Sheet 3 Exercise 4

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The formula in matrix notation:

$$\begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_{30} \end{pmatrix} = \begin{pmatrix} 1 & x_{1,1} & \cdots & x_{1,5} \\ 1 & a_{2,1} & \cdots & x_{2,5} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & a_{30,1} & \cdots & x_{30,5} \end{pmatrix} \begin{pmatrix} \beta_0 \\ \vdots \\ \beta_5 \end{pmatrix} + \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_{30} \end{pmatrix} \tag{1}$$

*: $rank(A'A) - rank(AA') = rank(A)$
+: $X(X'X)^{-1}X' = H, rank(H) = trace(H) = trace(X'X(X'X)^{-1}I) = trace(I(p-1)) = p+1$

Table 1:

	with intercept		without intercept	
	Dimension	Rank	Dimension	Rank
Y	30 x 1	1	30 x 1	1
	(n x 1)	1	(n x 1)	1
X	30 x 6	6	30 x 5	5
	(n x (p+1))	p+1	n x p	p
β	6 x 1	1	5 x 1	1
	((p+1) x 1)	1	p x 1	1
ε	30 x 1	1	30 x 1	1
	(n x 1)	1	(n x 1)	1
$(X'X)$	6 x 6	6	5 x 5	5
	((p+1) x (p+1))	p+1^{*}	(p x p)	p
$X(X'X)^{-1}X'$	30 x 30	5	30 x 30	6
	(n x n)	p+1^{+}	(n x n)	p