Practice Midterm

Mathematics 099

Complete Problems. All work must be shown to receive credit.

1. Solve: $\left|\frac{2}{5} + 3x\right| = \frac{2}{3}$

   Solution:

   Case 1: $\frac{2}{5} + 3x = \frac{2}{3} \Leftrightarrow x = \frac{4}{45}$

   Case 2: $\frac{2}{5} + 3x = -\frac{2}{3} \Leftrightarrow x = -\frac{16}{45}$.

   So solutions are: $x = -\frac{16}{45}, \frac{4}{45}$

2. Write set in interval notation: $\{x|x \geq -2\}$

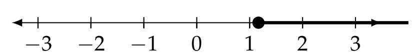
   Solution:

   Any x-value that is equal or greater than -2.

   So in interval notation: [-2,∞)

3. Solve and graph.

   Write the answers using interval notation: $-\frac{3}{4}\ x \leq -\frac{7}{8}$

   Solution:

   $$-\frac{3}{4}\ x \leq -\frac{7}{8} \quad \Leftrightarrow \quad (-\frac{4}{3})(-\frac{3}{4}\ x) \leq (-\frac{4}{3})(-\frac{7}{8})$$
   $$\Leftrightarrow \quad x \geq \frac{7}{6}$$

4. Arrange the polynomial in descending order:

   $-9 - 8p^2 + 6p^9 + 3p^3 - 2p$

   Solution:

   Rearrange using power of exponents from highest to lowest.

   Thus, $-9 - 8p^2 + 6p^9 + 3p^3 - 2p$ = $6p^9 + 3p^3 - 8p^2 - 2p - 9$

5. Evaluate $5x^3 + 6x^2 + 25$, when $x = -2$

   Solution:

   $5(-2)^3 + 6(-2)^2 + 25$ = $-40 + 24 + 25$ = 9

6. Substract: $(-8x^5 + 8x^7 - 6 + 7x^6) - (1 - 2x^6 + 2x^7 + 2x^5)$

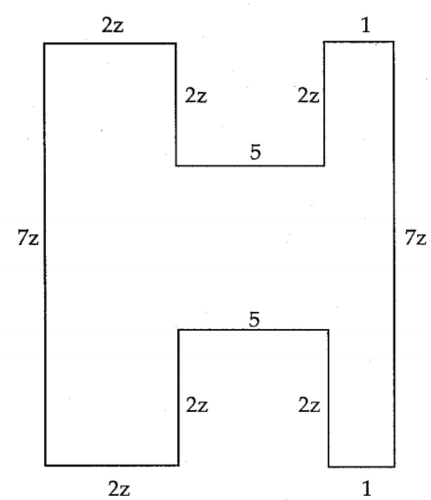
   Solution:

   $-8x^5 + 8x^7 - 6 + 7x^6 - 1 + 2x^6 - 2x^7 - 2x^5$

   = $8x^7 - 2x^7 + 7x^6 + 2x^6 - 8x^5 - 2x^5 - 6 - 1$

   = $6x^7 + 9x^6 - 10x^5 - 7$

7. Find a polynomial for the perimeter of the figure.



Solution:

Perimeter of a figure is the sum of the lengths of its sides.

$$\begin{aligned} \text{Perimeter} &= 7z + 2z + 2z + 5 + 2z + 1 + 7z + 1 + 2z + 5 + 2z + 2z \\ &= 26z + 12 \end{aligned}$$

8. Multiply: $12x^6(-4x^7 + 2x^4 - 12)$

Solution:

$12x^6(-4x^7 + 2x^4 - 12)$

$= -48x^{13} + 24x^{10} - 144x^6$

9. Multiply: $(x + \frac{1}{5})(3x^3 + 2x^2 - 5x - 2)$

Solution:

$(x + \frac{1}{5})(3x^3 + 2x^2 - 5x - 2)$

$= x\,(3x^3 + 2x^2 - 5x - 2) + \frac{1}{5}\,(3x^3 + 2x^2 - 5x - 2)$

$= 3x^4 + 2x^3 - 5x^2 - 2x + \frac{3}{5}x^3 + \frac{2}{5}x^2 - x - \frac{2}{5}$

$= 3x^4 + \frac{13}{5}x^3 - \frac{23}{5}x^2 - 3x - \frac{2}{5}$

10. Multiply: $(9m + 8)^2$

Solution:

$(9m + 8)^2$

$= (9m + 8)(9m + 8)$

$= 9m(9m + 8) + 8(9m + 8)$

$= 81m^2 + 144m + 64$

The FOIL method is also applicable here as well. But remember, FOIL is simply the result of the distributive property.

