DETECTION OF VISUAL EVOKED POTENTIALS USING RAMANUJAN PERIODICITY TRANSFORM FOR REAL TIME BRAIN COMPUTER INTERFACES

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ABSTRACT

Repetitive visual stimuli induce periodic Visual Evoked Potentials (VEPs) in the brain that can be potentially identified in an EEG trace. The ability to distinguish frequencies and patterns due to different stimuli is the basis for brain computer interfaces (BCIs) used for communication and control of neurologically disabled patients. Since such responses are recorded in presence of high levels of noise from background brain processes, the detection task is rather challenging. In this work, we propose a detection approach for VEPs based on Ramanujan Periodicity Transform matrices (RPT), which have shown promise in detecting periodicities in data. Our results show that the RPT-based approach can outperform conventional spectral techniques and the state-of-the-art correlation analysis, and is more compatible with real-time BCIs which have to work with short duration EEG epochs. The proposed approach is fairly robust to unknown natural latencies in brain response.

***Index Terms*—** Ramanujan Subspace, Ramanujan Periodicity Transform, Nested Periodic Matrices, Steady-state visual evoked potentials, Brain computer interface

1. INTRODUCTION

Steady State Visual Evoked Potentials (SSVEPs) are the brain responses to external visual stimuli (e.g. rapidly flashing fields), which can be recorded over the occipital scalp regions. Due to the electrical activity of the neurons, especially in the neighborhood of the occipital region, high levels of noise from the background brain processes are also recorded and regarded as noise in detection of SSVEPs.

Differentiating patterns in the brain's response to different stimuli is the basis for brain computer interfaces (BCIs) used for communication and control of locked-in and neurologically disabled patients. The common practice to extract event related potentials is to average several EEG epochs recorded over long durations. However, to realize their full potential, real time BCIs have to work with much shorter epochs. As such, detection based on shorter duration trials has received attention in recent years [1].

Various SSVEP detection methods were developed to detect the underlying frequencies in the brain response. Power Spectral Density Analysis (PSDA) is a popular spectral-based technique, which estimates the peak magnitude of the PSD at each stimulation frequency for classification [2]. However, to have a decent frequency resolution longer time windows are required, which may not be compatible with real time BCIs. Liavas et al. proposed a periodogram-based method to detect the SSVEPs, in which the pre-stimulus data is used to whiten the post-stimulus data [3]. Lin et al. proposed a method based on Canonical Correlation Analysis (CCA) that maximizes the correlation between the set of observations and a set of sinusoids that model the stimuli at different frequencies. The maximum correlation obtained from CCA is used to identify the underlying frequency [1].

There is evidence that the brain produces a response (resonance) at a frequency that matches that of the stimulus [4], and in many works the SSVEPs are modeled as sinusoids with the corresponding frequencies [3]. Hence, SSVEPS are expected to feature some periodicities. In this work we propose a new method to detect the underlying periodicity based on a family of matrices called Ramanujan Periodicity Transform (RPT) [5, 6], which to the best of our knowledge is used in this context for the first time. Tenneti and Vaidyanathan introduced the RPT matrices and established their potential in estimating periods directly from the data, and in overcoming the limitations of spectral-based techniques [5]. They evaluated the RPT on real world problems such as Protein Repeats and Electrocardiography. Furthermore, using numerical examples they have shown that RPT is more robust to noise and time shifts. We find this feature particularly useful for SSVEP detection, since brain responses can be modeled as sinusoids with a latent phase variable capturing inter-subject variability.

We introduce the RPT matrices in Section 2. In Section 3, we describe the detection problem and the proposed RPT based approach. In Section 4 we present results using real SSVEP data. A brief discussion and conclusion are provided in Section 5.

