Statistical Mechanics

Worksheet 5

May 18th, 2023

1 Two Containers And a Spring

Let us consider a system made up of two cylindrical containers A and B, with transversal section S and height h, connected by a wall with a tap and immersed in a reservoir at temperature T (See fig 1). The container A, which has rigid walls, contains n moles of an ideal gas at a pressure p_A. Container B is closed by a sliding piston without friction and of negligible mass, connected on the outside to a spring with elastic constant k and length at rest h. Given the action of the spring, the initial volume of the empty cylinder B is zero. Then, the tap is opened, and the gas flows slowly into container B, in such a way that the temperature remains constant. Using this, determine:

1. The final equilibrium pressure.
2. The work done by the gas.
3. The entropy variations of the gas, of the reservoir and of the universe.

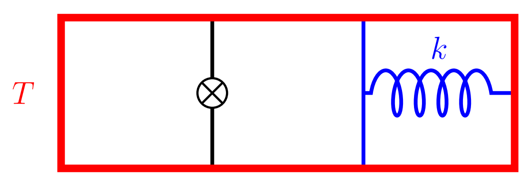

Figure 1: Representation of the two connected containers, the blue line represents the movable piston.

2 Two Gases with Changing Volume

An adiabatic cylinder of length l and with transversal section S is divided into three parts by two mobile adiabatic partitions connected by a spring with elastic constant k and length at rest l_0. Both A and B contain $n = 1$ mole of a diatomic ideal gas at temperature T and pressure p_0. The length of the spring under these conditions is l_s. In the central area where the spring is placed, there is no gas.

1. Calculate the values of T_0 and P_0.

2. After, the gas in A is electrically heated very slowly until the length of the spring becomes l'_s. Calculate the volume and temperature of the gas in A and B and the heat transferred to the gas.

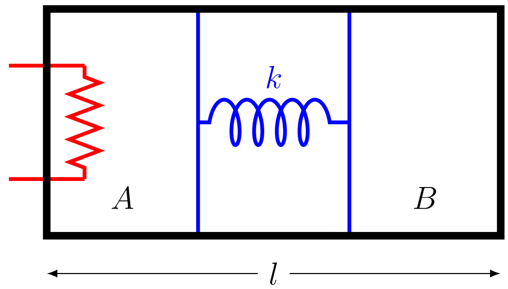

Figure 2: Representation of the three containers, the blue line represents the movable piston and the red the heating mechanism.

3 Cycles with Ideal Gases

One mole of diatomic ideal gas completes the cycle shown in the fig 3 , where the transformations AB and CD are reversible isotherms, the transformation DA is a reversible adiabatic, while BC is an irreversible adiabatic. Using just p_A, T_A, p_B, V_C and T_C as known variables, determine:

1. The change in the internal energy of the gas in the transformation BC and over the entire cycle.
2. the work obtained in the cycle.
3. cycle efficiency.
4. The efficiency of a Carnot cycle, which uses the same sources at temperatures T_A and T_C.
5. The work of the Carnot cycle with the same transformation AB.
6. The change of the entropy of the gas for the BC transformation and for the entire cycle.

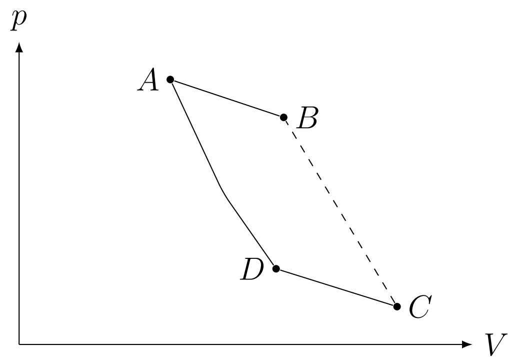

Figure 3: Representation of the cycle described.

4 Perverse initial conditions

If we start a gas of classical spherical particles in a square box all in a vertical line, all moving exactly in the vertical direction, they will bounce back and forth vertically forever. Does that mean a gas of particles in a square box is not *ergodic*? Why or why not?

5 Counting

1. A bus has nine seats facing forward and eight seats facing backward. In how many ways can seven passengers be seated if two refuse to ride facing forward and three refuse to ride facing backward?
2. Find the number of ways in which eight persons can be assigned to two rooms (A and B) if each room must have at least three persons in it.
3. In how many ways can five red balls, four blue balls, and four white balls be placed in a row so that the balls at the ends of the row are the same color?
4. Various six digit numbers can be formed by permuting the digits 666655 . All arrangements are equally likely. Given that a number is even, what is the probability that two fives are together?