CS405 - Quantum Computation

Quantum Random Walks

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Abstract

In recent years, quantum random walks (QRWs) have emerged as a powerful quantum computation and simulation tool. QRWs are a quantum mechanical analog of classical random walks, where a particle moves on a graph according to a set of rules. Unlike classical random walks, QRWs can exhibit quantum interference effects, leading to exponential speedups in certain computational tasks.

One interesting application of QRWs is their implementation on graphs with specific topologies, such as the c8 cyclic graph. This graph has a high degree of symmetry and can be used to model various physical systems, such as molecular vibrations and electronic transport.

In this project, we have tried to study and understand the concept of quantum random walks and their implementation on the C8 cyclic graph on AWS Braket. We have tried to simulate the circuit for a large number of steps and study the effects of noise on the results.

Contents

Chapter 1

Quantum random walk

1.1 Introduction

Quantum Random Walks are quantum counterparts of classical random walks. While classical random walks involve the walker occupying definite states and the randomness is a result of the stochastic transition between these states, quantum random walks involve randomness arising through three distinct mechanisms:

- quantum superposition of states
- non-random, reversible unitary evolution
- collapse of the wave function resulting from state measurements

These features provide scope for a richer and more complex range of behaviors and are the foundation for many applications in quantum information science.

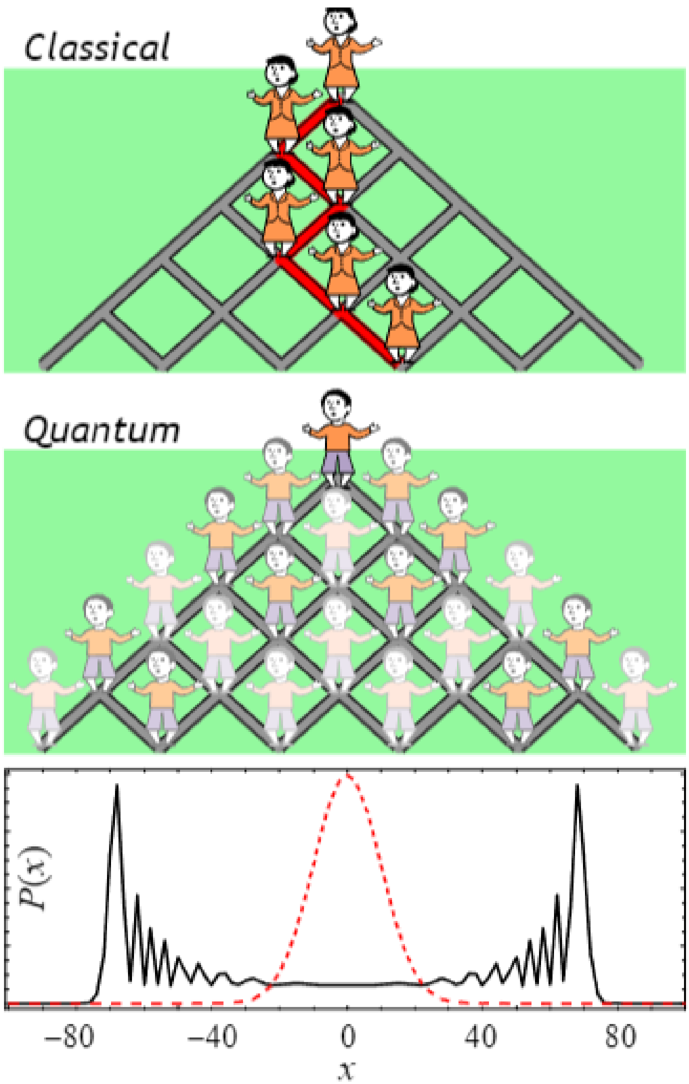

Figure 1.1: A comparison of quantum vs. classical random walk. A classical random walk leads to a Gaussian distribution (dotted) while its quantum counterpart produces a solid distribution which expands very rapidly.

1.2 Types of Quantum Random Walk

The key idea behind QRW is to use a unitary operator to evolve a quantum state, which represents the walker's position on a graph, in a way that leads to interesting and potentially useful behaviors. The unitary operator typically consists of a coin operation, which rotates the particle's internal degrees of freedom or "coin state", followed by a conditional shift operation, which moves the particle to its neighboring vertices on the graph depending on the outcome of the coin operation.

Now, there are several different types of QRW:

- **Coined Quantum Random Walk:** In a coined QRW, the particle moving on the graph has an internal degree of freedom or "coin" that is used to determine the probability amplitude of the particle moving to its neighboring vertices. The evolution of the coin and the particle's position on the graph are determined by a sequence of coin operations and conditional shifts.
- **Continuous time Quantum Walks:** In a continuous-time QRW, the evolution of the particle's state is governed by a Hamiltonian that describes the energy of the system. The Hamiltonian is typically defined in terms of the adjacency matrix of the graph, and the evolution of the particle's state is described by the Schrödinger equation.
- **Discrete time quantum walks:** In a discrete-time QRW, the evolution of the particle's state is determined by a unitary operator that acts on the particle's state after each discrete time step. The

unitary operator typically consists of a coin operation followed by a conditional shift on the graph.

