Statistical Mechanics

Worksheet 6

May 25th, 2023

1 Maxwell's Demon

The Maxwell's Demon is a thought experiment proposed by physicist James Clerk Maxwell in 1867. In the idea of Maxwell's demon, there is a box divided into two chambers. The chambers are separated by a partition with a small gate that can be opened and closed. The demon possesses knowledge about the momentum and position of all the particles in the system and can manipulate the door, allowing particles with specific conditions to cross from one side to the other.

Szilard's engine utilizes these ideas to extract work from the system. The setup of Szilard's engine involves a system consisting of a single molecule of ideal gas enclosed in a chamber of volume V, which is in contact with a reservoir at temperature T. A movable wall, controlled by the demon, can be removed and inserted without any energy cost. The demon places the movable wall in the middle of the box when the particle is on the left-hand side, as shown in the figure 1.

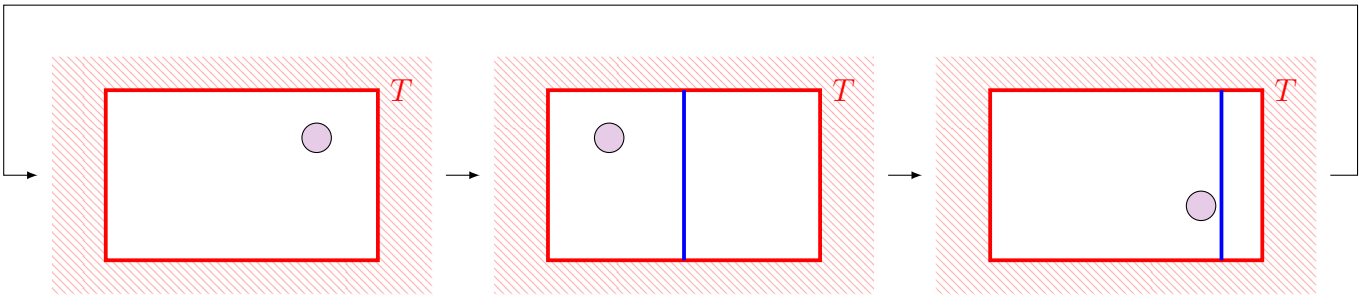

Figure 1: The Szilard's engine set up. (left) No movable wall when the particle is in the right side. (center) The movable wall is set once the particle is in the left-hand side of the box. (right) The particle moves the wall making work. The black lines represent the cycle.

1. During the expansion, as the wall is moved from the middle of the box to the end, the system performs work. Use the ideal gas equation of state to compute this work extracted from the system.

2. Calculate the change in entropy of the system, the bath and the universe.

3. How is it possible that this procedure extracts work from the system every repetition of the cycle?

2 Ergodic system

Can an ideal gas be considered an ergodic system?

3 Interstitial Lattice

A lattice contains N normal lattice sites and N interstitial lattice sites. The lattice sites are distinguishable. N Identical atoms sit on the lattice, M on the interstitial sites, and $N-M$ on the normal sites ($N \gg M \gg 1$). If an atom occupies a normal site, It has energy $E = 0$. If an atom occupies an interstitial site, it has energy $E = \varepsilon$. Compute the internal energy and heat capacity as a function of the temperature for this lattice.

4 3 states Lattice

Consider a lattice with N spin-1 atoms with magnetic moment μ. Each atom can be in one of 3 spin states, $S_z = -1, 0, +1$. Let n_{-1}, n_0 and n_1 denote the respective number of atoms in each of those spin states. Find the total entropy and the configuration which maximizes the total entropy. What is the maximum entropy?
Assume that no magnetic field is present, so all atoms have the same energy. Also assume that atoms on different lattice sites cannot be exchanged, so they are distinguishable.

5 Classical harmonic oscillators

consider N harmonic oscillators with coordinates and momenta $\{q_i, pi\}$, and subject to a Hamiltonian

$$\mathcal{H}(\{q_i, pi\}) = \sum_{i=1}^{N} \left[\frac{p_i^2}{2m} + \frac{m\omega^2 q_i^2}{2} \right] \tag{1}$$

1. Calculate the entropy S, as a function of the total energy E.
   Hint By appropriate change of scale, the surface of constant energy can be deformed into a sphere. You may then ignore the difference between the surface area and volume for $N \gg 1$. A more elegant method is to implement this deformation through a canonical transformation.
   Hint You might find useful the integrals performed by Kardar in Sec.4.4

2. Calculate the energy E, and heat capacity C, as functions of temperature T, and N.