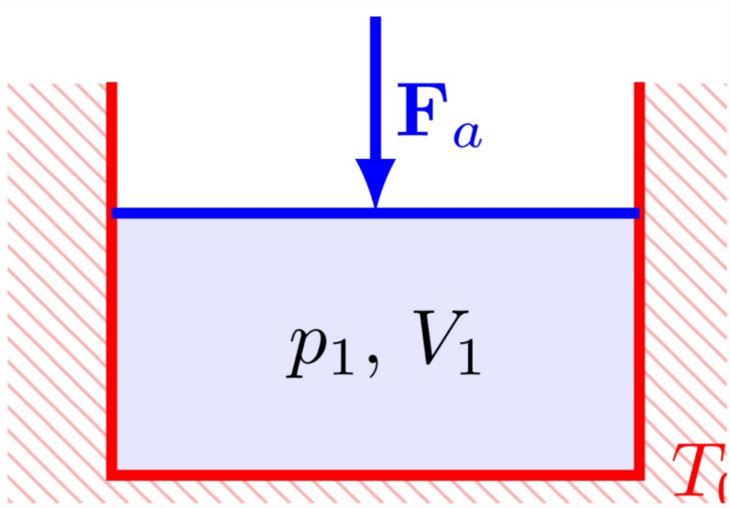
Worksheet 1.

1. Isothermal expansion.

Greiner. Example 1.3 page 23-24

Consider the following situation:

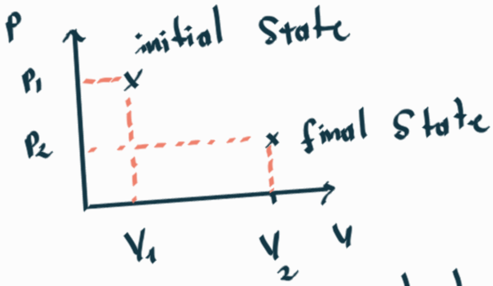


The system is in thermal contact with a reservoir at Temperature T.

If the external force $\vec{F_a}$ is removed suddenly, the gas will expand in a fast and non controlled way up to a volume V_2

Such a process is never going to be reversed by itself. This means that the gas is never (very unlikely) compress itself to the initial state.

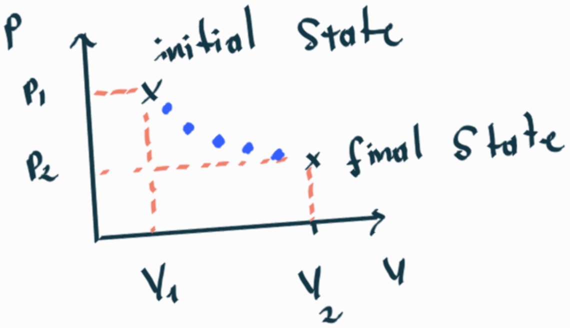
The path AB is therefore unclear so we can only use the initial and final state



- Increase of volume
- Decrease of Pressure.
- Both are equilibrium states.

For the free expansion here described there is no change in temperature as the reservoir keeps it constant, and there is no exchange of heat, so $\Delta Q = 0$, the system also does not perform work as the expansion is free, hence $\Delta U = 0$