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2. BACKGROUND

The authors of [5] introduced a novel method inspired by Ramanujan Periodicity Transform for detecting periodicties in data that, unlike traditional spectral methods, directly targets period estimation [6]. They introduced a family of matrices, dubbed Nested Periodic Matrices (NPM), with special properties that could be leveraged to detect such periodicities. Ramanujan sums are defined as [7]

$$c_q(n) = \sum_{\substack{k=1 \\ \gcd(k,q)=1}}^{q} e^{(j2\pi kn)/q}, \tag{1}$$

where $\gcd(k, q)$ is the greatest common divisor of k and q. A periodic signal with period P repeats every P samples, i.e. $x(n + P) = x(n)$ for all $n \in \mathbb{N}$. We briefly review the construction of an NPM in [5]. In general, the period P has k divisors, and for each divisor $d_i, i = 1, \dots, k$, we construct a matrix $\mathbf{C}_{d_i}$. Each $\mathbf{C}_{d_i}$ is a $P \times \phi(d_i)$ matrix, where $\phi(n)$ is the Euler totient function defined as the number of integers from 1 to n that are coprime to n. Each column of the matrix is a sequence of length P with period d_i, where $d_i \leq P$. Therefore, we have k sub-matrices one for each divisor of P, which are used to construct an NPM matrix as

$$\mathbf{A} = \begin{bmatrix} \mathbf{C}_{d_1} & \mathbf{C}_{d_2} & \dots & \mathbf{C}_{d_k} \end{bmatrix}. \tag{2}$$

A family of matrices with the characteristics of an NPM matrix were introduced in [5], such as natural basis matrices, DFT matrices and the Ramanujan Periodicity Transform (RPT) matrices. This paper focuses on the use of RPT matrices as they were shown to exhibit more robustness to noise and time delays. For each divisor q of P, we construct a sub-matrix of the RPT matrix. In particular, based on the Ramanujan sum equation in (1), for each divisor q we obtain the sequence c_q with length P and period q, and define the column

$$\mathbf{c}_q = \begin{bmatrix} c_q(0) & c_q(1) & \dots & c_q(P-2) & c_q(P-1) \end{bmatrix}^T. \tag{3}$$

We generate the remaining $\phi(q) - 1$ columns as circularly downshifted versions of $\mathbf{c}_q$. Then, $\mathbf{c}_q^{(1)}$ is the circulary downshifted version of $\mathbf{c}_q$, i.e.,

$$\mathbf{c}_q^{(1)} = \begin{bmatrix} c_q(P-1) & c_q(0) & c_q(1) & \dots & c_q(P-2) \end{bmatrix}^T \tag{4}$$

Hence, the sub-matrix $\mathbf{C}_q$ can be constructed by concatenating these columns as

$$\mathbf{C}_q = \begin{bmatrix} \mathbf{c}_q & \mathbf{c}_q^{(1)} & \dots & \mathbf{c}_q^{(\phi(q)-1)} \end{bmatrix}. \tag{5}$$

As an example, for $c_5(n) = \{4, -1, -1, -1, -1\}^T$, the submatrix $\mathbf{C}_5$ is given by

$$\mathbf{C}_q = \begin{bmatrix} 4 & -1 & -1 & -1 \\ -1 & 4 & -1 & -1 \\ -1 & -1 & 4 & -1 \\ -1 & -1 & -1 & 4 \\ -1 & -1 & -1 & -1 \end{bmatrix}. \tag{6}$$

Now we construct the RPT matrix $\mathbf{A}$ as in (2) using all the k sub-matrices corresponding to the divisors of P. For $P = 5$, $d_1 = 1, d_2 = 5$, thus the RPT matrix is

$$\mathbf{A} = \begin{bmatrix} 1 & 4 & -1 & -1 & -1 \\ 1 & -1 & 4 & -1 & -1 \\ 1 & -1 & -1 & 4 & -1 \\ 1 & -1 & -1 & -1 & 4 \\ 1 & -1 & -1 & -1 & -1 \end{bmatrix}. \tag{7}$$

The first submatrix has only 1 column since $\phi(1) = 1$, and the second submatrix consists of 4 columns since $\phi(5) = 4$. For any divisor q of P, there are precisely q columns in the NPM matrix with period q or a divisor of q as established in [5, Lemma 1]. Furthermore, any $P \times 1$ vector $\mathbf{y}$ with period q can be spanned by these q columns. It also follows that any $P \times 1$ vector $\mathbf{y}$ with period q that is a divisor of P can be written as $\mathbf{y} = \mathbf{A}\mathbf{c}$, where $\mathbf{A}$ is an NPM matrix, and $\mathbf{c}$ is only supported on the set of columns of $\mathbf{A}$ with periods equal to a divisor of q, i.e. only the entries of $\mathbf{c}$ corresponding to such columns are non zero. For complete statements of the lemmas and their proofs, we refer the reader to [5], [6], [8].

