Sessió Dubtes BAT_MAT2

Lliurament 2

3/11/2021 16:00 h

$$\begin{cases} x+2y-\lambda z=1 \\ -x+y+z=\lambda \\ 3x-y+3z=0 \end{cases}$$

rang A = rang A^* = n

S.C.D.

«Calcular Rangs»

$$A = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 1 & 2 & 3 & 4 \end{pmatrix}$$

rang $A = 1$

1ª) rang $B = 0$

$\Downarrow$

$B = \emptyset$

«Discutir Rang»

$$B = \begin{pmatrix} 1 & 4 & 0 \\ 2 & 5 & -1 \\ -1 & -1 & m \end{pmatrix}$$

menor $\begin{vmatrix} 1 & 4 \\ 2 & 5 \end{vmatrix} = 5 - 8 = -3 \neq 0$

$$\begin{pmatrix} + & - & + \\ - & + & - \\ + & - & + \end{pmatrix}$$

$$\begin{vmatrix} 1 & 4 & 0 \\ 2 & 5 & -1 \\ -1 & -1 & m \end{vmatrix} = 1 \cdot \begin{vmatrix} 1 & 4 \\ -1 & -1 \end{vmatrix} + m \begin{vmatrix} 1 & 4 \\ 2 & 5 \end{vmatrix} =$$

$$= -1 + 4 + m(5 - 8) =$$

$$= 3 - 3m$$

$|B| = 3 - 3m = 0$

$3m = 3 \rightarrow m = 1$

Discussió

3x4

$\circ$ Si $m \neq 1 \rightarrow |B| \neq 0 \rightarrow \boxed{\text{Rang } B = 3}$

rang "Max" = min (n, m)

$\circ$ Si $m = 1 \rightarrow \boxed{\text{rang } B = 2}$

Teorema de Rouché

«Discutir un sistema»

$n=3$

Termes independents.

$$\left\{\begin{array}{l} 1\boxed{x}+1\boxed{y}+2\boxed{z}=3 \\ x-y+z=2 \\ -x+3y+z=0 \end{array}\right.$$

Matriu del sistema

$$M=\begin{pmatrix} 1 & 1 & 2 \\ 1 & -1 & 1 \\ -1 & 3 & 1 \end{pmatrix}$$

rang M

Matriu ampliada

$$M^*=\begin{pmatrix} 1 & 1 & 2 & 3 \\ 1 & -1 & 1 & 2 \\ -1 & 3 & 1 & 0 \end{pmatrix}$$

rang M^*

$\boxed{\text{rang } M = \text{rang } M^*} \Rightarrow$ Compatible $\begin{cases} \text{rang } M = n \rightarrow \text{S.C.D.} \\ \text{rang } M < n \rightarrow \text{S.C.I.} \end{cases}$

$\text{rang } M \neq \text{rang } M^* \Rightarrow$ Incompatible

Volem discutir aquest sistema d'equacions lineals

$$\begin{cases} x+y+2z=3 \\ x-y+z=2 \\ 2x \quad +3z=5 \end{cases}$$

«La 3a eq. s'obté de les dues primeres»

$$\operatorname{rang} \overbrace{\begin{pmatrix} 1 & 1 & 2 \\ 1 & -1 & 1 \\ 2 & 0 & 3 \end{pmatrix}}^{M} = 2$$

«Només hi ha 2 files independents»

$$\begin{vmatrix} 1 & 1 & 2 \\ 1 & -1 & 1 \\ 2 & 0 & 3 \end{vmatrix} = 0$$

$$\operatorname{rang} M^* = \operatorname{rang} \begin{pmatrix} 1 & 1 & 2 & 3 \\ 1 & -1 & 1 & 2 \\ 2 & 0 & 3 & 5 \end{pmatrix} = 2$$

n. incògnites

$$\operatorname{rang} M = \operatorname{rang} M^* = 2 < 3$$

S.C. Indeterminat

∞ solucions.

Per resoldre el S.C.I. emprarem el mètode de Gauss

$$\begin{cases} x+y+2z=3 \\ x-y+z=2 \\ \cancel{2x+3z=5} \end{cases} \rightarrow \left(\begin{array}{ccc|c} 1 & 1 & 2 & 3 \\ 1 & -1 & 1 & 2 \\ 2 & 0 & 3 & 5 \end{array}\right) \begin{array}{l} \rightarrow \text{igual} \\ \rightarrow F_2 \rightarrow F_2 - F_1 \\ \rightarrow F_3 \rightarrow F_3 - 2F_1 \end{array}$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 2 & 3 \\ 0 & -2 & -1 & -1 \\ 0 & -2 & -1 & -1 \end{array}\right) \begin{array}{l} \\ \\ \rightarrow F_3 \rightarrow F_3 - F_2 \end{array} \qquad \left(\begin{array}{ccc|c} 1 & 1 & 2 & 3 \\ 0 & -2 & -1 & -1 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$5-2\cdot 3$
$5-6$

$$\left.\begin{array}{l} x+y+2z=3 \\ -2y-z=-1 \\ \boxed{\cancel{0=0}} \end{array}\right\} \rightarrow \left.\begin{array}{l} x+y+2z=3 \\ 2y+z=1 \end{array}\right\} \rightarrow x+2z=3-y \rightarrow$$

aïllar x

$$z=1-2y$$

$$x=3-y-2z=3-y-2(1-2y)=$$
$$=3-y-2+4y=3y+1$$

solució:

$$\begin{cases} x=1+3y \\ y=y \\ z=1-2y \end{cases}$$

S.C.I.

SISTEMES HOMOGENIS

Termes independents són zero

$n.i = 3$

$$\begin{cases} x \quad - z = \boxed{0} \\ 2x - y \quad = 0 \\ -3y + 2z = 0 \end{cases}$$

$$M = \begin{pmatrix} 1 & 0 & -1 \\ 2 & -1 & 0 \\ 0 & -3 & 2 \end{pmatrix} \qquad M^* = \left(\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 2 & -1 & 0 & 0 \\ 0 & -3 & 2 & 0 \end{array}\right)$$

SEMPRE SÓN COMPATIBLES!

Casos

→ $\text{rang } M = 3 \longrightarrow \text{rang } M^* = 3$

S.C.D. → $x = 0$, $y = 0$, $z = 0$

→ $\text{rang} = 2 \longrightarrow \text{rang } M^* = \begin{cases} \cancel{0} \\ \cancel{1} \\ 2 \end{cases}$

$\text{rang } M = \text{rang } M^* = 2 < 3$

S.C.I.