Value at Risk for a portfolio of stocks using Historical Simulation and Monte Carlo Simulation

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***Abstract*—In this B.Tech project, we have found the VaR(Value at Risk) of a portfolio consisting of 4 different stocks using two different approaches: Historical simulation and Monte Carlo simulation using geometric Brownian motion. These have then been compared to evaluate the accuracy of Monte Carlo simulation when compared with actual historical returns.**

***Index Terms*—Stock, Portfolio, VaR, Monte Carlo simulation, Random walks, geometric Brownian Motion.**

I. Introduction

A portfolio consists of a grouping of financial assets like stocks or bonds. Portfolios are held directly by investors and managed by financial professionals. Investors construct an investment portfolio according to their investing objectives and risk tolerance. Hence, investors are interested in finding out the risks that are associated with investing in different stocks. There are a few methods to find out the risk which include finding out the standard deviation of the portfolio, calculating the beta, finding the VaR and Conditional VaR of the portfolio.

In this project, we aim to find the risk associated with investing in 4 different stocks namely Apple, Facebook, Citigroup and Walt Disney. For this, we will calculate the Value at Risk(VaR) associated with the portfolio by using two different approaches: Historical Simulation and Monte Carlo simulation. In the Monte Carlo simulation, we have assumed that the stock prices follow a geometric Brownian motion.

II. Value at Risk

Value at Risk (VaR) attempts to provide a single number that summarizes the total risk in a portfolio of stocks. It is a statistic that quantifies the level of financial risk within a portfolio. It aims to make a statement of the following kind:
”I am X percent certain there will not be a loss of more than V dollars in the next N days.”
VaR modelling helps to find the VaR(the value V above) with probability/confidence level (X) for that loss to occur over a pre-defined period N[3].

There are 3 main methods to find out the VaR of a portfolio. They have been described briefly in the following subsections:

A. Historical Simulation

This approach uses past data to guide what can happen in the future. In this method, we use previous data to create different scenarios that can happen on the next day. These scenarios are then ordered according to the losses incurred. The required VaR is then found using the given confidence level. For example, if 40 different scenarios are created, to find the VaR for the next day with 95 percent confidence level, we take the 3rd highest loss as the VaR as 5 percent of 40 is 2 and hence the 3rd highest loss is taken as VaR for 95% confidence level.

B. Monte Carlo simulation

Monte Carlo simulation uses a model to predict the future value of a portfolio. Multiple trials are run using the model to predict the future prices of the stocks in the portfolio. Various scenarios can be created using the underlying model which helps us to find the behaviour of our portfolio in different market conditions. Based on these future values, we predict the VaR of our portfolio. In this project, we have used geometric Brownian motion as the underlying model which predicts the future prices of our portfolio. This has been discussed in detail in a later section.

C. Variance Covariance Method

This method assumes that stock returns are normally distributed. In this method, we find out the mean and standard deviation of stock prices using historical data. We find the covariance and correlation between different stocks in our portfolio and find the cumulative distribution function. We then use the specified confidence level to find the final value of stock prices in our portfolio and then calculate the VaR with the specified levels of confidence.

III. Historical simulation

In our project, we have used the historical data of the following 4 stocks in our portfolio: Apple, Facebook, Citigroup and the Walt Disney Company. For this, we used the repository from Yahoo Finance[2] which contains the data of various stocks and their daily prices. We then used the pandas library in python for data analysis required to calculate the Value At Risk.
The portfolio of our stocks consists of 100000 rupees that are distributed between the four stocks according to the assigned weights. The weights assigned to the four stocks are 0.25, 0.3, 0.15, 0.3 for Apple, Facebook, Citigroup and Walt Disney

group respectively, i.e investment of 25000 , 30000, 15000, and 30000. The historical data has been taken starting from 1st January 2012 till the present day. This creates around 2000 scenarios to find out the change in the portfolio value on the next day. The next day value of the each stock is taken as:

Value under ith scenario = $V_n * \frac{v_i}{v_{i-1}}$

This has been done for each of the stock in the portfolio. The changes are then added up to obtain the net change of the portfolio in each of the 2000 scenarios. These values basically provide us with the returns obtained from our portfolio on the next day.

Each of these data points are then plotted as a histogram to obtain the distribution of the returns.

