FoF 2022 Workshop: Primordial Black Holes I

Constraints on Extended PBH Mass Functions

Hands On Session

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April 18, 2022

Introduction

In this Hands on Session, you will learn how to define extended mass functions and calculate constraints on their abundance using observational data. To achieve this, we will study one example to then let you implement other kind of mass function to the already available code.

Requirements

To start working, you will need to have a version of Python3 installed with the following modules available:

- numpy
- scipy
- pandas
- matplotlib
- jupyter

If any of these modules is missing, it should suffice to do

```
pip install "module"
```

and the missing module should be installed.

The next thing you will need is the source code for the mass function calculations available at: github.com/jmsureda/PBH-Workshop.

Exercise 1

Now that you have seen how to implement a mass function is done, consider now the Fixed Conformal Time (FCT) scenario presented in Sureda et al. (2021). In this scenario, the energy density at formation time is given by

$$\rho_{fct} = \left(\frac{\rho_{\mathrm{DM},0}}{a_{\mathrm{fct}}^3} + \frac{\rho_{\mathrm{r},0}}{a_{\mathrm{fct}}^4} \right), \tag{1}$$

where a_{fct} is the formation scale factor for the PBH population. Using the Press-Schechter formalism presented by Nelson Padilla, one is able to define the FCT mass function as

$$\boxed{\left(\frac{\mathrm{d}n}{\mathrm{d}M} \right)_{\mathrm{fct}} = \begin{cases} \left(\frac{\mathrm{d}n}{\mathrm{d}M} \right)_{\mathrm{fct}}^{\mathrm{brk}} & \text{for } M < M_{\mathrm{piv}}, \\ \left(\frac{\mathrm{d}n}{\mathrm{d}M} \right)_{\mathrm{fct}}^{\mathrm{std}} & \text{for } M \geq M_{\mathrm{piv}}, \end{cases}} \tag{2}$$

where M_{piv} is given by

$$M_{\mathrm{piv}} \equiv (C_{\mathrm{fct}}/k_{\mathrm{piv}})^3 f_m, \tag{3}$$

and the mass functions for each part of the piecewise function are given by

$$\left(\frac{\mathrm{d}n}{\mathrm{d}M} \right)_{\mathrm{fct}}^{\mathrm{std}} = \frac{\rho_{\mathrm{DM}}(a)}{\sqrt{2\pi}} \frac{n_{\mathrm{s}} + 3}{3M^2} \left(\frac{M}{M_*} \right)^{(n_{\mathrm{s}}+3)/6} \exp\left[-\frac{1}{2} \left(\frac{M}{M_*} \right)^{(n_{\mathrm{s}}+3)/3} \right], \tag{4}$$

$$\left(\frac{\mathrm{d}n}{\mathrm{d}M} \right)_{\mathrm{fct}}^{\mathrm{brk}} = \frac{S_2 \alpha \rho_{\mathrm{DM}}(a)}{\sqrt{2\pi} M^{\alpha+2}} \frac{\left(S_1 f_m^{-\alpha} + S_2 M_*^{-\alpha} \right)^{1/2}}{\left(S_1 f_m^{-\alpha} + S_2 M^{-\alpha} \right)^{3/2}} \times \exp\left[-\frac{S_1 f_m^{-\alpha} + S_2 M_*^{-\alpha}}{2 \left(S_1 f_m^{-\alpha} + S_2 M^{-\alpha} \right)} \right]. \tag{5}$$

These expressions use some auxiliary definitions given by

$$C_{\mathrm{fct}} \equiv a_{\mathrm{fct}} \left(\frac{32\pi^4 \rho_{\mathrm{fct}}}{3} \right)^{1/3} \tag{6}$$

$$\alpha \equiv \frac{n_b + 3}{3} \tag{7}$$

$$S_1 \equiv (n_b - n_s) \left(\frac{C_{\mathrm{fct}}}{k_{\mathrm{piv}}} \right)^{-3\alpha}, \tag{8}$$

$$S_2 \equiv (n_s + 3). \tag{9}$$

For this activity:

1. Create a FCT `Massfunction` class (from Eq. (2)) that inherits all properties from the general `Massfunction` class. This will allow to immediately calculate f_{PBH}) by using the method

```
        FCT.compute_f()

```

   where FCT is your instance of a FCT mass function. Plot your resulting mass function for a given choice of parameters.

2. Use your FCT class to compute $f_{\mathrm{PBH}}(M_*)$ for $-15 \leq \log_{10}(M_*/M_\odot) \leq 10$ for a fixed $n_b = 3.5$ value.

For simplicity, assume that $f_m = 1$ and use a value of $k_{\mathrm{piv}} = 10\,\mathrm{Mpc}^{-1}$ and $a_{\mathrm{fct}} \sim 10^{-26}$.

Exercise 2

At some point one might be interested in exploring the effects of different constraints on the fraction $f_{\rm PBH}$. The code is already prepared to achieve this by passing the desired constraint to the `compute_f()` method as a string. In the Exercise notebook you will find a table with the corresponding names and a category for a number of constraints. With

Low	**Medium**	**High**
EGB	Neutron Star	LSS
INTEGRAL	SUBARU	Radio Sources
GRB	MACHOS	Dynamical
White Dwarfs	EROS	Wide Binaries
	OGLE	X-ray Binaries
	Accretion	GC Disruption
	GW	Galaxy Disruption
		Disk Heating
		CMB-Dip

Table 1: Constraint Names and mass regimes they act on

this, one could, for instance, compute the $f_{\rm PBH}$ for MACHOS, EROS and OGLE by doing

```
FCT.compute_f('MACHOS','EROS','OGLE')
```

which can be done also by placing them in a `list` and passing it to the method

```
Lensing = ['MACHOS','EROS','OGLE']
FCT.compute_f(Lensing)
```

The idea of this exercise is that you explore how constraints acting different mass regimes will affect the final value for $f_{\rm PBH}$.

1. Study what happens in the same M_* regime as in the previous exercise.
2. What happens if you change the n_b value?

You can also make your own categories, therefore you can explore what happens for each constraint individually or for certain groups of them.

Additional Exercises

- Remember that we fixed the values of n_b, f_m and $k_{\rm piv}$. You can explore what happens by changing those parameters.
- The code is built in order to be general for any kind of mass function. You can try to create another mass function and implement it.
- The data used for each constraint is just tabulated data in the form of ($M_{\rm PBH}$, $f(M_{PBH})^{-1}$). You can add new constraints by including them in `data/gM` and adding a identifier in the dictionary in the file `constraint.py`. Try creating a new constraint and implementing it!

References

Sureda J., Magaña J., Araya I. J., Padilla N. D., 2021, Monthly Notices of the Royal Astronomical Society, 507, 4804