Worksheet 3. Solution.

1) Probability of transfer of heat.

Let us take the definition of the diferential of entropy as

$$dS = \frac{dQ}{T}$$

$$\Delta S_{universe} = \Delta S_{u} = -\frac{Q}{T_2} + \frac{Q}{T_1} = \left[\frac{-1}{301} + \frac{1}{300}\right] J/K$$

Mind the signs.

$$= \frac{301 - 300}{301 \cdot 300} J/K$$

$$= \frac{1}{903000} J/K$$

$$\approx 1 \; 10^{-5} \; J/K$$

But, the entropy is also proportional, to the logarithm of the number of accesible states Ω

$$S = k_B \ln(\Omega)$$

Hence

$$\Delta S = k_B \ln\left(\frac{\Omega_{final}}{\Omega_{initial}}\right)$$

$$\ln\left(\frac{\Omega_{final}}{\Omega_{initial}}\right) = \frac{\Delta S}{k_B} \rightarrow \frac{\Omega_{final}}{\Omega_{initial}} = \exp\left(\Delta S / k_B\right)$$

$$\frac{\Omega_{final}}{\Omega_{initial}} \simeq \exp\left(10^{-5}\,\cancel{J/K} \Big/ 10^{-23}\,\cancel{J/K} \right) = e^{10^{18}}$$

look that, the reverse process, meaning the cold reservoir giving 1J of Heat to the hot one is very unlikely.

2) The Solar System

a) The motion and organization of planets is much more ordered than the original dust Cloud.

- Why this does not violate the second law of thermodynamics?

The dust configuration seems to have higher entropy, but given the nature of the gravitational interaction (Attractive), the configuration maximazing the entropy has some structure.

The potential energy is converted to kinetic energy that at the end is released in form of photons which carry entropy with it.

b) The nuclear processes of the sun convert protons to heavier elements such as carbon.

Does this further organization lead to a reduction of entropy?

- The formation of heavier elements releases large amounts of energy carried out by photons. The entropy carried away by the photons compensate any kind of ordering given by the packing of nucleons into heavier nuclei.

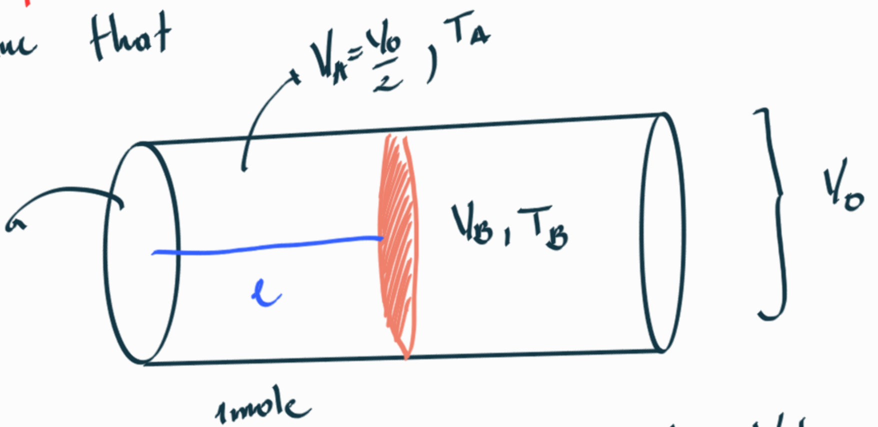
c) The evolution of life and intelligence requires even further levels of organization.

How is this achieved on earth without violating the second law?

There is usage of energy by the organisms that converts more ordered forms of energy to less ordered ones.

3 Ideal Gases I.

We have that



The Volume $V_A = S \times \ell$.

then the equilibrium condition reads

$$P_A S = P_B S + F \quad (1)$$

Tension of the wire.

as we have that for a mole of gas

$$P_A = \frac{R T_A}{V_A}$$

$$P_B = \frac{R T_B}{V_B}.$$

Then (1) becomes

$$\frac{R T_A S}{V_A} - \frac{R T_B S}{V_B} = F$$

$$\frac{2R}{V_0} S \left(T_A - T_B\right) = F.$$

b) the Force to break the wire F^* is calculated by

$$P_A^* S - P_B S = F^*$$

$$\downarrow$$

$$T_A^* = \frac{P_A^* V_A}{R} = \frac{F^* + P_B S}{2SR.} V_0$$

c) The heat;

$$Q = \Delta U = c_v \left(T_A^* - T_A\right).$$

after the wire breaks, then we may have that the cycle is complete;

$$\Delta U_A + \Delta U_B = 0$$

$$= c_v \left(T_A' - T_A^*\right) + c_v \left(T_B' - T_B\right)$$

where T_A' and T_B' are the final temperatures of A and B.