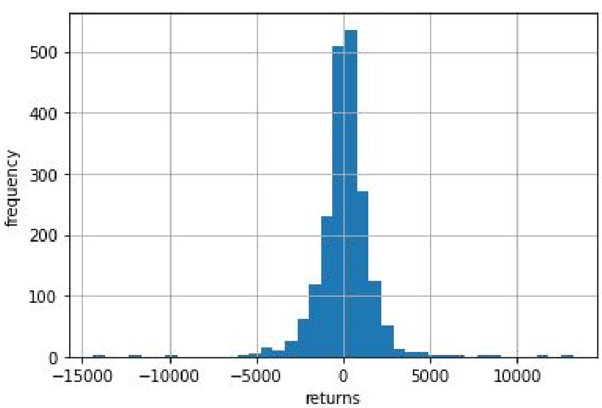

Figure 1. Historical returns of portfolio

The figure above shows the historical returns of 2000 scenarios created for our portfolio. These returns are mostly distributed around the 0 returns marks which shows that the daily historical returns are not very volatile for our portfolio. Diversification in a portfolio also helps to mitigate the effects of volatility. This is because the correlation between different stocks that are present in a portfolio might be negative which means that if there is a fall in the prices of one stock, the other stock would give positive returns and thus help in reducing the losses.

To calculate the Value at Risk with a given confidence level, we take the quantile function to find out the value present at that percentile in our data-set. Since VaR values are losses that will be sustained, the returns are hence negative. The magnitude of these values provide us with the required VaR.

Once we get the required value at risk for the next day, the n-day VaR is then calculated by using the formula:

$$N\_dayVaR = 1\_DayVaR * \sqrt{N} \tag{1}$$

The 99%ile value obtained by the quantile function is used to plot the VaR for the first day in Fig.3. The subsequent values are obtained by using the formula in equation (1).

Confidence Level value	Hist_VaR
90%	1484.68
95%	2112.57
99%	4173.52

Figure 2. VaR values at different confidence levels

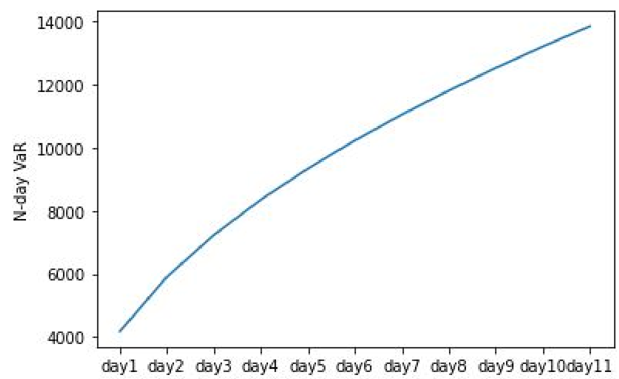

Figure 3. N-day VaR with 99% confidence

IV. Monte Carlo Simulation

The model building approach for calculating VaR can be implemented by using Monte Carlo simulation. It helps to generate a probability distribution function of returns. To calculate 1-Day VaR:
(i) We start with the initial investment in each stock of the portfolio.
(ii) Using the existing data and sampling the random shock of prices from a random number obtained from Gaussian distribution, we calculate ΔS.
(iii) Using the obtained ΔS for each stock, we value our portfolio on the next day.
(iv) ΔP i.e. return is calculated by subtracting the initial investment.
(v) Above steps are repeated again and again to obtain the probability distribution of returns of our portfolio [3].

In our project, the underlying model used for Monte Carlo simulation is the geometric Brownian motion for the stock prices. To understand the concept of geometric Brownian motion, we have first researched about random walks which is explained in the next subsection.

A. Random Walks

As the name suggests, random walk implies that a person standing at a point can move in any direction and it does not depend on where he is standing right now. The probability of his movement in various directions can very in case of asymmetric random walks. Similarly, in finance, random walk theory suggests that the change in stock price from today to tomorrow does not depend on the price today and can be in any direction depending upon the market conditions[1]. However, on a large time horizon, the markets generally seem to always move in the upward direction, i.e. the probability or weight of the stock price going upward has a higher value than that of the stock getting devalued. This is known as the drift of the market.

B. Geometric Brownian Motion

In geometric Brownian motion (GBM), the logarithm of the randomly varying quantity follows a Brownian motion with drift. It is also called exponential Brownian motion and is a continuous-time stochastic process. This model of stock price simulation consists of a few assumptions:

(i) Returns over a short period are normally distributed.

$$\frac{\Delta S}{S} \sim N(\mu, \sigma) \tag{2}$$

(ii) Expected return is independent of the stock price.

$$E(\frac{\Delta S}{S}) = \mu \Delta t \tag{3}$$

(iii) The volatility of stock is also independent of the stock price.

The equation for the discrete time version of the GBM is:

$$\Delta S = S(\mu \Delta t + \sigma z \sqrt{\Delta t}) \tag{4}$$

where μ and σ can be calculated from historical data and z is a random shock.