11. Divide: $\dfrac{6x^2y^4 + 6x^4y^8 - 6x^5y^5}{2x^2y^4}$

<u>Solution:</u>

$\dfrac{6x^2y^4 + 6x^4y^8 - 6x^5y^5}{2x^2y^4}$

$= \dfrac{6x^2y^4}{2x^2y^4} + \dfrac{6x^4y^8}{2x^2y^4} - \dfrac{6x^5y^5}{2x^2y^4}$

$= -3x^3y + 3x^2y^4 + 3$

12. Divide: $\dfrac{x^4 + 5x^2 + 6}{x^2 + 1}$

<u>Solution:</u>

$$\begin{array}{r r}
 & x^2 + 4 \\ \cline{2-2}
x^2 + 1 \big) & x^4 + 5x^2 + 6 \\
 & -x^4 \quad -x^2 \quad\; \\ \cline{2-2}
 & 4x^2 + 6 \\
 & -4x^2 - 4 \\ \cline{2-2}
 & 2
\end{array}$$

Therefore, $\dfrac{x^4 + 5x^2 + 6}{x^2 + 1} = x^2 + 4 + \dfrac{2}{x^2 + 1}$

Caution: Signs may look different when compared to textbook, as the implied negative sign is distributed here. But the idea is same.

13. Factor: $40x^7y^9 - 80x^5y^6 + 56x^3y^4$

<u>Solution:</u>

Determine GCF amongst terms (here: $8x^3y^4$), then factor it out.

Thus, $40x^7y^9 - 80x^5y^6 + 56x^3y^4 = 8x^3y^4(5x^4y^5 - 10x^2y^2 + 7)$

14. Factor by grouping: $2x^3 - 6x^2 - 6x + 18$

<u>Solution:</u>

$2x^3 - 6x^2 - 6x + 18$

$= (2x^3 - 6x^2) + (-6x + 18)$

$= 2x^2(x - 3) - 6(x - 3)$

$= (2x^2 - 6)(x - 3)$

15. Factor completely: $147x^2 + 42x + 3$

<u>Solution:</u>

$147x^2 + 42x + 3$

$= 3(49x^2 + 14x + 1)$

$= 3(7x + 1)(7x + 1) = 3(7x + 1)^2$

16. Factor completely: $875x^3 + 189$
    Solution:
    $875x^3 + 189$
    $= 7(125x^3 + 27) = 7((5x)^3 + 3^3)$
    $= 7(5x+3)((5x)^2 - (5x)(3) + 3^2)$
    $= 7(5x+3)(25x^2 - 15x + 9)$

17. Factor completely: $x^3 + 5x^2 - 9x - 45$
    Solution:
    $x^3 + 5x^2 - 9x - 45$
    $= (x^3 + 5x^2) - (9x + 45)$
    $= x^2(x+5) - 9(x+5)$
    $= (x^2 - 9)(x+5)$
    $= (x+3)(x-3)(x+5)$

18. Factor completely: $80x^2 + 40x + 5$
    Solution:
    $80x^2 + 40x + 5$
    $= 5(16x^2 + 8x + 1)$
    $= 5(4x+1)^2$

19. Factor completely: $54x^2 - 144x + 96$
    Solution:
    $54x^2 - 144x + 96$
    $= 6(9x^2 - 24x + 16)$
    $= 6(3x-4)^2$

20. Factor completely: $2x^3 + 6x^2y - 20xy^2$
    Solution:
    $2x^3 + 6x^2y - 20xy^2$
    $= 2x(x^2 + 3xy - 10y^2)$
    $= 2x(x+5y)(x-2y)$

21. Solve by factoring and using the principle of zero products:
    $6k^2 - 41k - 7 = 0$
    Solution:
    $6k^2 - 41k - 7 = 0 = (6k+1)(k-7) = 0$

