

Worksheet 3. Solution.

1) Probability of transfer of heat.

Let us take the definition of the differential of entropy as

$$dS = \frac{dQ}{T}$$

$$\Delta S_{\text{universe}} - \Delta S_u = -\frac{Q}{T_2} + \frac{Q}{T_1} = \left[-\frac{1}{301} + \frac{1}{300} \right] J/k$$

Mind the signs.

$$= \frac{301 - 300}{301 \cdot 300} J/k$$

$$= \frac{1}{903000} J/k$$

$$\approx 1 \cdot 10^{-5} J/k$$

But, the entropy is also proportional to the logarithm of the number of accessible states ω

$$S = k_B \ln(\omega)$$

Hence

$$\Delta S = k_B \ln \left(\frac{\omega_{\text{final}}}{\omega_{\text{initial}}} \right)$$

$$\ln \left(\frac{\omega_{\text{final}}}{\omega_{\text{initial}}} \right) = \frac{\Delta S}{k_B} - \frac{\omega_{\text{final}}}{\omega_{\text{initial}}} = \exp \left(\frac{\Delta S}{k_B} \right)$$

$$\frac{\sigma_{\text{final}}}{\sigma_{\text{initial}}} \approx \exp \left(\frac{10^{-5} \cancel{T} \cancel{k}}{10^{-23} \cancel{T} \cancel{k}} \right) = e^{10^{18}}$$

look that, the reverse process, meaning giving δJ of heat to the hot one is very unlikely. the cold reservoir

2) The Solar System

a) The motion and organization of planets is much more ordered than the original dust Cloud.

- Why this does not violate the second law of thermodynamics?

The dust configuration seems to have higher entropy, but given the nature of the gravitational interaction (Attractive), the configuration maximizing the entropy has some structure.

The potential energy is converted to kinetic energy that at the end is released in form of photons which carry entropy with it.

b) The nuclear processes of the sun convert protons to heavier elements such as carbon.

Does this further organization lead to a reduction of entropy?

- The formation of heavier elements releases large amounts of energy carried out by photons. The entropy carried away by the photons compensate any kind of ordering given by the packing of nucleons into heavier nuclei.

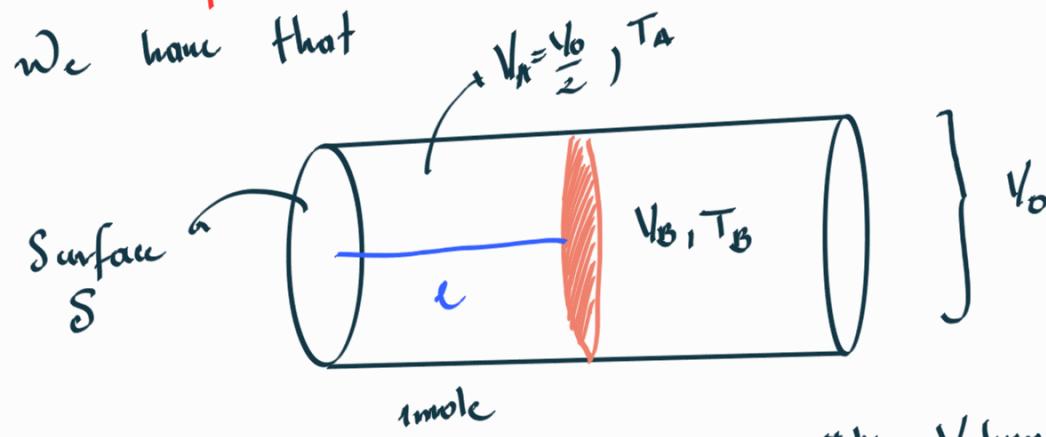
c) The evolution of life and intelligence requires even further levels of organization.

How is this achieved on earth without violating the second law?

There is usage of energy by the organisms that converts more ordered forms of energy to less ordered ones.

