Statistical Mechanics

Worksheet 11

June 29th, 2023

1 Fermi-Dirac Distribution $T \to \infty$

How does the distribution function of fermions look like at infinite temperature?
Comment on your result.

2 Fermi-Dirac Distribution

Show that the entropy for an ideal Fermi-Dirac gas (neglecting spin) can be written in the form

$$S = -k_B \sum_l \{\langle n_l \rangle \ln(\langle n_l \rangle) + (1 - \langle n_l \rangle) \ln(1 - \langle n_l \rangle)\} \tag{1}$$

where $\langle n_l \rangle = \left(e^{\beta(\epsilon_l - \mu) + 1}\right)^{-1}$

3 Identical particle pair

let $Z_1(m)$ denote the partition function for a single quantum particle of mass m in a volume V.

1. Calculate the partition function of two such particles, if they are bosons, and also if they are (spinless) fermions.
2. Use the classical approximation $Z_1(m) = V/\lambda^3$ with $\lambda = h/\sqrt{2\pi m k_B T}$. Calculate the corrections to the energy E, and the heat capacity C, due to bose or fermi statistics.
3. At what temperature does the approximation used above break down?

4 Generalized ideal gas

Consider a gas of non-interacting identical (spinless) quantum particles with an energy spectrum $\epsilon = |\vec{p}/\hbar|^s$, contained in a box of "volume" V in d dimensions.

1. Calculate the grand potential $\mathcal{G}_\eta = -k_B T \ln(\mathcal{Q}_\eta)$, and the density $n = N/V$, at a chemical potential μ. Express your answers in terms of s, d, and $f_m^\eta(z)$, where $z = e^{\beta\mu}$, and

$$f_m^\eta(z) = \frac{1}{\Gamma(m)} \int_0^\infty \frac{dx x^{m-1}}{z^{-1}e^x - \eta} \tag{2}$$

Hint Use integration by parts on the expression for $\ln(\mathcal{Q}_\eta)$.

2. Find the ratio PV/E, and compare it with the classical result obtained previously.

3. For *fermions*, calculate the dependence of E/N, and P, on the density $n = N/V$, at zero temperature. ***Hint*** $f_m(z) \to (\ln(z))^m/m!$ as $z \to \infty$.

4. For *bosons*, find the dimension $d_\ell(s)$, below which there is no bose condensation. Is there condensation for $s = 2$ at $d = 2$?