3. PROBLEM DESCRIPTION AND APPROACH

Repetitive visual stimuli, e.g. flashing illuminated fields at certain frequencies, can induce evoked potentials in the brain, that can be identified in EEG traces. Resonance spikes are often obvious in the EEG power spectrum with the same frequency of the stimulus [3], therefore SSVEP signals are expected to exhibit periodicities. A primary focus of BCI research lies in developing advanced signal processing techniques to efficiently detect the selection of stimulus (among several rapidly-flashing visual stimuli) in a subject's EEG to increase the utility of SSVEP-based BCI used for control and communication for neurologically disabled patients.

Brain response classification: In this work, we propose a new approach to identify the underlying frequencies of SSVEPs using the RPT matrices. Due to inter-subject variability in the delay of the brain response to an external stimulus, we model the brain response using sinusoids with known frequencies (corresponding to the different stimuli) but unknown phase to capture the unknown latency. For simplicity, we consider a binary hypothesis testing problem based on two possible stimuli

$$\begin{aligned} H_0 &: y(t) = \cos(2\pi f_0 t + \theta) + n(t) \\ H_1 &: y(t) = \cos(2\pi f_1 t + \theta) + n(t) \end{aligned} \tag{8}$$

where $y(t)$ denotes samples from the EEG trace, n additive noise modeling the background neuronal noise, and θ the unknown phase modeled as a uniform random variable with distribution

$$f_\theta(\theta) = \begin{cases} \frac{1}{2\pi} & 0 \leq \theta \leq 2\pi \\ 0 & \text{otherwise.} \end{cases} \tag{9}$$

RPT based classifier: We outlined the fundamentals of generating a Ramanujan subspace in Section 2. Since an observation vector can be expressed as $\mathbf{y} = \mathbf{A}\mathbf{c}$, where $\mathbf{A}$ is a $P \times P$ RPT matrix, the vector $\mathbf{c}$ provides an alternate representation of the vector $\mathbf{y}$. For each of the divisors d_i of P, one can obtain the energy $E(d_i)$ of the coefficients that correspond to the columns of $\mathbf{A}$ with period d_i as

$$E(d_i) = \sum_{k=K+1}^{K+\phi(d_i)} |c(k)|^2, \quad K = \sum_{\substack{p|P \\ p<d_i}} \phi(p) \tag{10}$$

where $p|P$ means that p is a divisor of P.

For example, consider a cosine wave with frequency $f_0 = 8$ Hz over the time interval $[0, 1]$ sampled at sampling frequency $f_s = 256Hz$, i.e.,

$$y(t) = \cos(2\pi f_0 t) + n(t). \tag{11}$$

Hence, the period $T_0 \triangleq f_s/f_0$ of the sequence of samples from the cosine wave will be $\frac{256}{8} = 32$ samples. To obtain $\mathbf{c}$ we form the vector $\mathbf{y}$ from the noisy data samples and construct the $P \times P$ RPT matrix $\mathbf{A}$. Since the length of the sequence is 256, we use $P = 256$. Note that the period $T = 32$ is a divisor of P. The Strength vs. Period plot representing the energy of the coefficients of the columns with periods that are divisors of P in (10), is depicted in Figure 1 (right). One can observe that the energy in period 32 has the largest value.

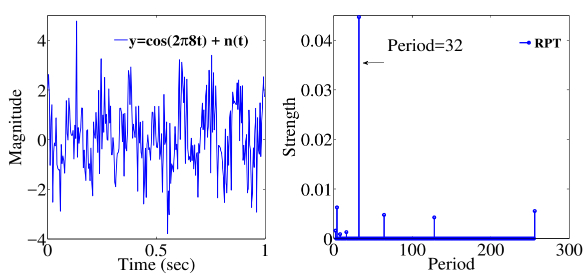

Fig. 1. Strength vs. Period for a cosine wave with period $T = 32$.