3 Ideal Gases I.

We have that



the Volume $V_A = S \times l$.

then the equilibrium condition reads

$$P_a S = P_b S + F \quad \textcircled{1}$$

Tension of the wire.

as we have that for a mole of gas

$$P_a = \frac{R T_a}{V_a}$$

$$P_b = \frac{R T_b}{V_b}$$

Then \textcircled{1} becomes

$$\frac{R T_a S}{V_a} - \frac{R T_b S}{V_b} = F$$

$$\frac{2R}{V_0} S (T_A - T_B) = F.$$

b) the force to break the wire F^* is calculated by

$$P_A^* S - P_B S = F^*$$

$$\downarrow$$

$$T_A^* = \frac{P_A^* V_0}{R} = \frac{F^* + P_B S}{2 S R} V_0$$

c) the heat;

$$Q = \Delta U = c_V (T_A^* - T_A),$$

after the wire breaks, then we may have that
the cycle is complete;

$$\Delta U_A + \Delta U_B = 0$$

$$= c_V (T_A' - T_A^*) + c_V (T_B' - T_B)$$

where T_A' and T_B' are the final temperatures
of A and B.

$$(T_A' - T_A^*) + (T_B' - T_B) = 0$$

$$T_A' + T_B' = T_A^* + T_B$$

Then using again the equation of state

$$P_B' V_B' = P_B' (V_0 - V_A') = P_B' V_0 - P_B' V_A' = R T_B'$$

$$P_B' V_0 = R T_B' + P_B' V_A'$$

But in equilibrium $P_A' = P_B'$ and

$$P_A' V_A' = R T_A'$$

$$P_B' = P_A' = \frac{R T_B' + R T_A'}{V_0}$$

$$P_B' = P_A' = \frac{R (T_A' + T_B')}{V_0}$$

$$= \frac{R (T_A^* + T_B)}{V_0}$$

Then

$$\frac{V_A'}{S} = \frac{R T_A'}{S P_A} = \frac{T_A' V_0}{S (T_A^* + T_B)} = l'$$

Ideal gases II

The initial temperature of the gas in BC goes as;

$$T_i = \frac{(V_B + V_c) P}{R} = \frac{(1.5 + 2)V_0 P}{R} = \frac{3.5 V_0 P}{R}$$

after the compression.

$$T_f = \frac{V_B P}{R} = 1.5 \frac{V_0 P}{R}$$

The change on the internal energy goes as

$$\Delta U_i = c_v (T_f - T_i) = \frac{c_v V_0 P}{R} (1.5 - 3.5)$$

$$= -2 \frac{c_v V_0 P_0}{R}$$

and the work.

$$W_i = P [V_B - (V_c + V_B)] = -P V_c = -2 P V_0.$$

So that the exchange in heat;

$$Q_i = \Delta U_i - W_i = -2 \frac{c_v V_0 P_0}{R} - 2 P V_0$$

$$= -2 V_0 P_0 \left(\frac{c_v}{R} - 1 \right)$$

for 1 mole of diatomic ideal gas $c_v = \frac{5}{2}$

$$Q_i = -2 V_0 P_0 \left(\frac{5}{2} - 1 \right) = -3 V_0 P_0$$

b) After closing the tap BC and opening AB, the expansion leads to;

$$V_B \longrightarrow (V_A + V_B)$$

$$T = \text{ctc.}$$

Then it cools down at constant pressure.

(Free gas expansion + irreversible isochoric)

$$Q_2 = c_V (T_{\text{room}} - T_f)$$

Then the total heat exchange

$$Q_{\text{tot}} = Q_1 + Q_2 = -V_0 P_0 + c_V (T_{\text{room}} - T_f).$$

The change of the entropy of the universe is the sum of the following transformations.

