Comparitive Analysis of Portfolio Optimisation Models During the COVID-19 Pandemic

Jayati Patel
201601073 DAIICT
Gandhinagar, Gujarat, India
201601073@daiict.ac.in

Gaurav Sofat
201601091 DAIICT
Gandhinagar, Gujarat, India
201601091@daiict.ac.in

***Abstract*—The project is based on portfolio optimisation techniques used by the market indices. We have used the pandemic of 2020 caused by the COVID-19, to exemplify the comparative performance of the models, namely, Monte Carlo Optimisation, the Markovitz model and the Black-Litterman model. We have analysed each graphical result and based on the analysis, obtained the ratio of the weights of the portfolio components which has the highest probability of giving the best result.**

I. Introduction

A portfolio is a grouping of financial assets such as stocks, bonds, commodities, currencies and cash equivalents, as well as their fund counterparts, including mutual, exchange-traded and closed funds [1]. Investors ideally want to invest in different assets and maximize their returns and hence want to optimize their portfolio. Different investors have different preferences or requirements. For example, one investor may want to maximize his returns and may be ready to play heavy risks while another may want to minimize the risks. Thus, different investors may have different optimum portfolios based on their individual requirements.

Optimising a portfolio can simply be understood as picking the best ratio or weights of the component assets by considering essential factors, expected return and financial risk being the major ones. One can analyze trends over a year or a month to select an optimum portfolio. Through the past, many models have emerged quite beneficial to the investors and have gained a lot of popularity. The approaches like Monte Carlo Simulation and Markovitz Model, generally consider higher expected returns and lesser risk as characteristics of the most profitable allocation. However, many models like the Black-Litterman Model have evolved which work when expected returns of an investment are not known.

The world is at a sensitive stage today as we face the COVID-19 pandemic. Various predictions have been made about how the economy faces a crisis because of the pandemic. We wanted to analyze better how much of an effect the pandemic has caused on the market. Our project considers a group of 30 stocks (BSE sensex30) as the desired sample space and a period of one year, starting with the financial year of 2019. We prove mathematically and graphically that the gain is comparatively higher (than sensex for example) over the total period, if the investors sink in according to the optimised portfolio composition, even though the period is struck by the economic crisis caused by COVID-19. We obtained the results using a python code which modeled the different methods we considered for our approach.

II. Approach I: Monte Carlo Simulation

Monte Carlo methods Simulations, are a broad class of computational algorithms that rely on repeated random sampling to obtain numerical results . This method uses randomness to solve a set of problems which could be classified as being deterministic in principle [2]. The Monte Carlo method is one of the more used method for portfolio optimisation [3]. We first assign weight to each stock in the portfolio randomly. After this, the mean daily return and the standard deviation of the return is to be calculated. Now, a portfolio can be considered to be optimized when we have the maximum return to risk ratio. It is important to note here that guessing and checking for a large number of stocks may not be a very efficient method computationally but is still one of the computationally easy ways to get an approximate composition of the portfolio. The method is explained in further detail with the help of some equations below:

- Initially, we have to calculate two main important factors that are expected return and standard deviation of the portfolio. We start by calculating daily returns of the assets after extracting stock prices for the target companies [4].

$$\begin{aligned} P_{i-1} &= yesterday's\ price\ of\ stock\ x \\ P_i &= today's\ price\ of\ the\ same\ stock \\ R_d(x,i) &= daily\ return\ of\ the\ stock\ x\ =\ \frac{P_i}{P_{i-1}} \end{aligned}$$

- Later we calculate mean return for each company:

$$\begin{aligned} N &= number\ of\ days\ for\ which \\ &\quad\ we\ have\ collected\ the\ data \\ T &= number\ of\ stocks\ =\ 30 \\ R_{mean}(x) &= mean\ of\ daily\ return\ of\ the\ stock\ x \\ &= \frac{\sum_{i=1}^{N} R_d(x,i)}{N} \end{aligned}$$

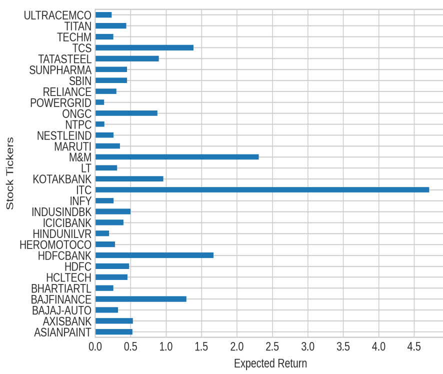

Fig. 1. Annualised expected returns of sensex components

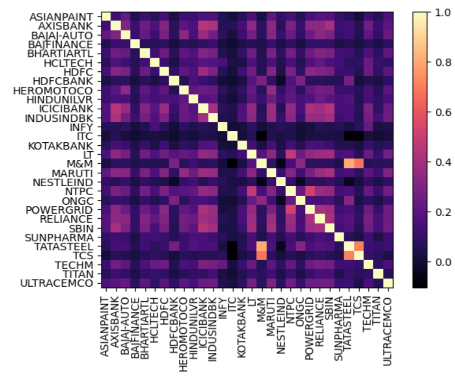

Fig. 2. Heat-Map of the Covariance Matrix

- We the use Monte Carlo Simulation to generate portfolios of random weights but with the condition that weights of the individual stocks would add up to one. Condition:

$$\sum_{i=1}^{T} w_i \; = \; 1$$

- The next obvious step is to calculate portfolio expected return by weighted sums of mean returns of each stock:

$$\begin{aligned} R_{portfolio} \; &= \; Expected\ Portfolio\ Return \\ &= \sum_{i=1}^{T} w_i \times R_{mean}(i) \end{aligned}$$