$$\left(T_A' - T_A^*\right) + \left(T_B' - T_B\right) = 0$$

$$T_A' + T_B' = T_A^* + T_B$$

Then using again the equation of state

$$P_B' V_B' = P_B' (V_0 - V_A') = P_B' V_0 - P_B' V_A' = R T_B'$$

$$P_B' V_0 = R T_B' + P_B' V_A'$$

But in equillibrium $P_A' = P_B'$ and

$$P_A' V_A' = R T_A'$$

$$P_B' = P_A' = \frac{R T_B' + R T_A'}{V_0}$$

$$P_B' = P_A' = \frac{R (T_A' + T_B')}{V_0}$$

$$= \frac{R (T_A^* + T_B)}{V_0}$$

Then

$$\frac{V_A'}{S} = \frac{R T_A'}{S P_A} = \frac{T_A' V_0}{S (T_A^* + T_B)} = l'$$

Ideal gases II

The initial temperature of the gas in BC goes as;

$$T_i = \frac{(V_B + V_c)\,P}{R} = \frac{(1{,}5 + 2)\,V_0\,P}{R} = \frac{3{,}5\,V_0 P}{R}$$

after the compression.

$$T_f = \frac{V_B\,P}{R} = \frac{1.5\,V_0\,P}{R}.$$

The change on the internal energy goes as

$$\Delta U_1 = C_v\,(T_f - T_i) = \frac{C_v V_0 P}{R}\,(1{,}5 - 3{,}5)$$

$$= -2\,\frac{C_v V_0 P_0}{R}$$

and the work.

$$W_1 = P\left[V_B - (V_c + V_B)\right] = -P V_c = -2 P V_0.$$

So that the exchange in heat;

$$Q_1 = \Delta U_1 - W_1 = -2\,\frac{C_v V_0 P_0}{R} - 2 P V_0$$

$$= -2 V_0 P_0 \left(\frac{C_v}{R} - 1\right)$$

for 1 mole ideal gas $C_v = \frac{3}{2} R$

$$Q_1 = -2 V_0 P_0 \left(\frac{3}{2} - 1\right) = -V_0 P_0$$

b) After closing the tap. BC and opening AB, the expansion leads to;

$$V_B \longrightarrow (V_A + V_B)$$

$$T = cte.$$

Then it cools down at constant pressure.

(Free gas expansion + irreversible isochoric)

$$Q_2 = c_v (T_{room} - T_f)$$

Then the total heat exchange

$$Q_{tot} = Q_1 + Q_2 = -V_0 P_0 + c_v (T_{room} - T_f).$$

The change of the entropy of the universe is the sum of the following transformations.

- free expansion
- irreversible isochoric cooling

Note: This is just considering the irreversible processes.

$$\Delta S_{gas} = nR \ln\left(\frac{V_A + V_B}{V_A}\right)$$

$$+ n c_v \ln\left(\frac{T_{room}}{T_f}\right)$$

$$\Delta S_{environment} = -\frac{Q_2}{T_{room}}$$

$$\Delta S_{universe} = nR \ln\left(\frac{V_A + V_B}{V_A}\right) + n c_v \ln\left(\frac{T_{room}}{T_f}\right) - \frac{c_v (T_{room} - T_f)}{T_{room}}$$

5 ideal Gases III

To obtain the final pressure we may use

$$P_f = P_0 + \frac{Mg}{S} \qquad (5,1)$$

$S \equiv$ transversal Surface

contribution of the extra mass

let us calculate S.

So, the equation of state

$$P_0 \underbrace{h_0 S}_{V_0} = RT_0 \longrightarrow \left(P_0 + \frac{Mg}{S}\right)(h_0 - h) S = R T_0$$

$$\cancel{P_0 h_0 S} - P_0 h S + Mg h_0 - Mgh = \cancel{RT_0}$$

$$-P_0 h S + Mgh - Mgh = 0$$

$$h_0 = \frac{RT_0}{P_0 S}$$

Then

$$P_0 h S^2 + Mgh\, S - Mg h_0 S = 0$$

$$P_0 h S^2 + Mgh S - Mg \cancel{S}\left[\frac{RT_0}{P_0 \cancel{S}}\right] = 0$$

$$P_0 h S^2 + Mgh S - \frac{MRT_0 g}{P_0} = 0$$

this is a cuadratic equation

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-Mgh \pm \sqrt{(Mgh)^2 + 4(\cancel{P_0}h)\left(\frac{MRT_0 g}{\cancel{P_0}}\right)}}{2P_0 h}$$

The only root that works is the positive

$$S = \frac{-Mgh \pm \sqrt{(Mgh)^2 + 4MRT_0 gh}}{2P_0 h}$$

then it can be replaced on $S,1$

b) The process is an irreversible isothermal compression where

$$V_{initial} = S h_0$$
$$V_{final} = S(h_0 - h)$$

The pressure goes on the piston

$$P = P_0 + \frac{Mg}{S}$$

so on the system

$$W = P\Delta V = -\left(P_0 + \frac{Mg}{S}\right) hS < 0$$

on the system

the isoterm for the ideal gas $dU = 0$.

