### **ALMAGESTO** - Technical Manual

A Free Pascal Astronomical Library

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## **List of Abbreviations**

IAU International Astronomical Union. 1

 ${\sf IERS}$  International Earth Rotation and Reference Systems Service. 1

### 1. Introduction

Almagesto is a free source-code library in pascal that provides common astronomical and astrometric quantities and coordinate systems transformations.

The algorithms used in Almagesto are based on a vector and matrix formulation that is rigorous and consistent with the recommendations by the International Astronomical Union (IAU) and with the conventions of the International Earth Rotation and Reference Systems Service (IERS).

The following sections describe the calculation methods that implement the models used in fundamental astronomy.

mathematics

# 2. Coordinate Systems

## 3. Time Scales

# 4. Coordinate Systems Transformations

# 5. Ephemerides and Star Catalogs

# 6. Map Projections

## **A.** Conversion Constants

## **B. IAU Resolutions**

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