- Investors are risk averse i.e they try to avoid risky investments. In this technique, volatility or risk is measured by calculating standard deviation as it can inform us of how the price of a stock deviates from its mean. As an instance, some of the stocks might be negatively correlated with each other, implying that as the value of the first stock moves down, the value of the negatively correlated stock increases. This shows us that the risk of a portfolio is not a simple sum of the risks of the individual assets [5]. The volatility is computed by calculating the standard deviation of the returns of each stock along with the covariance between each pair of the stocks by using this formula:

$$\begin{aligned} \sigma_{portfolio} \; &= \; Standard\ Deviation\ of\ Portfolio \\ &= \left[\sum_{i=1}^{T} \sum_{j=1}^{T} w_i w_j \sigma_{ij} \right]^{\frac{1}{2}} \\ \sigma_{ij} \; &= \; \rho_{ij}\ \sigma_i\ \sigma_j \\ \sigma_i \; &= \; Standard\ Deviation\ of\ stock_i \\ \sigma_j \; &= \; Standard\ Deviation\ of\ stock_j \\ \rho_{ij} \; &= \; Correlation\ coefficient\ between \\ & \quad the\ return\ on\ stock\ i\ and \\ & \quad the\ return\ on\ stock\ j \end{aligned}$$

- Now we could have invested our money into a savings account (risk-free) and earned a return without taking any risk. How can we decide whether the return we earned is worth the risk we took?- This is why we calculate Sharpe Ratio. Sharpe Ratio is the amount of excess return over the risk-free rate as the relevant measure of risk. As we have calculated the other essential factors, now we are left to calculate Sharpe Ratio:

$$\begin{aligned} Sharpe\ Ratio \; &= \; \frac{R_{portfolio} \; - \; R_{rf}}{\sigma_{portfolio}} \\ R_{rf} \; &= \; Return\ over\ risk\ free\ rate \end{aligned}$$

- In a risk-return graph, with return on the y-axis and standard deviation on the x-axis, we figure out that for an investor with maximum Sharpe Ratio as the basis for getting optimised portfolio, the most north-west point of the curve will give the desired ratio. So we plot the risk-return graph and get the proportions of the composite stocks of the optimised portfolio:

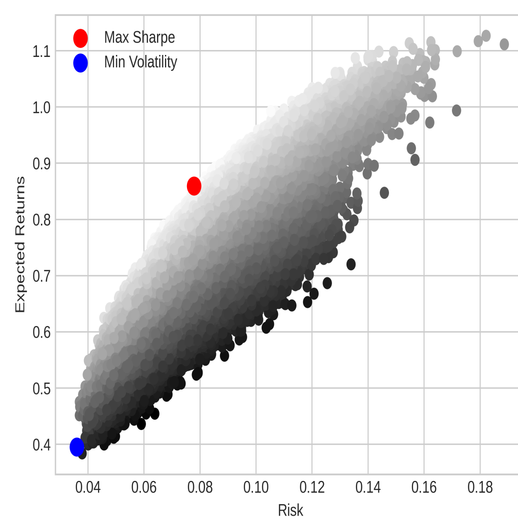

Fig. 3. Risk-Return Graph with optimal solution

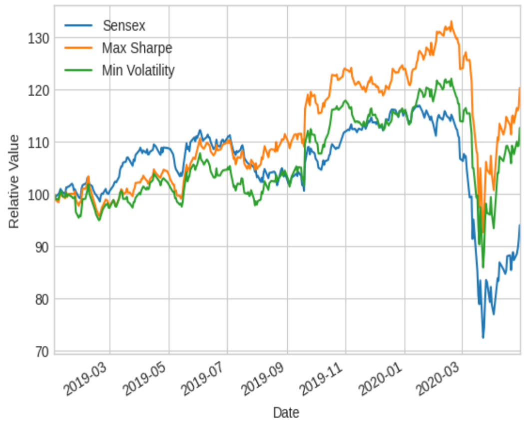

Fig. 4. Relative value of optimal portfolio obtained by Monte-Carlo Simulation compared to benchmark market index