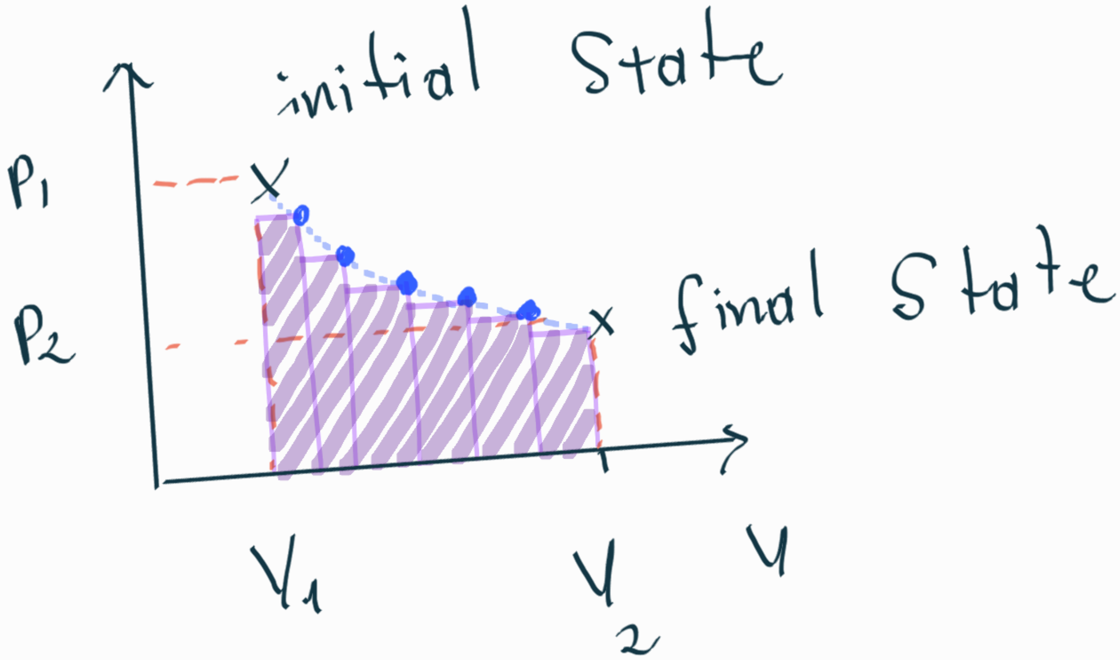
This process can also be done decreasing in a finite number of steps. This is a quasireversible process, meaning that even if the system jumps from state to state, the jump is followed by an equilibration



Consider n small steps between A and B

$$W_{AB} = \sum_{i=A}^{n-1} \Delta W_{i,i+1}$$

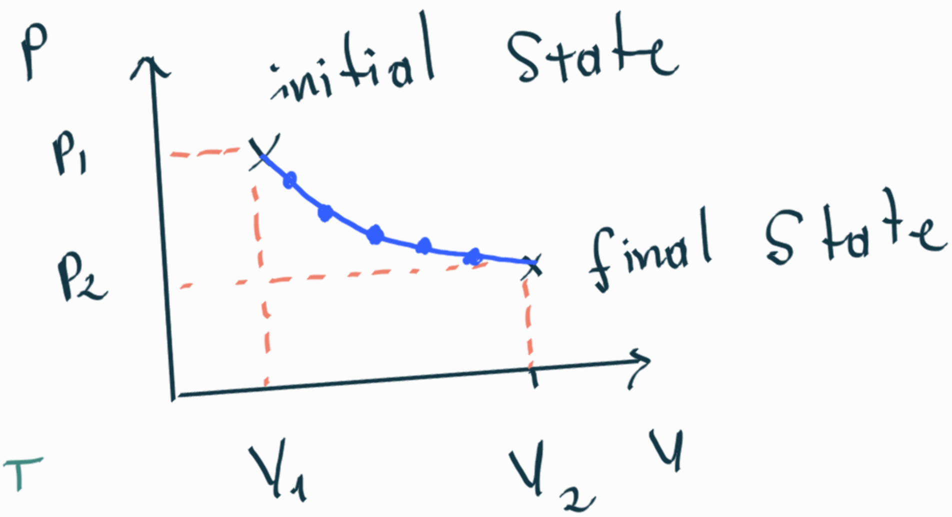
$$W_{AB} = N k_B T \sum_{i=A}^{n-1} \frac{\Delta V_i}{V_i} \, p$$



when $n \longrightarrow \infty$ the system becomes reversible.

The work in the limit is maximum, let us take the limit $n \rightarrow \infty$

$$W_{AB} \approx \int_A^B dW$$



for an ideal gas $PV = N k_B T$

$$W_{AB} = -N k_B T \int_A^B dV = -N k_B T \ln\left(\frac{V_2}{V_1}\right)$$

work done by the system against the external force $\vec{F}_a$.