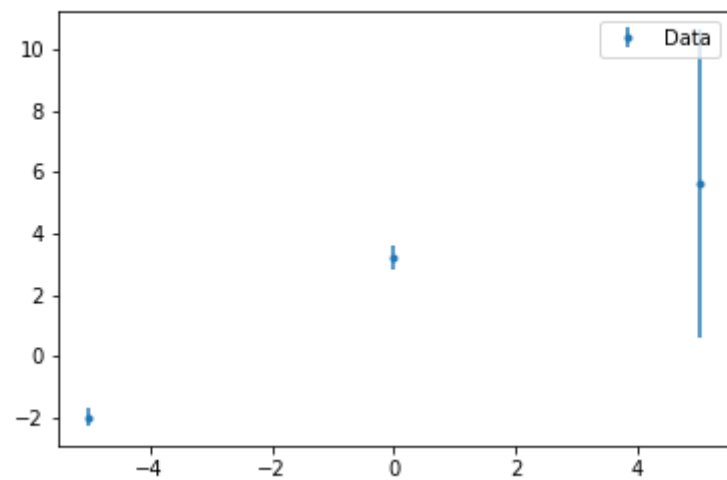
That means that

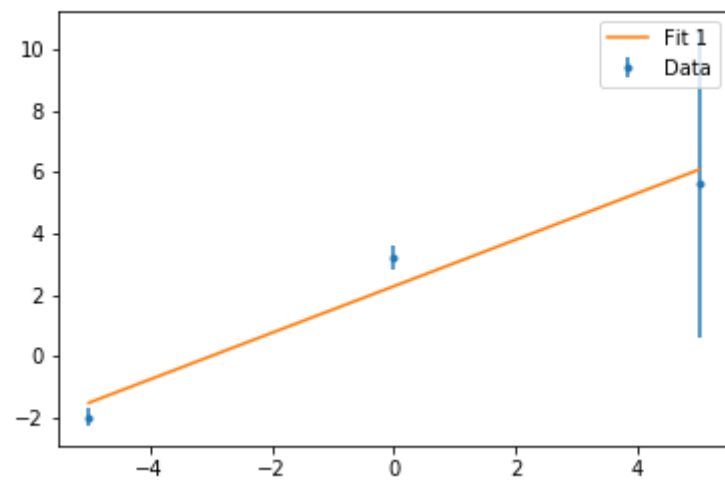
$$d_{\text{Green lines}} = \sum_i d_i (y - f(x))^2$$

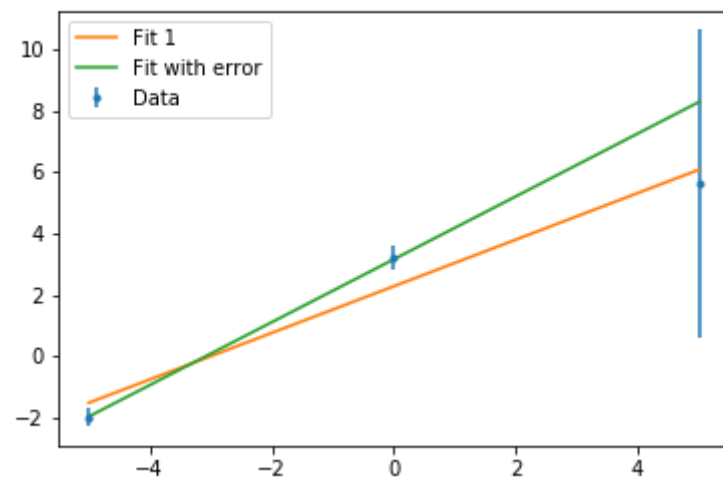
such that $d_{\text{Green lines}}$ takes its minimum value.











```
In [55]: from scipy.optimize import curve_fit
```