Ticker	Max Sharpe	Min Volatility
ASIANPAINT	7.34%	1.97%
AXISBANK	0.86%	7.92%
BAJAJ-AUTO	3.81%	4.73%
BAJFINANCE	6.12%	0.61%
BHARTIARTL	0.46%	6.39%
HCLTECH	2.09%	2.02%
HDFC	3.61%	2.1%
HDFCBANK	4.04%	0.31%
HEROMOTOCO	6.36%	6.42%
HINDUNILVR	0.14%	8.36%
ICICIBANK	4.36%	2.45%
INDUSINDBK	0.45%	0.24%
INFY	8.04%	3.5%
ITC	5.37%	0.53%
KOTAKBANK	2.49%	1.52%
LT	0.53%	2.61%
M&M	2.68%	0.78%
MARUTI	0.22%	4.59%
NESTLEIND	6.08%	5.2%
NTPC	1.52%	4.61%
ONGC	5.44%	1.68%
POWERGRID	3.26%	4.83%
RELIANCE	1.56%	6.8%
SBIN	5.65%	1.73%
SUNPHARMA	4.9%	6.42%
TATASTEEL	0.71%	1.1%
TCS	6.35%	0.88%
TECHM	2.48%	2.58%
TITAN	0.17%	0.04%
ULTRACEMCO	2.92%	7.1%

Fig. 5. Table consisting of the composition of Monte-Carlo Simulated optimal portfolio

III. Approach II: Markovitz's model

Harry M. Markowitz is considered as the pioneer of Modern Portfolio Theory. He put forward this new concept in his paper 'Portfolio Selection' and has also received a Nobel Prize for the same. [6] The Markowitz model or the Modern Portfolio Theory argues that a portfolio should be evaluated as a whole and not just the individual risks and returns for each asset. How an individual asset fits in the entire portfolio is more important than its risks and returns alone. Thus, this model uses statistical analysis for measurement of risk and mathematical programming for selection of assets in a portfolio in an efficient manner.

The Modern Portfolio Theory has some underlying assumptions associated with it. Some of the assumptions made by the model are:

- Investors are rational and want to maximize their utility for a given level of money.
- Investors can access fair and correct information on risk and returns freely.
- Markets are efficient and the information fed to a market is absorbed quickly and perfectly.
- Investors want to minimize the risk and maximise the return i.e. they are risk averse.
- Investors make decisions based on expected returns and the variance of such returns.
- Investors prefer higher returns to lower returns for a given level of risk i.e. the expected returns for investors increases as the risk increases.

Thus, a portfolio with the above stated assumptions is considered optimum or efficient if we can find no other portfolio which offers a greater return with a lower or equal risk or a lower risk with greater or equal return. One method which helps in achieving the state objectives is diversification of securities.

A. *Diversification of portfolio:*

Every security in the market is different than others and hence, the risk for each security is also different than that of others. We can create a selection of different securities to arrive at a balance where the risk of one security is offset by others. For example, we can reduce unsystematic and company related risk by selecting various different securities and assets with different variability such that they offset each other. In this scenario, we want these assets to be negatively correlated or unrelated. Thus, we need to consider the relationships among different securities by considering the covariance along with the variance of each security. Notwithstanding, we avoid using this traditional estimator of volatility because it is known to be replete with estimation errors around the extremes, in particular when the number of historical observations are comparable to the number of assets under consideration. We will use a transformation of the sample co-variance matrix, constructed using the Ledoit-Wolf Shrinkage Estimator [7].

Thus, by offsetting, we try to reduce the standard deviation of the entire portfolio to zero, if possible or as close to zero as possible. We also want as negative an interactive effect as possible among the different securities with the portfolio. The last factor we need to consider is the coefficient of correlation. The coefficient of correlation reflects how two securities or assets are related. Positive correlation indicates that the securities are directly related i.e. if the returns for one increase, the returns for the other also increase and if the returns for one decrease, the returns for the other also decreases. Zero correlation indicates an absence of correlation altogether. Negative correlation indicates that the securities are inversely related i.e. if the returns for one increase, the returns for the other decrease and vice versa. Now, according to the Modern Portfolio Theory, it is advisable to have as low a coefficient of correlation as possible, since a lower coefficient indicates lower risk as the risks of a single security or asset are offset by others. This effectively results in diversification and Markowitz has shown that a portfolio selected this way will have a lower risk while not lowering the return. It also needs to be noted, however, that each investor will have a different level of diversification better suited for that particular investor.

B. *Efficient Frontier*

To find a diversification level which is tailored for an investor, we use a concept known as Efficient Frontier. The Efficient Frontier is a graphical representation of all possible combinations of securities and assets. We can find the optimal return for a given level of risk using the Efficient Frontier. At each level of return, a portfolio can be created that offers the lowest risk and similarly, for every level of risk, a portfolio that offers the highest return can be created. A portfolio that falls outside the Efficient Frontier is considered suboptimal. This is because we can either create a portfolio that offers a lower risk relative to the same return on the Frontier or we can create a portfolio with a greater return relative to the same risk. A portfolio that lies below the frontier doesn't provide optimum return compared to the level of risk associated with it and a portfolio that lies right to the frontier doesn't optimise risk for the return associated with it.