$$Q = -W$$

the change on the entropy of the universe

$$\Delta S_{reservoir} = \frac{|Q|}{T_0}$$

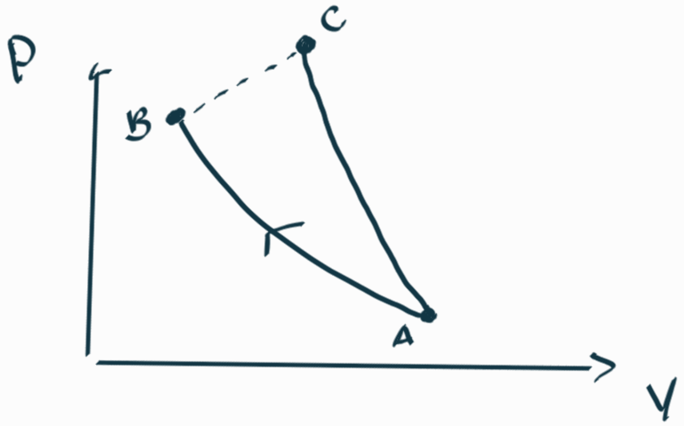
$$= \left(P_0 + \frac{Mg}{S}\right)\frac{hS}{T_0}$$

$$\Delta S_{gas} = R \ln\left(\frac{P_i}{P_f}\right) = R \ln\left(\frac{P_0}{P_0 + \frac{Mg}{S}}\right)$$

$$\Delta S_u = S_{reservoir} + \Delta S_{gas}$$

6 Ideal Gases IV

The process goes



We have from the equaton of state

$$n_A = \frac{P_A V_A}{R T_A}$$

With the information we have, we could compute the amount of gas.

Then, for the isothermal compression

$$Q_{AB} = n_A R T_A \ln\left(\frac{V_B}{V_A}\right)$$

$$Q_{total} = -W \Rightarrow \Delta U = 0$$ as it is a cycle.

$$Q_{total} = Q_{AB} + Q$$

Heat absorbed

$$= n_A R T_A \ln\left(\frac{V_B}{V_A}\right) + Q$$

The efficiency

$$\eta = \frac{W}{Q_{absorbed}} = 1 + \frac{n_A R T_A}{Q} \ln\left(\frac{V_B}{V_A}\right)$$

To know if BC transformation is reversible

$$\left(\Delta S_{environment} + \Delta S_{gas}\right)^{BC}\Big|_{rev} = 0$$

or

$$\Delta S^{BC}_{environment} = -\Delta S^{BC}_{gas.} \quad \text{if reversible.}$$

as the whole cycle

0 → adiabatica reversible.

$$\Delta S_{AB} + \Delta S_{BC} + \Delta S_{CA} = 0$$

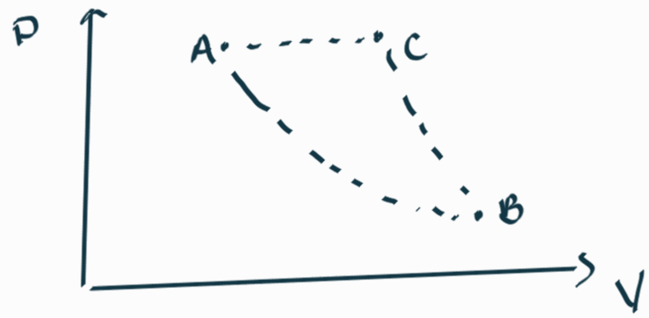
we need to see ΔS_{gas}^{AB}

$$\Delta S^{AB}_{gas} = n_A R \ln\left(\frac{V_B}{V_A}\right) = -\Delta S^{BC}_{gas}$$

Then it is reversible.

7) ideal Gases V

we have the following cycle;



From the free expansion, the temperature keeps constant $T_A = T_B$ and the work done by the compressor;

$$W_{BC} = -\Delta U_{BC} = C_V (T_B - T_C) = C_V (T_A - T_C).$$

$$T_C = T_A - \frac{W_{BC}}{C_V}$$

and from the equation of state

$$V_C = \frac{RT_C}{P_A} = \frac{R}{P_A}\left(T_A - \frac{W_{BC}}{C_V}\right)$$

The Heat absorbed

$$Q = -Q_{gas}^{CA} = C_P (T_C - T_A).$$

Lastly the change on the entropy of the universe

$$\Delta S_U = \frac{Q}{T_A} = \frac{C_P (T_C - T_A)}{T_A}$$

$$= \frac{7}{2} R \left(\frac{T_C}{T_A} - 1\right).$$