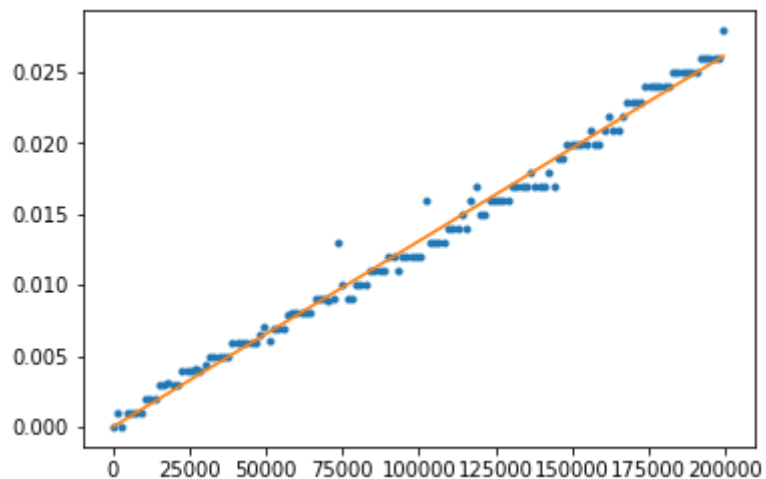
```
In [56]: def f(x,a):  
         return a*x
```

```
In [57]: popt,pcov=curve_fit(f,t,times)
```



```
In [58]: plt.plot(t,times,'.')  
plt.plot(t,f(np.array(t),*popt))
```

```
Out[58]: [<matplotlib.lines.Line2D at 0x22c31123588>]
```



In [59]: `print(pcov)`

```
[[3.19685454e-19]]
```

In [60]: `print(popt)`

```
[1.30912978e-07]
```

In [61]: `print(np.sqrt(pcov.diagonal()))`

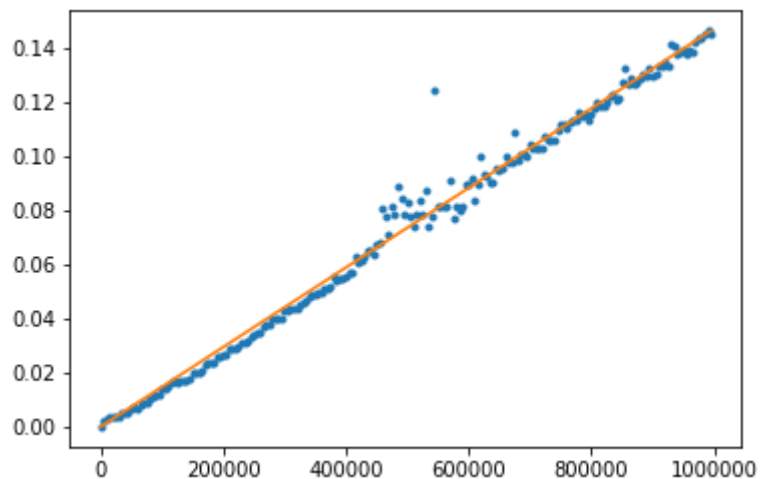
```
[5.65407335e-10]
```

```
In [62]: times=[]  
t1=[]  
for i in range(0,1000000,5000):  
    t1.append(i)  
    times.append(list_sum(i))  
t1=np.array(t1)
```

```
In [63]: popt,pcov=curve_fit(f,t1,times)
```

```
In [64]: plt.plot(t1,times,'.')  
plt.plot(t1,f(t1,*popt))
```

```
Out[64]: [<matplotlib.lines.Line2D at 0x22c31158048>]
```

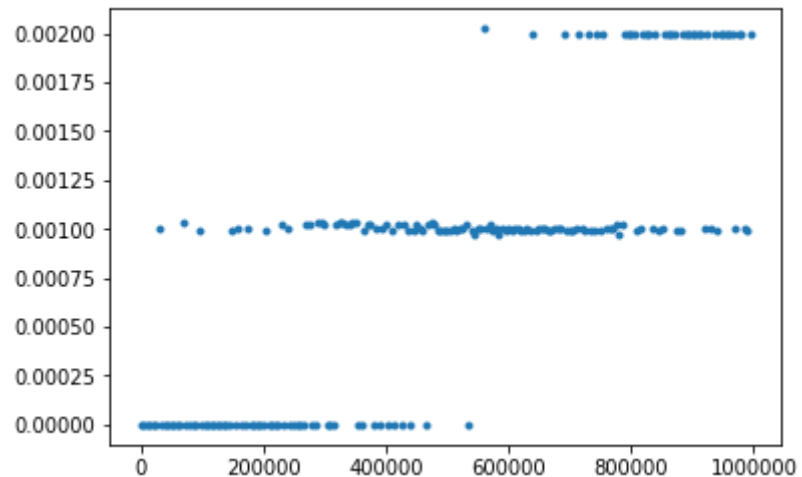


```
In [65]: def arr_sum(N):  
          x=np.arange(N)  
          y=np.arange(N)  
          b=time.time()  
          z=x+y  
          c=time.time()  
          return c-b
```

```
In [66]: times=[]  
         t2=[]  
         for i in range(0,1000000,5000):  
             t2.append(i)  
             times.append(arr_sum(i))  
         t2=np.array(t2)
```

```
In [67]: plt.plot(t2,times,'.')
```