C. *Formulation*

The model is explained in further detail with the help of equations and graphs below:

- As in the case of Monte Carlo, we calculate the daily return for each stock taken into consideration

$$\begin{aligned} r_d(x,i) &= daily\ retrun\ of\ stock\ x \\ &= \frac{p_i - p_{i-1}}{p_{i-1}} \end{aligned}$$

- Then we calculate expected daily return for each stock

$$\bar{r_d}(x) = \frac{\sum_{i=1}^{N} {}_d(x,i)}{N}$$

- We now calculate annual expected return in a way similar to how we generally formulate annual quantities when working financial assets, by compounding $E[R_d]$ over a period of n_N:

$$\begin{aligned} r(x) &= (1+\bar{r_d}(x))^{n_N} - 1, \quad where \\ r(x) &= annualised\ expected\ daily\ returns\ of\ stock \\ n_N &= average\ number\ of\ days\ in\ a\ financial\ year \end{aligned}$$

- Solving these above mentioned computations for all assets in our asset universe of size T, we get:

$$X = \{x_1, x_2, \dots x_T\}$$

$$\bar{r} = \begin{pmatrix} \bar{r}(x_1) \\ \bar{r}(x_2) \\ \vdots \\ \bar{r}(x_T) \end{pmatrix}$$

$$\bar{r} = the\ expected\ return\ matrix$$

- Now, let us consider an asset x with daily closing prices $\{p_0(x), p_1(x), \dots p_N(x)\}$ over the period of analysis (i.e five financial years, in our case). We can use the daily closing prices to compute the daily returns $\{r_2(x), r_3(x) \dots r_N(x)\}$ over our period of analysis. We can subsequently use these returns to compute $\bar{r}(x)$, the mean daily returns of asset x.

$$\begin{aligned} r_k(x) &= \frac{p_k(x) - p_{k-1}(x)}{p_{k-1}(x)}, \ 2 \le k \le N \\ \bar{r}(x) &= \frac{\sum_{i=1}^{N} r_i(x)}{N} \end{aligned}$$

In our asset universe, $X = \{x_1, x_2, \dots x_T\}$, where T is the size of the universe, we now need to consider the daily return matrices of the form $\{r_1, r_2, \dots r_N\}$ and the mean daily return matrix $\bar{r}$.

$$r_i = \begin{pmatrix} r_i(x_1) \\ r_i(x_2) \\ \vdots \\ r_i(x_T) \end{pmatrix} \quad \bar{r} = \begin{pmatrix} \bar{r}(x_1) \\ \bar{r}(x_2) \\ \vdots \\ \bar{r}(x_T) \end{pmatrix}$$

- We can now compute the sample co-variance matrix of daily returns, S_d, for our asset universe. We then perform an element-wise compounding operation on this co-variance matrix S_d which gives us the annualised co-variance matrix of our asset universe S. We do this as we generally work with per annum values.

$$\begin{aligned} S_d &= \frac{\sum_{i=1}^{N} (r_i - \bar{r})(r_i - \bar{r})^T}{N-1} \\ S &= (1+S_d)^{\circ(n_N - 1)} \end{aligned}$$

- The Ledoit-Wolf Shrinkage Estimator uses a Shrinkage constant $\delta \in [0,1]$ and a structured estimator F (also known as the shrinkage target) to transform the sample co-variance matrix S and make it closer to the true (or population) co-variance matrix. We use the *constant correlation* Ledoit-Wolf shrinkage model and accordingly, the shrinkage target is based on our estimate that all pairwise correlations are identical. This identical value is equal to the mean of all sample correlations. Our

shrinkage target is the implied using this value and the previously obtained sample variances [8].
Let typical entries of sample co-variance matrix S be defined as $\sigma_{ij},\ i,j \in X$ where X is the asset universe with size T as defined earlier. Then the sample co-relation between two assets i and j $(i,j \in X)$ denoted by ρ_{ij}.

$$\begin{aligned} \rho_{ij} &= \frac{\sigma_{ij}}{\sqrt{\sigma_{ii}\sigma_{jj}}} \\ \bar{\rho} &= average\ sample\ correlation\ across \\ &\quad the\ sample\ co-variance\ matrix\ S \\ &= \frac{2}{T(T-1)} \sum_{i=1}^{T-1} \sum_{j=i+1}^{T} \rho_{ij} \end{aligned}$$

Correspondingly if the elements of the shrinkage target matrix F are given by f_{ij} then —

$$f_{ij} = \begin{cases} \bar{\rho}\sqrt{\sigma_{ii}\sigma_{jj}} & \text{if } i \neq j \\ \sigma_{ii} & \text{otherwise} \end{cases}$$

Now, we can obtain our Ledoit-Wolf shrinkage estimation of the co-variance matrix Σ by performing a convex linear combination of the constant co-relation estimator F, the sample co-variance matrix S and the optimal shrinkage constant δ^*. We use this matrix as an input for the Markowitz model.

