MATEMÀTIQUES II

IEDIB

Solucions

MICROTASCA D'APRENENTATGE BAT_MAT2 2.1

MÈTODE DE GAUSS

i

RANGS DE MATRIUS

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1. Resoleu aquest sistema $\begin{cases} x+y-z+\quad t=4 \\ \quad y+z-t \quad =3 \\ \qquad z+2t \quad =1 \end{cases}$

$$\left.\begin{array}{r} x+y-z+\ t=4 \\ y+z-\ t=3 \\ z+2t=1 \end{array}\right\} \quad \left.\begin{array}{r} x+y-z=4-\ t \\ y+z=3+\ t \\ z=1-2t \end{array}\right\} \longrightarrow z=1-2t$$

$$y=3+t-z=3+t-1+2t=2+3t$$

$$x=4-t-y+z=$$

$$4-t-2-3t+1-2t=3-6t$$

$$x=3-6t,\ y=2+3t,\ z=1-2t,\ t=t\ ;\quad t\in\mathbb{R}$$

(S.C.I.)

2. Resoleu el sistema $\begin{cases} 3x + 4y - z = 3 \\ 6x - 6y + 2z = -16 \\ x - y + 2z = -6 \end{cases}$ pel mètode de Gauss. Classificau-lo.

$$\left.\begin{aligned} 3x + 4y - z &= 3 \\ 6x - 6y + 2z &= -16 \\ x - y + 2z &= -6 \end{aligned}\right\} \left(\begin{array}{ccc|c} 3 & 4 & -1 & 3 \\ 6 & -6 & 2 & -16 \\ 1 & -1 & 2 & -6 \end{array}\right) \to \begin{array}{l} (3.^a) \\ (2.^a):2 \\ (1.^a) \end{array} \left(\begin{array}{ccc|c} 1 & -1 & 2 & -6 \\ 3 & -3 & 1 & -8 \\ 3 & 4 & -1 & 3 \end{array}\right) \to$$

$$\to \begin{array}{l} (1.^a) \\ (2.^a) - 3 \cdot (1.^a) \\ (3.^a) - 3 \cdot (1.^a) \end{array} \left(\begin{array}{ccc|c} 1 & -1 & 2 & -6 \\ 0 & 0 & -5 & 10 \\ 0 & 7 & -7 & 21 \end{array}\right) \to \begin{array}{l} (1.^a) \\ (2.^a):(-5) \\ (3.^a):7 \end{array} \left(\begin{array}{ccc|c} 1 & -1 & 2 & -6 \\ 0 & 0 & 1 & -2 \\ 0 & 1 & -1 & 3 \end{array}\right) \to$$

$$\to \left.\begin{aligned} x - y + 2z &= -6 \\ z &= -2 \\ y - z &= 3 \end{aligned}\right\} \begin{aligned} &y = 3 + z = 3 - 2 = 1 \\ &x = -6 + y - 2z = -6 + 1 + 4 = -1 \end{aligned}$$

Solució: $(-1, 1, -2)$
$\quad x \quad y \quad z$

S. Compatible Determinat.

3. Calcula raonadament el rang de la matriu $D = \begin{pmatrix} 1 & 2 \cdot 2 & 0 & 3 \\ 0 & 1 \cdot 3 & -1 & -2 \\ 2 & 7 & -3 & 0 \end{pmatrix}$.

$$D = \begin{pmatrix} 1 & 2 & 0 & 3 \\ 0 & 1 & -1 & -2 \\ 2 & 7 & -3 & 0 \end{pmatrix}$$

$$F_3 = \underbrace{\alpha}_{2} F_1 + \underbrace{\beta}_{3} F_2$$

Tomamos un menor de orden 2 distinto de cero: $\begin{vmatrix} 1 & 2 \\ 0 & 1 \end{vmatrix} \neq 0$

Las dos primeras filas son linealmente independientes.

Veamos si la 3.ª fila depende linealmente de las dos primeras:

$$F_3 = 2F_1 + 3F_2$$

$$\begin{vmatrix} 1 & 2 & 0 \\ 0 & 1 & -1 \\ 2 & 7 & -3 \end{vmatrix} = -3 - 4 + 7 = 0$$

$$\begin{vmatrix} 1 & 2 & 3 \\ 0 & 1 & -2 \\ 2 & 7 & 0 \end{vmatrix} = -8 - 6 + 14 = 0$$

La 3.ª fila depende linealmente de las otras dos.