    For $6k + 1 = 0$: $\quad 6k + 1 = 0 \Leftrightarrow 6k = -1 \Leftrightarrow k = -\frac{1}{6}$
    For $k - 7 = 0$: $\quad k - 7 = 0 \Leftrightarrow k = 7$

    Therefore, $k = -\frac{1}{6}, 7$

FACTORING SUMS OR DIFFERENCES OF CUBES

$A^3 + B^3 = (A+B)(A^2 - AB + B^2),$
$A^3 - B^3 = (A-B)(A^2 + AB + B^2)$

Identities above useful for Problem 16. These represent the sums and differences of cubes respectively

FACTORING A DIFFERENCE OF SQUARES

$A^2 - B^2 = (A+B)(A-B)$

Difference of Squares Identity is useful for Problem 17 to factor problem completely.

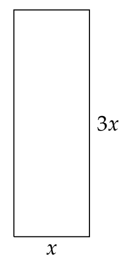
22. Solve the problem:
   A number is 56 less than its square. Find all such numbers.
   Solution:
   Let x be the "number".
   Need to solve: $x = x^2 - 56$

   $x = x^2 - 56 \Leftrightarrow x^2 - x - 56 = 0$
   $(x-8)(x+7) = 0$

   For $x - 8 = 0$: $x - 8 = 0 \Leftrightarrow x = 8$
   For $x + 7 = 0$: $x + 7 = 0 \Leftrightarrow x = -7$

   Therefore, $x = -7, 8$

23. Solve the problem:
   A rectangular garden is three times as long as it is wide. If the area of the garden is 1875 ft^2, find the length and width of the garden.
   Solution:
   Using the information given to us, the rectangle on the right was constructed to represent width and length of garden.



The constructed rectangle represents given information in Problem 23

   Thus, length = $3x$ and width = x.

   We also know that formula for Area = $(3x)(x)$ = $3x^2$

   So if $3x^2 = 1875$:
   $3x^2 = 1875 \Leftrightarrow x^2 = 625 \Leftrightarrow x = -25, 25$
   Length can't be negative so $x = 25$

   If $x = 25$:
   width = 25 ft
   length = 3(25) = 75 ft

24. Two cars leave an intersection. One car travels north; the other east. When the car traveling north had gone 9 miles, the distance between the cars was 3 miles more than the distance traveled by the car heading east. How far had the eastbound car traveled?

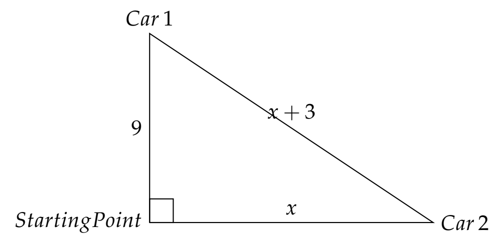
Solution:

Using the Pythagorean Theorem, we know the following:

$(x+3)^2 = x^2 + 9^2 \;\Leftrightarrow\; x^2 + 6x + 9 = x^2 + 81 \;\Leftrightarrow\; 6x = 72$

Thus, $x = 12$

Therefore, the eastbound car has travelled 12 miles.



25. A ladder is resting against a wall. The top of the ladder touches the wall at a height of 15 feet. Find the length of the ladder if the length is 5 feet more than its distance from the wall.

Solution:

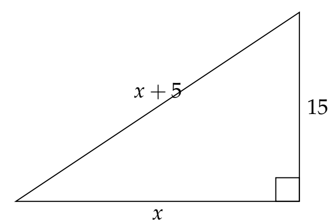
Using the Pythagorean Theorem, we know the following:

$(x+5)^2 = x^2 + 15^2 \;\Leftrightarrow\; x^2 + 10x + 25 = x^2 + 225 \Leftrightarrow 10x = 200$

Thus, $x = 20$

As demonstrated above, the length of ladder is $(x+5)$ ft.

Since $x = 20$, ladder length is 25 ft.



26. In a sports league of n teams in which each team plays every other team twice, the total number N of games to be played is given by $N = n^2 - n$. What is the number of games to be played in a football league having 13 teams?

Solution:

$N = 13^2 - 13 \Leftrightarrow N = 169 - 13 \Leftrightarrow N = 156$