$$\Sigma = \delta^* F + (1 - \delta^*) S$$

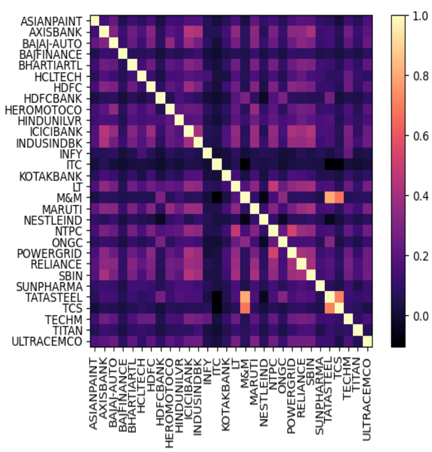

Fig. 6. Heat-Map of Ledoit-Wolf Shrinkage Covariance Matrix

- The *optimal portfolio* is different for every investor. It can be seen as a subjective term which largely depends on the investor outlook and risk-tolerance. As discussed before, one of the optimal portfolios best suited to a risk-averse investor would be the minimum volatility portfolio. We can model this problem as a quadratic programming problem. Now, we need to find an optimal weight allocation matrix $w = [\ w_{x_1}\ w_{x_2}\ w_{x_3}\ \ldots\ w_{x_T}\ ]$. Here, w_i is the proportion of the total amount of funds that are invested in asset i from our asset universe $X = \{x_1, x_2, \ldots x_T\}$ where T is the size of the asset universe. For a feasible solution, we have $\sum_{i=1}^{T} w_i = 1$. This equation simply reflects the property that the sum of proportions of funds allocated to each asset should equal the total amount of funds to be invested.

$$\begin{aligned} &\underset{w}{max}\ \frac{1}{2} w^T \Sigma w \\ &s.t \sum_{i=1}^{T} w_i = 1 \end{aligned}$$

We can solve this problem by known convex quadratic programming methods.
- As we mentioned before, we use the Markowitz model to find two different optimal portfolios. The second one here is the portfolio which has the maximum reward-to-risl ratio or the Sharpe Ratio [9]. As computed above, $\bar{r}$ represents the expected returns for all assets and Σ represents the co-variance matrix. We make the assumption here that such an optimal portfolio allocation $\hat{w}$, with $\hat{w}\bar{r} > r_f$ exists. Here, r_f represents the risk-free rate of return available in the market. A quadratic programming problem can now be formulated based on this to find the desired portfolio which has the maximum Sharpe ratio.

$$\begin{aligned} &\underset{w}{max}\ \frac{w\bar{r} - r_f}{(w^T \Sigma w)^{1/2}} \\ &s.t \sum_{i=1}^{T} w_i = 1 \end{aligned}$$

- We need to transform this problem to an equivalent QP problem as it is not solvable in its present form [10]. Based on our assumption, $\exists\ \hat{w},\ \hat{w}\bar{r} > r_f$, we can limit our search to only those w for which $w(\bar{r} - r_f I) > 0$. Here, I is a conformable identity matrix. We can now change our variables as follows:

$$\begin{aligned} \kappa &= \frac{1}{w(\bar{r} - r_f I)} \\ y &= \kappa w \\ \sqrt{w^T \Sigma w} &= (1/\kappa)\sqrt{y^T \Sigma y} \end{aligned}$$

We can rewrite the objective function of the original QP as $\sqrt{y^T \Sigma y}$ using the new variables. We also have,

$$w(\bar{r} - r_f I) > 0, x \in \chi \iff \kappa > 0,\ \frac{y}{\kappa} \in \chi$$
$$\kappa = \frac{1}{w(\bar{r} - r_f I)} \iff w(\bar{r} - r_f I) = 1,\ \frac{y}{\kappa} = x$$

The transformation of our original QP problem to a problem which can be solved by some known convex quadratic programming methods is now complete.

$$\begin{aligned} &\underset{y}{min}\ y^T \Sigma y \\ &s.t\ w(\bar{r} - r_f I) = 1 \\ &\kappa > 0,\ (y, \kappa) \in \chi \end{aligned}$$

D. Result

After computing the model, we get the following graphs:

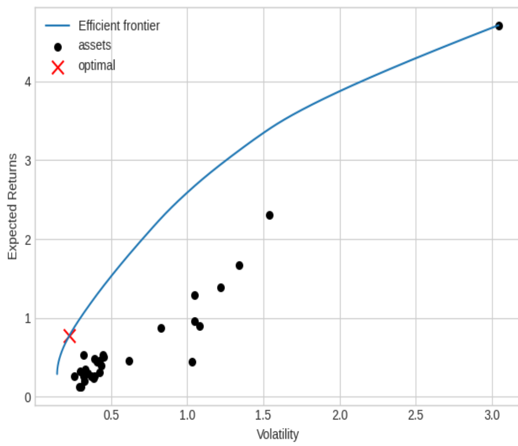

Fig. 7. Efficient Frontier obtained by Markowitz Theory

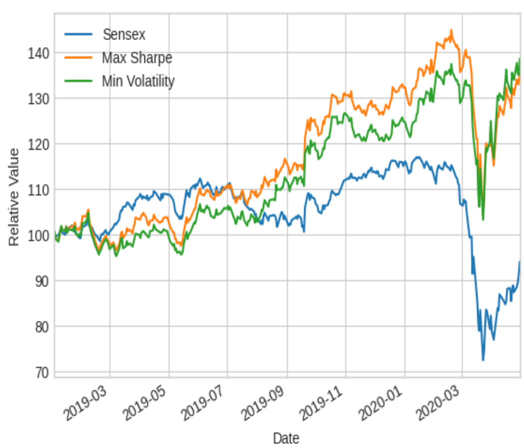

Fig. 8. Relative value of optimal portfolio obtained by Markowitz Model compared to benchmark market index