Por tanto, $ran\,(D) = 2$

4. Estudia el rang de la matriu $A = \begin{pmatrix} t & 1 & 1 & 2 \\ 2 & t & t^2 & 1 \\ 2 & 1 & 1 & 2 \end{pmatrix}$ pels diferents valors del paràmetre t.

rang $A \leq 3$.

$$\begin{vmatrix} t & 1 & 1 \\ 2 & t & t^2 \\ 2 & 1 & 1 \end{vmatrix} \xrightarrow{F_1 \to F_1 - F_3} = \begin{vmatrix} t-2 & 0 & 0 \\ 2 & t & t^2 \\ 2 & 1 & 1 \end{vmatrix} = (t-2) \cdot \begin{vmatrix} t & t^2 \\ 1 & 1 \end{vmatrix} =$$

$$= (t-2) \cdot (t - t^2) = (t-2) \cdot t \cdot (1-t) = 0 \quad \begin{cases} t = 0 \\ t = 1 \\ t = 2 \end{cases}$$

- Si $t \neq 0,\ t \neq 1,\ t \neq 2 \rightarrow$ rang $A = 3$
- Si $t = 0$ $\begin{pmatrix} 0 & 1 & 1 & 2 \\ 2 & 0 & 0 & 1 \\ 2 & 1 & 1 & 2 \end{pmatrix}$ $\quad \begin{vmatrix} 0 & 1 & 2 \\ 2 & 0 & 1 \\ 2 & 1 & 2 \end{vmatrix} = 2 \neq 0 \rightarrow$ rang $A = 3$

4. Estudia el rang de la matriu $A = \begin{pmatrix} t & 1 & 1 & 2 \\ 2 & t & t^2 & 1 \\ 2 & 1 & 1 & 2 \end{pmatrix}$ pels diferents valors del paràmetre t.

- $t = 1$

$$\begin{pmatrix} 1 & 1 & 1 & 2 \\ 2 & 1 & 1 & 1 \\ 2 & 1 & 1 & 2 \end{pmatrix} \qquad \begin{vmatrix} 1 & 1 & 2 \\ 2 & 1 & 1 \\ 2 & 1 & 2 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 2 \\ 2 & 1 & 1 \\ 1 & 0 & 0 \end{vmatrix} = 1 \cdot \begin{vmatrix} 1 & 2 \\ 1 & 1 \end{vmatrix} = -1 \neq 0$$

rang $A = 3$

- $t = 2$

$$\begin{pmatrix} 2 & 1 & \cancel{4} & 2 \\ 2 & 2 & \cancel{4} & 1 \\ 2 & 1 & \cancel{1} & 2 \end{pmatrix} \qquad \begin{vmatrix} 2 & 1 & 2 \\ 2 & 2 & 1 \\ 2 & 1 & 2 \end{vmatrix} = 0 \longrightarrow \text{rang } A = 2$$

- $t \neq 2 \longrightarrow$ rang $A = 3$
- $t = 2 \longrightarrow$ rang $A = 2$

5. Resoleu el sistema d'equacions $\begin{cases} x+y+z+u &= 0 \\ -2x-3y-z-u &= 7 \\ -x-y+2z-u &= 15 \\ x-z+2u &= -8 \end{cases}$ pel mètode de Gauss . Classificau-lo.

$$\left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ -2 & -3 & -1 & -1 & 7 \\ -1 & -1 & 2 & -1 & 15 \\ 1 & 0 & -1 & 2 & -8 \end{array}\right)\begin{array}{l} \to \\ \to +2F_1 \\ \to +F_1 \\ \to -F_1 \end{array} \quad \left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 0 & -1 & 1 & 1 & 7 \\ 0 & 0 & 3 & 0 & 15 \\ 0 & -1 & -2 & 1 & -8 \end{array}\right)\begin{array}{l} \\ \\ \\ \to -F_2 \end{array}$$

$$\left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 0 & -1 & 1 & 1 & 7 \\ 0 & 0 & 3 & 0 & 15 \\ 0 & 0 & -3 & 0 & -15 \end{array}\right)\begin{array}{l} \\ \\ \to :3 \\ \to +F_3 \end{array} \quad \left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 0 & -1 & 1 & 1 & 7 \\ 0 & 0 & 1 & 0 & 5 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)\begin{array}{l} \\ \\ \\ \to 0=0 \end{array}$$

5. Resoleu el sistema d'equacions $\begin{cases} x+y+z+u &= 0 \\ -2x-3y-z-u &= 7 \\ -x-y+2z-u &= 15 \\ x-z+2u &= -8 \end{cases}$ pel mètode de Gauss . Classificau-lo.

$$\begin{cases} x+y+z+u=0 \\ -y+z+u=7 \\ z=5 \end{cases} \qquad u=\lambda \qquad \rightarrow \qquad \begin{cases} x+y+z=-\lambda \\ -y+z=7-\lambda \\ z=5 \end{cases} \rightarrow y=5-7+\lambda=-2+\lambda$$

$$x=-\lambda-y-z=-\lambda+2-\lambda-5=-3-2\lambda$$

$$\begin{cases} x=-3-2\lambda \\ y=-2+\lambda \\ z=5 \\ u=\lambda \end{cases} \qquad \lambda \in \mathbb{R} \qquad \underline{S.C.I.}$$