- free expansion
- irreversible isochoric cooling

Note: This is just considering the irreversible processes.

$$\Delta S_{\text{gas}} = nR \ln \left(\frac{V_A + V_B}{V_A} \right)$$

$$+ n c_V \ln \left(\frac{T_{\text{room}}}{T_f} \right)$$

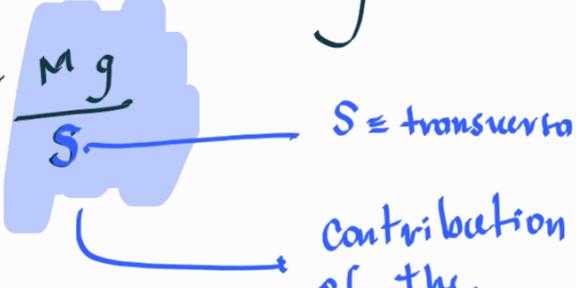
$$\Delta S_{\text{environment}} = - \frac{Q_2}{T_{\text{room}}}$$

$$\Delta S_{\text{universe}} = nR \ln \left(\frac{V_A + V_B}{V_A} \right) + n c_V \ln \left(\frac{T_{\text{room}}}{T_f} \right) - c_V \frac{(T_{\text{room}} - T_f)}{T_{\text{room}}}$$

5 ideal Gases III

To obtain the final pressure we may use

$$P_f = P_0 + \frac{Mg}{S}$$


S = transversal Surface
(5,1)

Let us calculate S.

So, the equation of state

$$P_0 \underbrace{h_0 S}_{V_0} = R T_0 \rightarrow \left(P_0 + \frac{Mg}{S} \right) (h_0 - h) S = R T_0$$

$$\cancel{P_0 h_0 S} - P_0 h S + Mg h_0 - Mg h = \cancel{R T_0}$$

$$- P_0 h S + Mg h - Mg h = 0$$

$$h_0 = \frac{R T_0}{P_0 S}$$

Then

$$P_0 h S^2 + Mg h S - Mg h_0 S = 0$$

$$P_0 h S^2 + Mg h S - Mg \cancel{S} \left[\frac{R T_0}{P_0 S} \right] = 0$$

$$P_0 h S^2 + Mg h S - \frac{M R T_0 g}{P_0} = 0$$

this is a quadratic equation

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\frac{-Mgh \pm \sqrt{(Mgh)^2 + 4(P_0h)\left(\frac{MRT_0g}{P_0}\right)}}{2P_0h}$$

The only root that works is the positive

$$S = \frac{-Mgh \pm \sqrt{(Mgh)^2 + 4MRT_0gh}}{2Mh}$$

then it can be replaced on

(5.1)

b) The process is an irreversible isothermal compression where

$$V_{\text{initial}} = S h_0$$

$$V_{\text{final}} = S (h_0 - h)$$

The pressure goes on the piston

$$P = P_0 + \frac{Mg}{S}$$

so on the system

$$W = P \Delta V = -\left(P_0 + \frac{Mg}{S}\right) hS < 0$$

↓
on the system

+ the isotherm for the ideal gas $dU=0$.

$$Q = -W$$

the change on the entropy of the reservoir

$$\Delta S_{\text{reservoir}} = \frac{|Q|}{T_0}$$

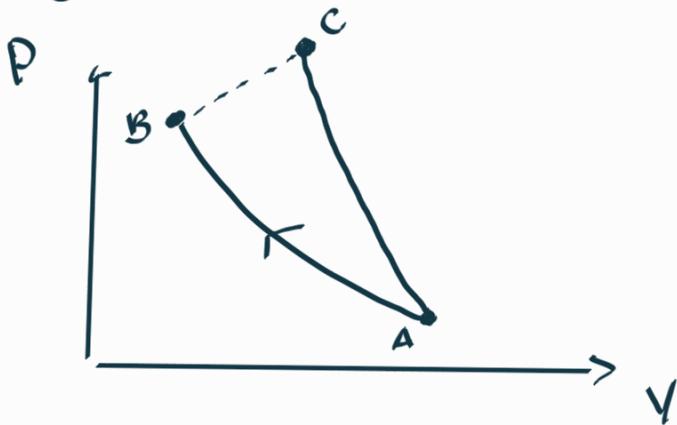
$$= \left(P_0 + \frac{Mg}{S}\right) \frac{hS}{T_0}$$