Ticker	Max Sharpe	Min Volatility
ASIANPAINT	25.82%	10.34%
AXISBANK	3.05%	0.0%
BAJAJ-AUTO	0.0%	6.87%
BAJFINANCE	6.99%	0.26%
BHARTIARTL	0.0%	3.78%
HCLTECH	0.67%	0.0%
HDFC	8.12%	1.36%
HDFCBANK	4.34%	0.0%
HEROMOTOCO	0.0%	6.45%
HINDUNILVR	0.0%	8.03%
ICICIBANK	0.0%	0.0%
INDUSINDBK	0.0%	0.0%
INFY	14.56%	25.73%
ITC	4.18%	0.0%
KOTAKBANK	3.91%	0.0%
LT	0.0%	0.0%
M&M	4.78%	0.0%
MARUTI	0.0%	1.04%
NESTLEIND	9.68%	13.49%
NTPC	0.0%	4.79%
ONGC	1.9%	0.37%
POWERGRID	0.0%	9.92%
RELIANCE	0.0%	0.0%
SBIN	1.21%	0.0%
SUNPHARMA	8.08%	4.92%
TATASTEEL	0.0%	0.0%
TCS	2.73%	0.04%
TECHM	0.0%	0.0%
TITAN	0.0%	0.02%
ULTRACEMCO	0.0%	2.58%

Fig. 9. Table consisting of the composition of optimal portfolio obtained by Markowitz Theory

IV. Approach III: Black-Litterman Model

A. Why this Model?

While the Markowitz model has its merits, it also has some drawbacks. For example, sometimes, the Markowitz model outputs the optimized portfolio which consists of a high concentration of some assets and many others are allocated zero concentration. Moreover, a single change in the input can cause a huge change in the output since the model is very input sensitive. Thus, the model can also be said to be unpredictable or unintuitive in this sense. Moreover, due to the high input dependence, some extreme values are also prone to estimation error maximisation. The Black-Litterman model is hence used by investors who want to avoid these drawbacks.

B. Introducing the Model

The Black-Litterman model was developed by Fischer Black and Robert Litterman and it aims to use a Bayesian approach. The aim of this approach is to incorporate investor views associated with asset returns in a prior return estimate. This in turn generates a mixed return of asset returns. The model also has the capability for investors to input their confidence level because of which subjective views of investors are also taken into consideration. Thus, the Black Litterman model aims to return more intuitive and sensible portfolio weights.

We discussed the drawback of the Markowitz model being highly input sensitive. To tackle this issue, researchers have explored alternatives such as using equal 'mean' returns for all assets and risk-adjusted equal mean returns among others, but this does not solve the problem of outputting extreme portfolio allocations. [11]

C. Implied Equilibrium return Vector

The Black Litterman model instead approaches this problem by using a neutral starting point of equilibrium returns. Equilibrium returns are the returns that just clear the market. [12] We will use reverse optimization methods involving:

$$\begin{aligned} T &= number\ of\ assets \\ w_{mkt} &= market\ capitalization\ weights\ of\ stocks \\ \Sigma &= sample\ covariance\ matrix \\ \lambda &= risk\ aversion\ coefficient \end{aligned}$$

The risk-aversion factor is a scaling factor that characterizes the rate at which an investor will forego expected return for less variance.

$$\begin{aligned}
\lambda &= \frac{E(R_{portfolio}) - r_f}{\sigma^2} \\
E(R_{portfolio}) &= expected\ benchmark\ returns \\
E(r_{rf}) &= risk\ free\ rate\ of\ investment \\
\sigma^2 &= sample\ variance\ of\ benchmark\ returns
\end{aligned}$$

Now we compute π, the implied equilibrium return vector:

$$\Pi = \lambda\, \Sigma\, w_{mkt}$$

D. Investor Views

The Black Litterman model also has the novel ability to take into consideration investor views along with prior estimates. This gives investors the ability to influence the implied excess return vector π with their personal subjective views.

