Group details

Group 10

Prathav Kevadiya - 202003020
Preet Mevawalla - 202003025
Prayag Patel - 202003048

Quantum Optimization Algorithm
CS405 Quantum Computation

May 20, 2023

Dhirubhai Ambani Institute of Information and Communication Technology

Quantum Approximate Optimization Algorithm(QAOA)

QAOA is a variational algorithm that uses unitary $U(\beta, \gamma)$ where (β, γ) are the parameters to prepare the quantum state $|\psi(\beta, \gamma)\rangle$. This algorithm aims to find $(\beta_{opti}, \gamma_{opti})$ such that the quantum state $\left|\psi\left(\boldsymbol{\beta}_{opt}, \gamma_{opt}\right)\right\rangle$ encodes the solution to the problem.

Max cut Problem

Problem Statement: A Max-Cut problem involves partitioning nodes of a graph into two sets, such that the number of edges between the sets is maximum. The example below has a graph with four nodes, and some of how it can be partitioned into two sets, "red" and "green" is shown.

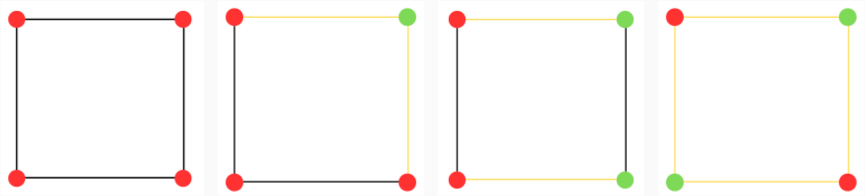


Figure 1: number of edges between two sets in the figure, as we go from left to right, are 0, 2, 2, and 4

Quantum Approximate Optimization Algorithm(QAOA)

In QAOA, the following two gates are used to evolve the system's quantum state and optimize the objective function:

1. Problem unitary
2. Mixer unitary

Preparing the Problem Unitary

Problem Unitary: The problem unitary is a quantum gate that encodes the objective function of the optimization problem being solved(such as the Max-Cut problem). It is given by:

$$U(\gamma) = e^{-i\gamma H_P}$$

Here,H_P is the problem Hamiltonian.

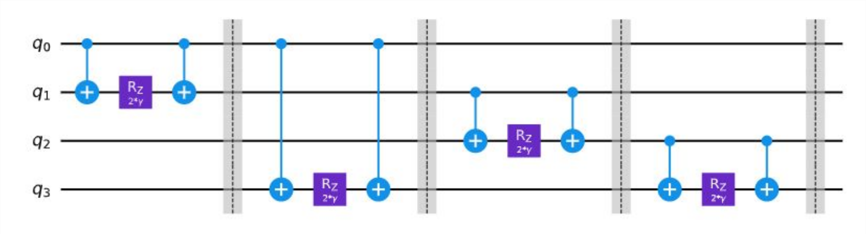
$$where, H_P = \frac{1}{2} \sum_{(i,j)\in E} Z_i \otimes Z_j$$

Problem Hamiltonian is a Hermitian matrix that represents the objective function in terms of Pauli-Z operators, i.e, for the edge between node 0 and node 1: $Z_0 \otimes Z_1 \otimes I_2 \otimes I_3$

Preparing the Problem Unitary

The effect of the problem unitary is to create a superposition of all possible solutions to the optimization problem, By applying the problem unitary multiple times with different values of γ.

$$U(\gamma) = e^{-i\gamma H_P} = e^{-i\gamma(Z_0Z_1+Z_0Z_3+Z_1Z_2+Z_2Z_3)}$$
$$= e^{-i\gamma Z_0Z_1}e^{-i\gamma Z_0Z_3}e^{-i\gamma Z_1Z_2}e^{-i\gamma Z_2Z_3}$$



Preparing the Mixer Unitary

Mixer Unitary: The mixer unitary is a quantum gate that "mixes" the quantum state of the system and helps break any symmetries in the problem Hamiltonian.

$$U(\beta) = e^{-i\beta H_B}$$

Here H_B is the mixer Hamiltonian.

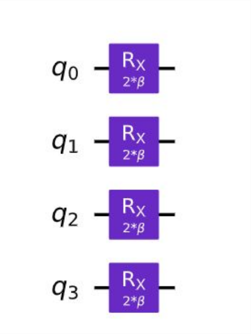
We can take mixer Hamiltonian as summation of tensor product between Pauli X gate of node and Identities of other nodes, for all nodes.