$$\Delta S_{\text{gas}} = R \ln \left(\frac{P_f}{P_i}\right) = R \ln \left(\frac{P_0}{P_0 + \frac{Mg}{S}}\right)$$

$$\Delta S_u = S_{\text{reservoir}} + \Delta S_{\text{gas}}$$

6 Ideal Gases IV

The process goes



We have from the equation of state

$$n_A = \frac{P_A V_A}{R T_A}$$

With the information
 we have, we could
 compute the amount of
 gas.

Then, for the isothermal compression

$$Q_{AB} = n_A R T_A \ln \left(\frac{V_B}{V_A} \right)$$

$Q_{\text{total}} = -W \Rightarrow \Delta U = 0$ as it is a cycle.

$$\begin{aligned}
 Q_{\text{total}} &= Q_{AB} + Q \\
 &= n_A R T_A \ln \left(\frac{V_B}{V_A} \right) + Q^{\text{heat absorbed}}
 \end{aligned}$$

The efficiency

$$\eta = \frac{W}{Q_{\text{absorbed}}} = 1 + \frac{n_A R T_A}{Q} \ln \left(\frac{V_B}{V_A} \right)$$

To know if BC transformation is reversible

$$(\Delta S_{\text{environment}} + \Delta S_{\text{gas}})_{\text{rev}}^{\text{BC}} = 0$$

or

$$\Delta S_{\text{environment}}^{\text{BC}} = -\Delta S_{\text{gas}}^{\text{BC}} \quad \text{if reversible.}$$

as the whole cycle

$$\Delta S_{AB} + \Delta S_{BC} + \Delta S_{CA} = 0 \quad \xrightarrow{0 \rightarrow \text{adiabatically reversible.}}$$

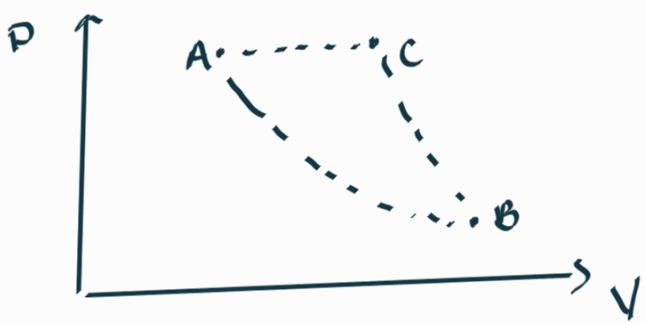
we need to see $\Delta S_{\text{gas}}^{AB}$

$$\Delta S_{\text{gas}}^{AB} = n_A Q \ln \left(\frac{V_B}{V_A} \right) = -\Delta S_{\text{gas}}^{\text{BC}}$$

Then it is reversible.

7) ideal Gases V

we have the following cycle;



From the free expansion, the temperature keeps constant $T_A = T_B$
and the work done by the compressor;

$$W_{BC} = -\Delta U_{BC} = C_V (T_B - T_C) = C_V (T_A - T_C).$$

$$T_C = T_A - \frac{W_{BC}}{C_V}$$

and from the equation of state

$$V_C = \frac{RT_C}{P_A} = \frac{R}{P_A} \left(T_A - \frac{W_{BC}}{C_V} \right)$$

The heat absorbed

$$Q = -Q_{gas}^{ca} = C_P (T_C - T_A).$$

Lastly the change on the entropy of the universe

$$\begin{aligned} \Delta S_u &= \frac{Q}{T_A} = \frac{C_P (T_C - T_A)}{T_A} \\ &= \frac{\gamma}{2} R \left(\frac{T_C}{T_A} - 1 \right), \end{aligned}$$