To build a classifier for the hypothesis testing problem in (8), we use the prior knowledge about the frequencies of the SSVEPs to construct a matrix $\mathbf{A}$, such that all the periods (i.e. T_0 and T_1) are divisors of P. To this end, we generate the $P \times P$ RPT matrix, where $P = \text{lcm}(T_0, T_1)$, the least common multiplier of T_0 and T_1. For instance, if $f_0 = 8Hz$ and $f_1 = 14Hz$ with sampling frequency $f_s = 256$, the corresponding periods will be $T_0 = 32$ samples and $T_1 = 18$ samples. Hence we use $P = \text{lcm}(32, 18) = 288$ samples. This sets a lower bound $\text{lcm}(T_0, T_1)/f_s$ on the length T (in seconds) of the observation vectors $\mathbf{y}$. In this example, we need to use EEG epochs with duration $T \geq 1.1250$ sec.

We classify the EEG trials by comparing the energies corresponding to each of the periods T_0 and T_1,

$$\frac{E(T_1)}{E(T_0)} \underset{T_0}{\overset{T_1}{\gtrless}} 1 \tag{12}$$

and choosing the period with the larger value. To assess the performance of this approach, we evaluate the Accuracy, Probability of Detection P_D and Probability of False Alarm P_F for different time windows as

$$\text{Accuracy} = \frac{TP + TN}{TP + TN + FN + FP} \tag{13}$$

$$P_D = \frac{TP}{TP + FN}, \quad P_F = \frac{FP}{TP + TP} \tag{14}$$

where TP denotes the True Positives, TN the True Negatives, FN the False Negatives and FP the False Positives.

4. RESULTS

Data: We evaluate the performance of the proposed approach using a publicly available SSVEP dataset [9]. The dataset contains the EEG recordings of four subjects and for each subject there are 5 trials for every stimulus frequency. The frequencies are 8, 14 and 28 Hz, so there are 20 trials for each frequency in total. The signals are sampled at 256 Hz. Each trial begins with a 5-second pre-stimulation period, followed by 15-second SSVEP in response to the stimulus, and ends with a 5 second post-stimulus. Since the SSVEPs are known to be detected over the visual cortex, we created a virtual visual electrode by averaging over the electrodes that are close to the visual cortex. The electrodes used in this work are A14, A15, A16, A21, A22, A23, A25, A27, A28, and A29. To remove the DC baseline of the EEG signals, a time average is computed and removed from all the trials. We only used two sets of data that are SSVEPs to 8 Hz and 14 Hz stimuli.

Although we have 15 seconds of post-stimulus SSVEPs, for real time BCI, we need shorter epochs to test the performance. To this end, we used time windows of duration $T = n \cdot \text{lcm}(T_0, T_1)/f_s$, for differet values of $n \in \mathbb{N}$. Therefore, the RPT matrix A would be of size $Tf_s \times Tf_s$. For example, for $n = 2$ we have $T = 2.250$ sec leading to a 576×576 matrix, and both $T_0 = 32$ and $T_1 = 18$ are divisors of 576. Figure 2 illustrates the Strength vs. Period plot for two trials with $T = 2.50$ sec, one from the 8 Hz and the other from the 14 Hz datasets.

Table 1 presents the performance of the RPT based classifier for different time windows based on the criteria in (13) and (14). The RPT based classifier is generally successful at detecting the true underlying frequency even over short time windows. Hence, this approach holds potential for real-time BCI, which must work with short and single-trial epochs.

Harmonics are often present in an EEG trace (the brain does not really behave as a linear system). As such, in the

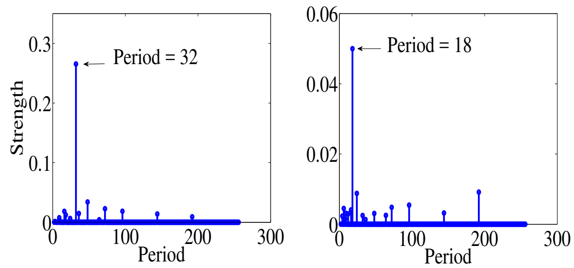

Fig. 2. Strength vs. Period plot for $T_0 = 32$ (left) and $T_1 = 18$ (right). This illustrates that the strength on period 32 and its divisors is greater than the strength on period 18 when the true underlying frequency is 8 Hz and vice versa for 14 Hz.

Table 1. Performance of RPT based classification using different time windows.