Preparing the Mixer Unitary

For example, for the node 0 $(X_0 \otimes I_1 \otimes I_2 \otimes I_3)$.

$$U(\beta) = e^{-i\beta H_B} = e^{-i\beta(X_0+X_1+X_2+X_3)} = e^{-i\beta X_0}e^{-i\beta X_1}e^{-i\beta X_2}e^{-i\beta X_3}$$

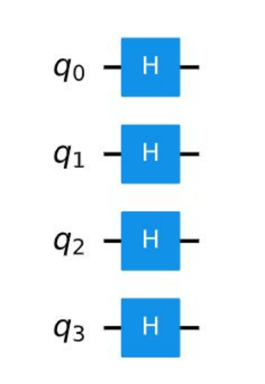
The effect of the mixer unitary is to create a superposition of all possible states that differ by a single bit flip (i.e., a single qubit being flipped from 0 to 1 or vice versa)



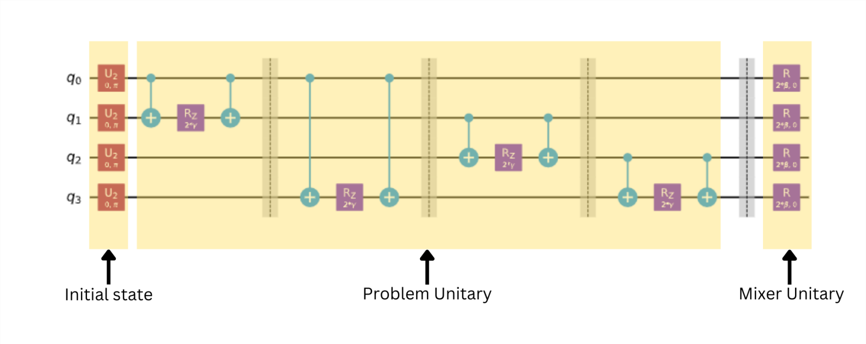
Initial state

The initial state used during QAOA is usually an equal superposition of all the basis states i.e.

$$|\psi_0\rangle = \left(\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)\right)^{\otimes n}$$



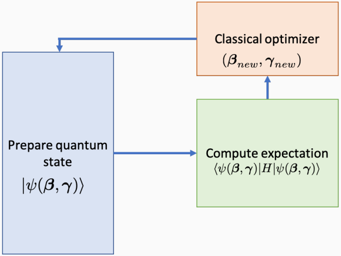
The QAOA circuit

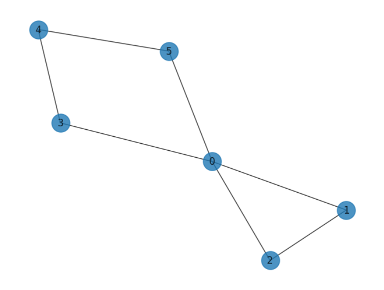


The next step is to find the optimal parameters $(\beta_{opti}, \gamma_{opti})$ such that the expectation value is minimum.

Algorithm

1. Initialize β and γ to suitable real values.
2. Repeat until some suitable convergence criteria is met:
   1. Prepare the state $|\psi(\beta,\gamma)\rangle$ using the QAOA circuit.
   2. Measure the state in standard basis.
   3. Compute $\langle\psi(\beta,\gamma)|H_P|\psi(\beta,\gamma)\rangle$.
   4. Find new set of parameters $(\beta_{new}, \gamma_{new})$ using a classical optimization algorithm.
   5. Set current parameters (β,γ) equal to the new parameters $(\beta_{new}, \gamma_{new})$.





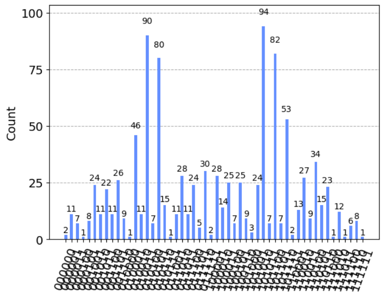


Figure 2: A graph with 6 nodes and its corresponding max-cut bit string

References

[1] Wolfe, Cameron R. "The Quantum Approximate Optimization Algorithm from the Ground Up." Medium, 16 July 2021, wolfecameron.medium.com/the-quantum-approximate-optimization-algorithm-from-the-ground-up-ba6e643b061d.

[2] "Qaoa" QAOA - Qiskit 0.43.0 Documentation, qiskit.org/documentation/stubs/qiskit.algorithms.minimumeigensolvers.QAOA.html. Accessed 14 May 2023.

Thank you!

Group 10
Prathav Kevadiya - 202003020
Preet Mevawalla - 202003025
Prayag Patel - 202003048