C. Monte Carlo Simulation using GBM

As with historical simulation, we have chosen the following 4 stocks: Apple, Facebook, Citigroup and Walt Disney group. The amount invested is also the same in each of the above. First we found out the average change and average volatility of the four stocks. 2000 scenarios were created using past data to find out the required average. This provided us with the μ and σ required according to the equation given above.

Then we have simulated 5 scenarios (for clarity in graph) to look what the future values of our portfolio looks like. For this, we have individually used geometric Brownian motion for each of the stock prices to find out the value of the portfolio for the next 10 days.

As is visible from Fig 4, most of the simulation have a rising curve because of the drift. Also, the graph shows random shocks.The random shock will be the standard deviation multiplied by a random number(z in the above equation). This is used to scale the standard deviation.

To obtain various values of returns, we then simulated 2000 trials for each of the 4 stocks with respective μ and σ of each stock and using a random shock with each of them. Each of these values are then added to obtain the total value of out portfolio on the next day in these 2000 scenarios.

The returns of our portfolio in each of the scenarios is calculated by subtracting the initial investment. Our GBM model assumes normality, i.e.price returns are normally distributed

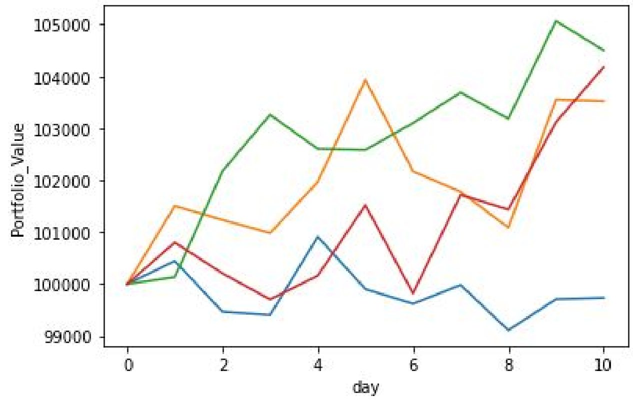

Figure 4. Future portfolio values using GBM

with expected return μ and standard deviation σ. However, the price levels are log normally distributed[4].

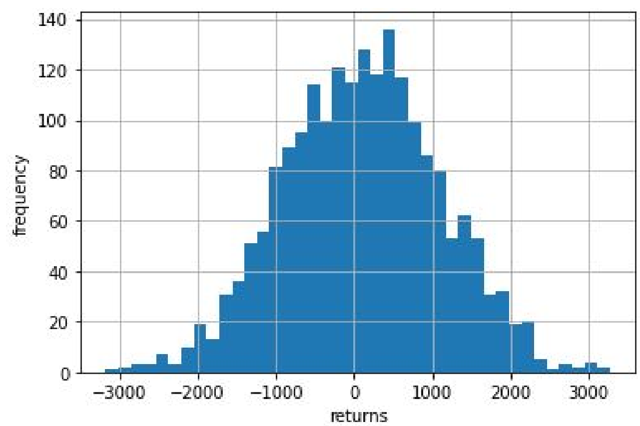

Figure 5. Returns using Monte Carlo Simulation

As seen above, the returns when plotted as a histogram show normal returns. As the number of trials increase, its shape would tend more and more towards the normal curve.

To obtain the Value at Risk of our portfolio with 99% confidence level, we use the quantile function. Similarly we also obtain the values at 90% and 95% confidence level.

Confidence Level value	Value at risk
90%	1270.09
95%	1669.88
99%	2336.42

Figure 6. VaR values with different confidence levels

The n-day VaR is then calculated by using the same formula as in historical simulation. The 1-Day VaR taken here is with 99% confidence level.

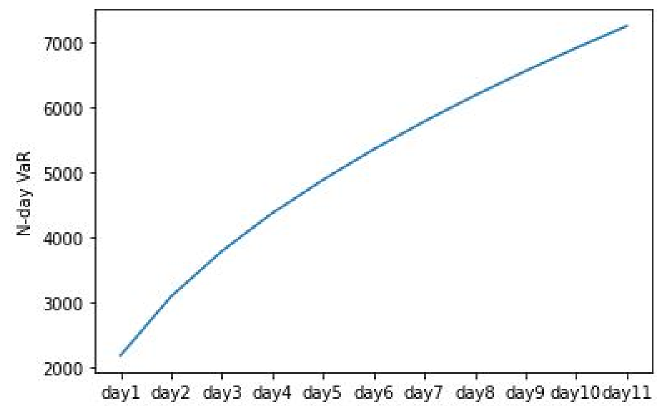

Figure 7. N-day VaR using Monte Carlo Simlation

V. Comparison of the two models

Historical simulation uses historical record of returns to simulate future possible outcomes. The method assumes that future performance can be predicted by past performances.