- Absolute views [eg. asset x will provide a return of 7% (with confidence = 30%)]
- Relative views [eg. asset y will outperform asset z by 2% (with confidence = 60%)]

The model includes these investor inputs by using a views matrix Q of dimensions $k \times 1$:

$$\begin{aligned}
k &= total\ number\ of\ investor\ views \\
Q + \varepsilon &= \begin{bmatrix} Q_1 \\ Q_2 \\ \vdots \\ Q_k \end{bmatrix}
\end{aligned}$$

The uncertainity due to the subjective inputs in the views matrix Q gives rise to a random, normally-distributed Error Term Matrix ε with:

$$\begin{aligned}
E\left[\varepsilon\right] &= 0 \\
\Omega &= covariance\ matrix
\end{aligned}$$

All error terms are non-zero quantities, unless the views are entered with 100% confidence. Instead of the error terms ε the variance of each error term ω is directly involved in the Black-Litterman formula through the covariance matrix Ω. The off-diagonal elements of $k \times k$ dimensional matrix Ω are zero, to enforce the assumption that all views are independent of one another while the variance terms express the uncertainty of the views.

$$\begin{aligned}
Q + \varepsilon &= \begin{bmatrix} Q_1 \\ Q_2 \\ \vdots \\ Q_k \end{bmatrix} + \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_k \end{bmatrix} \\
\Omega &= \begin{bmatrix} \omega_1 & 0 & \dots & 0 \\ 0 & \omega_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \omega_k \end{bmatrix}
\end{aligned}$$

Now we link the value-based views and certainty in each one with the respective asset returns, so that a mixed estimate can be computed. A matrix P helps us out with this. With T assets under consideration and k views, each view contributes to a $1 \times T$ row in the matrix P of dimension $k \times T$.

$$P = \begin{bmatrix} p_{1,1} & p_{1,2} & \cdots & p_{1,T} \\ p_{2,1} & p_{2,2} & \cdots & p_{2,T} \\ \vdots & \vdots & \ddots & \vdots \\ p_{k,1} & p_{k,2} & \cdots & p_{k,T} \end{bmatrix}$$

There are different methods of populating P such as those based on assign percentages to assets , an equal weightage system, etc. In this article, we avoid using different methods for populating P (such as those based on assign percentages to assets , an equal weightage system etc [13]), in favor of one based on the market capitalization of assets [11]. Accordingly, the relative weightage of an asset in a view is proportional to the market capitalization of the particular asset with respect to the aggregate market capitalization of assets falling in the same category (i.e total market cap of either out-performing or under-performing assets). After populating P,

$$\begin{aligned}
& the\ variance\ of\ a\ particular\ view \\
&= p_i\, \Sigma\, p_i^T \\
p_i &= 1 \times T\ size\ matrix\ which\ is \\
& \quad part\ of\ the\ i^{th}\ subjective\ investor\ view
\end{aligned}$$

E. Posterior Expected Returns

$$E[R] = \left[(\tau\Sigma)^{-1} + P^T\Omega^{-1}P\right]^{-1} \left[(\tau\Sigma)^{-1}\Pi + P^T\Omega^{-1}Q\right]$$

Except the scalar τ, we have all the quantities essential to calculate the posterior expected returns $E[R]$. We can probably say that τ should be inversely proportional to the relative weight given to π, however there is lack of information about the estimating τ across Black-Litterman literature. There have been many methods of estimating it [14], but for the purpose of this study we assume $\tau = 0.05$, which is a conventional value.

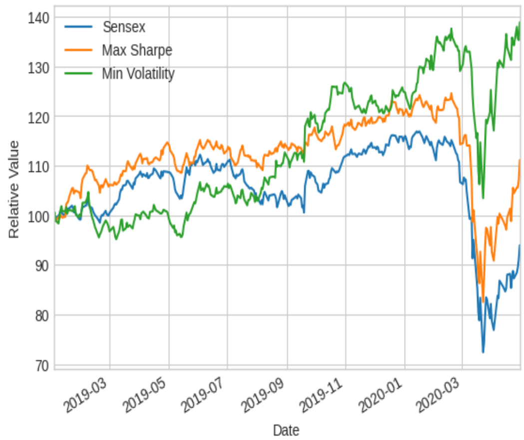

Fig. 10. Relative value of optimal portfolios obtained by Black-Litterman Model compared to benchmark market index

Ticker	Max Sharpe	Min Volatility
ASIANPAINT	0.0%	10.37%
AXISBANK	0.0%	0.0%
BAJAJ-AUTO	0.0%	6.76%
BAJFINANCE	0.0%	0.32%
BHARTIARTL	0.0%	3.78%
HCLTECH	2.17%	0.0%
HDFC	0.0%	1.36%
HDFCBANK	12.9%	0.0%
HEROMOTOCO	0.0%	6.43%
HINDUNILVR	3.62%	8.0%
ICICIBANK	0.0%	0.0%
INDUSINDBK	0.0%	0.0%
INFY	37.65%	25.82%
ITC	6.12%	0.04%
KOTAKBANK	4.49%	0.0%
LT	0.0%	0.0%
M&M	0.0%	0.0%
MARUTI	0.0%	0.86%
NESTLEIND	0.0%	13.58%
NTPC	0.0%	4.58%
ONGC	0.01%	0.45%
POWERGRID	0.0%	9.91%
RELIANCE	16.85%	0.0%
SBIN	0.0%	0.0%
SUNPHARMA	0.96%	4.98%
TATASTEEL	0.0%	0.0%
TCS	7.62%	0.1%
TECHM	6.25%	0.0%
TITAN	1.34%	0.08%
ULTRACEMCO	0.0%	2.58%

Fig. 11. Composition of portfolio obtained by applying Black-Littermann Model

V. Analysis and Conclusion

In this project, we have implemented and analysed the working of the three famous models of portfolio optimisation by coding [15]. We get the most efficient composition of the portfolio as the result of the analysis. The table given below summarises the performance of all these optimal portfolios by stating the Compound Annual Growth Rate (CAGR) and Maximum Drawdown (MDD) of all portfolios obtained, over the investment period.

