Statistical Mechanics

Worksheet 3

May 4th, 2023

1 Probability of transfer of heat

When two objects at different temperature are set in thermal contact, they reach thermal equilibrium by transferring heat from the hot source to the cold one. Consider the case of two sources, one a t$T_2 = 301$ K and another $T_1 = 300$ K Q. What would it be the ratio of the probability of observing a transfer of $Q = 1$ J between the two sources?.

1. Calculate the change of entropy between the two states.
2. Compare this with the expression for the amount of microstates possible at each temperature.

2 The Solar System

Originated from a dilute gas of particles, sufficiently separated from other such clouds to be regarded as an isolated system. Under the action of gravity the particles coalesced to form the Sun and planets.

1. The motion and organization of planets is much more ordered than the original dust cloud. Why does this not violate the second law of thermodynamics?
2. The nuclear processes of the Sun convert protons to heavier elements such as carbon. Does this further organization lead to a reduction in entropy?
3. The evolution of life and intelligence requires even further levels of organization. How is this achieved on Earth without violating the second law?

3 Ideal Gases I

Consider an adiabatic cylinder with fixed walls with a volume V_0. Such a volume is divided into two parts A and B by a friction-less adiabatic piston. Initially the piston was set in the middle, locked by an inextensible wire of length ℓ_0 connected in the other end to the base of A. Each part contains the same amount 1 mole of an ideal gas and two different temperatures $T_A > T_B$. As represented in Figure 1

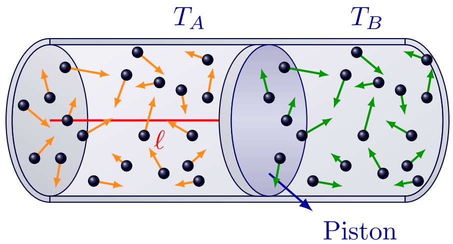


Figure 1: Representation of the cylinder composed with two ideal gases and a piston

1. Compute the tension on the wire.
2. If we take F^* the breaking force of the wire, compute the temperature T_A so the wire will break.
3. Compute the amount of Heat Q required in the total process.

4 Ideal Gases II

Consider now the case of a cylinder divided in three parts A,B and C, with volumes $V_A = V_0$, $V_B = 1.5V_0$ and $V_C = 2V_0$, respectively. Initially B and C contain 1 mole of diatomic ideal gas and are connected by an open tap so that both parts are set to have a pressure $P_C = P_B = P_0$. A is set initially empty. Consider the following processes

- Reversible compression of the C section to B at constant pressure, exchanging an amount of heat Q_1.
- After closing the tap between B and C and opening the one between A and B, the gas expands.
- Lastly, the gas reaches room temperature T_{room} as is not perfectly isolated (It is in contact with a thermal Bath).

Compute the heat exchanged Q, the change of inner energy and the change of the entropy of the universe.

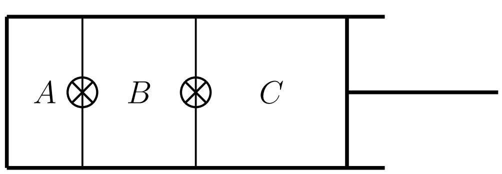
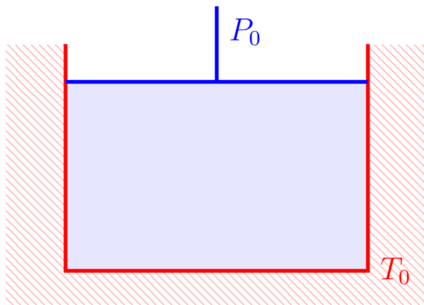


Figure 2: Representation of the cylinder with the three compartments. The $\otimes$ represent the opening taps.

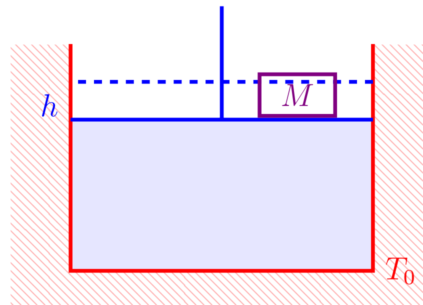
5 Ideal Gases III

A mole of monoatomic ideal gas on a cylinder placed vertically with a frictionless piston. The gas can exchange heat with a reservoir at temperature T_0. The initial pressure of the piston on the gas is P_0. This means that the gas is set to have the pressure imposed by the piston.

After the system has equilibrated, a mass M is set on the piston. As a consequence of the mass, the gas is compressed so that the position of the piston is decreased by h (see the figure). Compute the final pressure P_f, heat transferred Q and the change of the entropy of the universe.



(a) Initial configuration



(b) After adding an extra mass M. The piston's height decreased by h

6 Ideal Gases IV

Consider a diatomic ideal gas is in thermodynamic equilibrium state $A = (P_A, V_A, T_A)$. The gas undergoes a cycle with the following steps

- A reversible isothermal compression to a volume V_B.
- After the state B it reaches a state C absorbing an amount of heat Q and changing the entropy of the environment ΔS.
- From C it reaches again the state A via a reversible adiabatic expansion. Take the change of entropy in the environment to be $\Delta S^{BC}_{\text{env}} = -nR\ln(V_B/V_A)$

Calculate the heat exchanged during the process and the efficiency of the cycle. Is the transformation BC reversible?

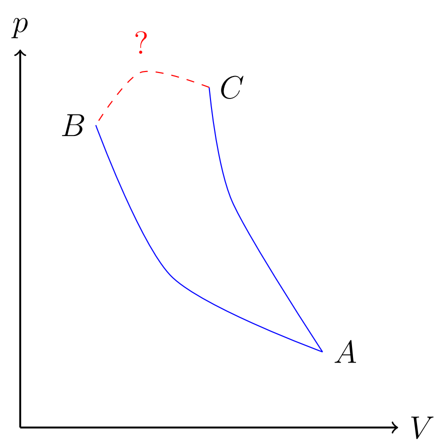


Figure 4: Representation of the cycle.

7 Ideal Gases V

Consider a mole of diatomic gas in thermodynamic equilibrium (P_A, T_A). After a free adiabatic expansion, it gets to a different thermodynamic state B (P_B, T_B). Then, after an irreversible adiabatic compression, it reaches the equilibrium C. During this last transformation BC there was an amount of work $W_{BC} > 0$ done on the system. Finally, the cycle is completed by getting the gas in contact with a reservoir at temperature T_A, hence it goes back to A following an isobaric transformation. Compute V_C and the change of the entropy of the universe $\Delta S_{\text{universe}}$.

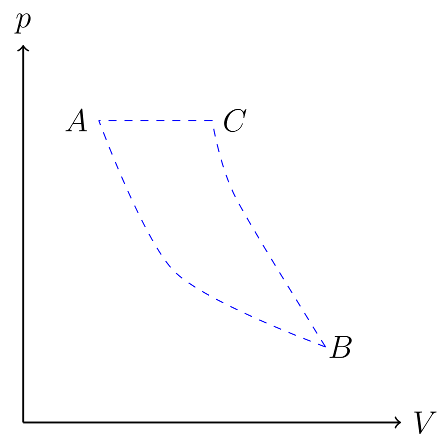


Figure 5: Representation of the cycle.