Time Windows(sec)	1.1250	2.25	3.3750	4.5
Accuracy	0.7625	0.8375	0.875	0.875
P_D $(f = 8Hz)$	0.6625	0.70	0.75	0.75
P_D $(f = 14Hz)$	0.8625	0.975	1	1
P_F $(f = 8Hz)$	0.1375	0.025	0	0
P_F $(f = 14Hz)$	0.3375	0.3	0.25	0.25

next experiment we added the strength of the second largest divisors ($T_0/2$ and $T_1/2$) to the energies of the main period components. Table 2 shows a general improvement in the overall performance. We note that these results show the average performance of all four subjects. However, for two of the subjects, the RPT reached an accuracy above 90 percent even with a time window of 1.1250 sec.

In our next experiment, we compare the performance of the proposed approach to conventional spectral techniques and CCA. PSDA is one of the most popular and widely used methods based on power spectral density analysis (PSDA). To this end, we used 256-point FFT, and to determine the peak frequency we used the method in [1]. We also consider two alternative decision rules. One rule is the Maximum Likelihood (ML) detection rule for the binary hypothesis testing problem (8) of sinusoids with unknown phase in additive noise [10]. The other decision rule, designated as ML-no phase, is an ML rule for a model that ignores the unknown phases, i.e. assumes $\theta = 0$ in (8). Figure 3 illustrates the results obtained from RPT, PSDA, CCA and the ML rules using four different time windows. We note that the length of the windows in the RPT and PSDA methods are not exactly equal, but the difference is negligible, as in RPT we use time windows for which the target periods are divisors. As shown, RPT generally outperforms the other approaches.

5. DISCUSSION AND CONCLUSION

In this work, we proposed a new method to classify the underlying frequency of SSVEPs based on RPT matrices. The

Table 2. Performance of RPT when the energies of the divisors of T_0 and T_1 are also used. Divisors of the period values can represent harmonics in this context.

Time Windows(sec)	1.1250	2.25	3.3750	4.5
Accuracy	0.7937	0.90	0.90	0.925
P_D $(f = 8Hz)$	0.75	0.85	0.80	0.90
P_D $(f = 14Hz)$	0.8375	0.95	1	0.95
P_F $(f = 8Hz)$	0.1625	0.05	0	0.05
P_F $(f = 14Hz)$	0.25	0.15	0.2	0.1

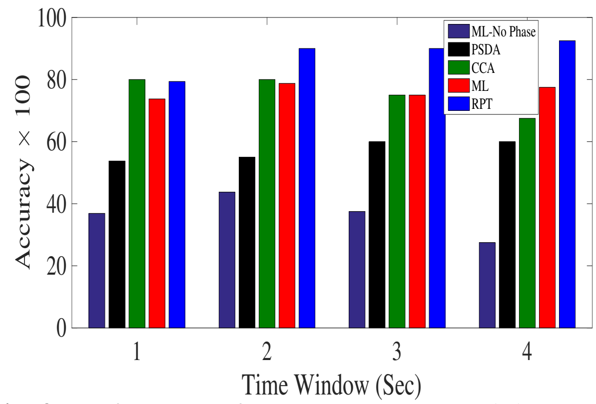

Fig. 3. Performance of RPT, PSDA, CCA and the two ML rules for real time applications.

work in [5] has shown that RPTs and other NPMs can identify periodicities directly from the sequences rather than from the PSD as for conventional spectral methods. Our results show that RPT-based methods can detect event related potentials in real data in presence of high levels of noise from background brain processes. The performance can further improve by accounting for energies in harmonics corresponding to energies of the divisors of the period values.

While statistical decision rules (such as ML) are highly sensitive to model mismatch – recalling the severe performance degradation when we dropped the unknown phase – the RPT based approach is fairly robust to phase shifts.

Conventional spectral methods such as PSDA are generally successful at detecting the underlying frequencies, albeit they rely on averaging over several epochs of data recorded over long experimental durations, which limits their applicability in real-time BCIs. In this context, we investigated the performance of detection methods with shorter duration trials. The comparison results indicate that leveraging RPT matrices is promising, and holds potential to improve over conventional spectral techniques, for real time applications.

In future work, we plan to further push the boundaries of real time SSVEP detection by studying means to dispense with the lower bound $T \geq \text{lcm}(T_0, T_1)/f_s$ on the length of the EEG trials that stems from our choice of P in the design of the matrix $\mathbf{A}$.

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