After collecting historical data of a time period of 5 financial years, we conclude that even with a huge economic downfall(during COVID-19 pandemic) an investor would gain over the total investment period of one year or so. We can also consider that the negative Sensex returns against net positive

Portfolio	CAGR	MDD
Sensex (Benchmark)	-4.584%	38.070%
Monte Carlo - Max Sharpe	14.851%	30.402%
Monte Carlo - Min Volatility	9.308%	29.645%
Markowitz - Max Sharpe	25.735%	28.137%
Markowitz - Min Volatility	27.692%	24.863%
Black Litterman - Max Sharpe	8.224%	33.775%
Black Litterman - Min Volatility	27.975%	24.831%

Fig. 12. CAGR and MDD of all the optimised portfolios and benchmark market index

portfolio returns points to the success of the techniques in diversifying risk across the different options of stocks to save huge lose during harsh situations like the pandemic. Another significant observation is that all portfolios undergo a fall as the market collapses under pandemic conditions but

a) the risk management techniques

b) the culmination of the profit during normal market cycles helps the portfolio optimisation to turn out to be profitable over the significant period of time, after some huge losses.

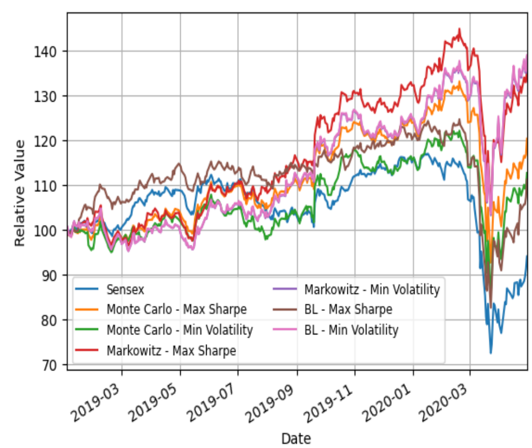

Fig. 13. performance comparison of the optimal portfolios obtained from all the model with benchmark market index

Acknowledgment

I would sincerely like to express my gratitude to Prof. Jaideep Mulherkar for his support and guidance throughout this project.

References

[1] "https://www.investopedia.com/terms/p/portfolio.asp."
[2] S. Raychaudhuri, "Introduction to monte carlo simulation," in *2008 Winter Simulation Conference*, 2008, pp. 91–100.
[3] D. P. Kroese, T. J. Brereton, T. Taimre, and Z. I. Botev, "Why the monte carlo method is so important today," *Wiley Interdisciplinary Reviews: Computational Statistics*, vol. 6, pp. 386–392, 2014.
[4] H. Ghodrati and Z. Zahiri, "A monte carlo simulation technique to determine the optimal portfolio," *Management Science Letters*, vol. 4, pp. 465–474, 03 2014.

[5] H. Wang, *Monte Carlo simulation with applications to finance*. CRC Press, 2012.
[6] H. Markowitz, "Portfolio selection," *The Journal of Finance*, vol. 7, no. 1, pp. 77–91, 1952.
[7] O. Ledoit and M. Wolf, "Improved estimation of the covariance matrix of stock returns with an application to portfolio selection," *Journal of empirical finance*, vol. 10, no. 5, pp. 603–621, 2003.
[8] ——, "Honey, i shrunk the sample covariance matrix," *The Journal of Portfolio Management*, vol. 30, no. 4, pp. 110–119, 2004.
[9] W. F. Sharpe, "The sharpe ratio," *Journal of portfolio management*, vol. 21, no. 1, pp. 49–58, 1994.
[10] G. Cornuejols and R. Tütüncü, *Optimization methods in finance*. Cambridge University Press, 2006, vol. 5.
[11] T. Idzorek, "A step-by-step guide to the black-litterman model: Incorporating user-specified confidence levels," in *Forecasting expected returns in the financial markets*. Elsevier, 2007, pp. 17–38.
[12] M. J. Best and R. R. Grauer, "On the sensitivity of mean-variance-efficient portfolios to changes in asset means: some analytical and computational results," *The review of financial studies*, vol. 4, no. 2, pp. 315–342, 1991.
[13] S. Satchell and A. Scowcroft, "A demystification of the black–litterman model: Managing quantitative and traditional portfolio construction," *Journal of Asset Management*, vol. 1, no. 2, pp. 138–150, 2000.
[14] C. Walters *et al.*, "The factor tau in the black-litterman model," *Available at SSRN 1701467*, 2013.
[15] "https://github.com/jmulherkar/btp2020/tree/master/gauravsofat$_jayatipatel.''$