On the other hand, Monte Carlo simulation relies on modeling the distribution of returns using a random number generator. The model is run thousands of time to provide the output which is recorded and ordered to estimate the probability distribution of the possible outcomes. Monte Carlo simulation can also be used to incorporate other models that factor in various interrelationships or correlations that might be present in a portfolio. It can also be used to simulate the effects of outlier events.

In the case of the geometric Brownian motion model that we have used, we assume simple market dynamics with no sudden changes. It assumes that markets tend to normalise in the long run and that stock values normally rise barring a few shocks.

Since normal market dynamics are assumed in this model, it should tend close to the historical model.

So first, we have compared the returns that are obtained in both the cases.

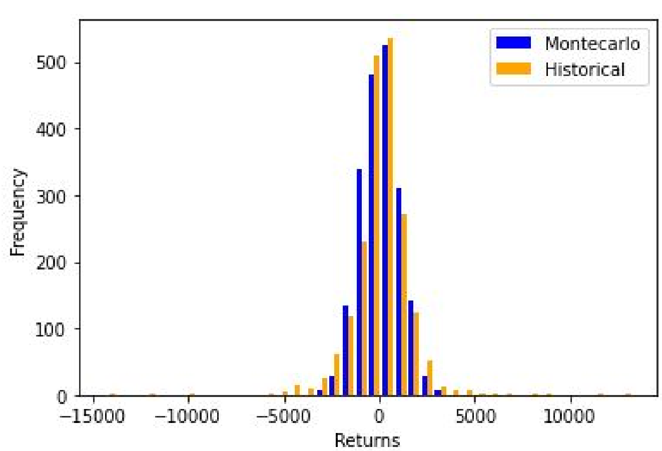

Figure 8. Comparison Of Returns in both models

The returns in both the cases have shown approximately the same values. Most of the models that simulate stock prices assume normal returns and these are based on historical evidence. Volatility of portfolios do not have very high values and thus the deviation of the value of portfolio from it's mean position is not very high. Thus most of the returns are clustered around the mean value which is also the case with normal distribution.

Moving forward, we then compared the VaR values that have been obtained from both the models at different confidence levels.

Confidence Level value	Hist_VaR	MC_VaR
90%	1484.68	1270.09
95%	2112.57	1669.88
99%	4173.52	2336.42

Figure 9. Comparison of VaR

As seen in Figure 9, the VaR values here differ by around a multiple of 2 at 99% confidence level. This is mainly due to the fact that the time period considered here in historical simulation consists of the time during which Covid-19 pandemic had struck. There was a historical fall in the stock prices around the world including the US. This resulted in the daily changes of stock prices to fall and thus give negative returns on our portfolio. In the case of Monte Carlo simulation, during the calculation of μ and σ, though the time period included the time of pandemic, the calculation was averaged out on an 8 year time horizon. Hence, the effects were reduced.

Since the N-day VaR is simply calculated as a multiple of 1-Day VaR, similar effects are seen on the N-day VaR graph.

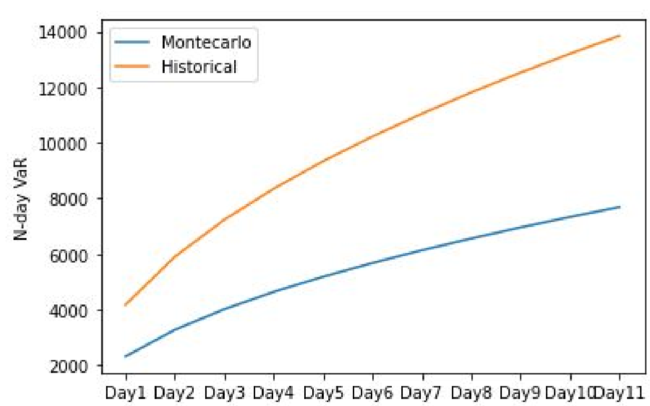

Figure 10. Comparison of N-day VaR

Note: The code and the final report of the project is available at https://github.com/jmulherkar/BTP2020/tree/master/ShikharDubey_ShivamAgarwal

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REFERENCES

[1] Varun Divakar. URL: https://blog.quantinsti.com/random-walk/.

[2] Yahoo Finance. *Stock prices of Companies*. URL: https://in.finance.yahoo.com/.

[3] John C Hull. *Options,Futures and other derivatives*. Pearson.

[4] Investopedia. URL: https://www.investopedia.com/articles/07/montecarlo.asp.