Statistical Mechanics from Quantum Thermodynamics

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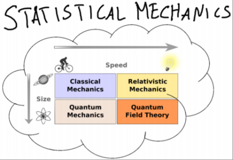
Thessaloniki, July 1, 2020



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Need for quantum foundations of statistical mechanics

- Quantum probability amplitudes are not consistent with the axiom of a priori equal probability(at least not on first sight).
- Absence of unitarity in the evolution of thermodynamic systems.
- Further applications.

Density Matrix

- The definition:

$$\rho = \sum_j p_j \left|\psi_j\right\rangle \left\langle\psi_j\right| \tag{1}$$

- Quantum Randomness vs Classical Randomness:

$$\sum p_j = 1 \tag{2}$$

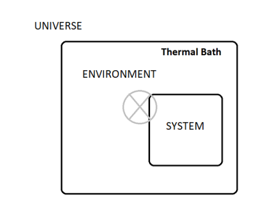
- The canonical state:

$$\Omega_S = (1-p)^k \sum_s \exp\left(-\frac{|s|B}{\mathrm{k}_B T}\right) |s\rangle\langle s| \propto \exp\left(-\frac{H_S}{\mathrm{k}_B T}\right) \tag{3}$$

Assumptions

- Restricted Subspace R

$$\mathcal{H}_R \subseteq \mathcal{H}_S \otimes \mathcal{H}_E \tag{4}$$



- Equiprobable:

$$\mathcal{E}_R = \frac{\mathbb{I}_R}{d_R} \tag{5}$$

- State of the system:

$$\Omega_S = \mathrm{Tr}_E\left(\mathcal{E}_R\right) \tag{6}$$

Typicality

- We compare with a pure state of the Universe.
- Typicality is a naturally statistical equality of an observable in the two cases (i.e. pure or maximally mixed state of the universe):

$$\rho_S = \mathrm{Tr}_E(|\psi\rangle\langle\psi|) \tag{7}$$

- It has been proven that for sufficiently large dimensions of the environment(i.e. possible states):

$$||\rho_S - \Omega_S|| \approx 0 \tag{8}$$

Envariance

- There are arguments that the symmetry of envariance can produce Born's rule and Thermal States.
- ψ_{SE} is called envariant under a unitary map $U_{\mathcal{S}} = u_{S} \otimes \mathbb{I}_{E}$ iff there exists another unitary $U_{E} = \mathbb{I}_{E} \otimes u_{E}$ such that:

$$\begin{aligned} U_S \left|\psi_{SE}\right\rangle &= (u_S \otimes \mathbb{I}_E) \left|\psi_{SE}\right\rangle = \left|\eta_{SE}\right\rangle \\ U_E \left|\eta_{S\mathcal{E}}\right\rangle &= (\mathbb{I}_S \otimes u_E) \left|\eta_{SE}\right\rangle = \left|\psi_{SE}\right\rangle \end{aligned} \tag{9}$$

Envariance

To illustrate the above we state a simple example. Let $\mathcal{S}$ and E be two level systems and assume that $|\psi_{SE}\rangle \propto |\uparrow\rangle_S \otimes |\uparrow\rangle_E + |\downarrow\rangle_S \otimes |\downarrow\rangle_E$.

$$|\uparrow\rangle_{\mathcal{S}}\otimes|\uparrow\rangle_E+|\downarrow\rangle_{\mathcal{S}}\otimes|\downarrow\rangle_E \xrightarrow{U_{\mathcal{S}}} |\downarrow\rangle_{\mathcal{S}}\otimes|\uparrow\rangle_E+|\uparrow\rangle_{\mathcal{S}}\otimes|\downarrow\rangle_E \quad (10)$$

$$|\downarrow\rangle_{\mathcal{S}}\otimes|\uparrow\rangle_E+|\uparrow\rangle_{\mathcal{S}}\otimes|\downarrow\rangle_E \xrightarrow{U_E} |\downarrow\rangle_{\mathcal{S}}\otimes|\downarrow\rangle_E+|\uparrow\rangle_{\mathcal{S}}\otimes|\uparrow\rangle_E \quad (11)$$

Conclusions

- Quantum Thermodynamics promises a lot about the foundations of Statistical Mechanics.
- Information Theoretic Approaches are quite convincing and they obviously have something to do with Statistical Mechanics in general.
- Quantum Entanglement is of a vital importance in the production of Statistical Mechanics Formulas via Hilbert Spaces.

The end

Thank you!!!!