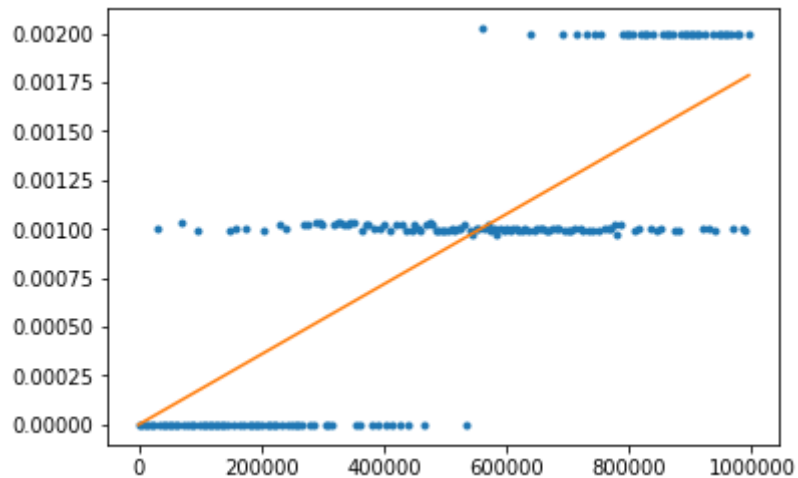
```
Out[67]: [<matplotlib.lines.Line2D at 0x22c31247fd0>]
```




```
In [68]: popt2,pcov2=curve_fit(f,t2,times)
```

```
In [69]: plt.plot(t2,times,'.')  
plt.plot(t2,f(t2,*popt2))
```

```
Out[69]: [<matplotlib.lines.Line2D at 0x22c31283978>]
```

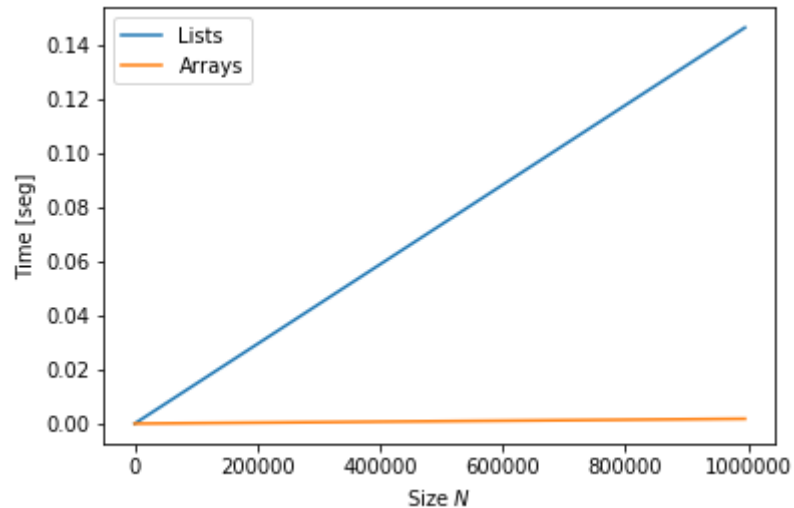


```
In [70]: print(pcov2.diagonal()**.5)
```

```
[5.18637054e-11]
```

```
In [71]: plt.xlabel("Size $N$")  
plt.ylabel("Time [seg]")  
plt.plot(t1,f(t1,*popt),label='Lists')  
plt.plot(t2,f(t2,*popt2),label='Arrays')  
plt.legend()
```

Out[71]: <matplotlib.legend.Legend at 0x22c31315dd8>



In [72]: `print(popt,popt2)`

`[1.47226817e-07] [1.79354508e-09]`