- **Inhomogeneous Quantum Walks:** In an inhomogeneous QRW, the coin and the shift operations that govern the particle's motion on the graph vary randomly from step to step. These types of walks can exhibit interesting statistical properties and have been used to model quantum search algorithms and other quantum computing tasks.

These are just a few examples of the different types of QRWs that have been studied in the literature, wherein, each type of QRW has its own unique properties and applications.

Chapter 2

Coined Quantum Random Walks

2.1 What are CQRW's?

Coined quantum random walks are a type of quantum walk that incorporate an internal "coin" degree of freedom for the walker, which can be thought of as a quantum analog of a classical coin flip. The coin is used to determine the probability amplitudes for the walker to move to its neighboring vertices on the graph.

2.2 Key Idea

The key idea behind coined quantum random walks is to use the internal coin state to control the walker's behavior in a way that generates interesting and potentially useful patterns of movement on the graph. Over here, the coin can be thought of as a quantum analog of a classical coin flip, with two or more possible outcomes that determine the probabilities for the walker to move to its neighboring vertices on the graph.

The unitary operator that evolves the walker's quantum state consists of two parts: the coin operation, which acts on the coin state, and the conditional shift operation, which moves the walker to its neighboring vertices depending on the outcome of the coin operation.

By choosing different types of coin operations, such as the Hadamard coin, one can generate different types of quantum random walks that exhibit different behaviors, such as enhanced localization or faster spreading on the graph.
For example, the Hadamard coin produces a symmetric quantum walk that spreads out rapidly and exhibits strong localization.

Overall, the key idea behind coined quantum random walks is to use the internal coin state to control the walker's movement in a way that generates complex, non-classical behaviors that can be harnessed for applications in quantum computing, cryptography, and more.

2.3 C8 Cyclic Graph

The C8 cyclic graph, also known as the cycle graph with eight vertices, is a simple graph consisting of eight vertices arranged in a circular fashion such that each vertex is adjacent to its two neighbors. The C8 graph is an example of a regular graph, meaning that all vertices have the same degree or number of neighbors.

The C8 graph is a common testbed for studying quantum walks because it exhibits some interesting quantum behavior. For example, when a quantum walk is performed on the C8 graph using the Hadamard coin, the walker's probability distribution becomes highly localized around certain vertices after a few steps, as opposed to the classical random walk which exhibits a uniform distribution over all vertices.

2.4 C8 Cyclic Graph and Quantum Random Walks

The C8 cyclic graph has several interesting properties that make it a useful testbed for studying quantum random walks:

- The C8 graph is a regular graph, meaning that all vertices have the same degree. In the case of the C8 graph, each vertex has degree two, since it is connected to two adjacent vertices.
  This regularity simplifies the analysis of quantum random walks on the graph, since the walker's behavior is more uniform and symmetric.
- The C8 graph is a small and simple graph, making it computationally tractable to simulate and study on a quantum computer.
  This simplicity allows us to gain valuable insights into the behavior of quantum walks on more complex graphs and networks.
- The C8 graph exhibits non-classical behavior in quantum walks.
  For example, when a quantum walk is performed on the C8 graph using the Hadamard coin, the walker's probability distribution becomes highly localized around certain vertices after a few steps, in contrast to the classical random walk which exhibits a uniform distribution over all vertices.
  This localization behavior has applications in quantum search and

optimization algorithms.

- The C8 graph can be used as a building block for constructing more complex graphs and networks. For example, the C8 graph can be connected to other graphs to form a larger, more complex graph. Studying quantum walks on such composite graphs can provide insights into the behavior of quantum walks on more realistic and practical networks.

Overall, the C8 cyclic graph is a useful tool for studying quantum random walks due to its regularity, simplicity, non-classical behavior, and its potential for use in constructing more complex graphs and networks.

2.5 Coined QRW on C8 Cyclic Graph

In a coined quantum random walk on the C8 graph, the coin operation and the conditional shift operation are applied successively in each time step. The coin operation acts on the internal coin state and determines the probabilities for the walker to move to its neighboring vertices. The conditional shift operation then moves the walker to the appropriate neighboring vertex based on the outcome of the coin operation.

Different types of coin operations can be used in a coined quantum random walk on the C8 graph, each of which can produce different types of behavior. For example, the Hadamard coin produces a symmetric walk that exhibits strong localization, while the Grover coin produces a more biased walk that can be used for search algorithms.

Coined quantum random walks on the C8 cyclic graph have been studied extensively in quantum computing and quantum information science. They have applications in developing quantum algorithms for search, optimization, and other problems. They also provide insights into the behavior of quantum walks on more complex graphs and networks.

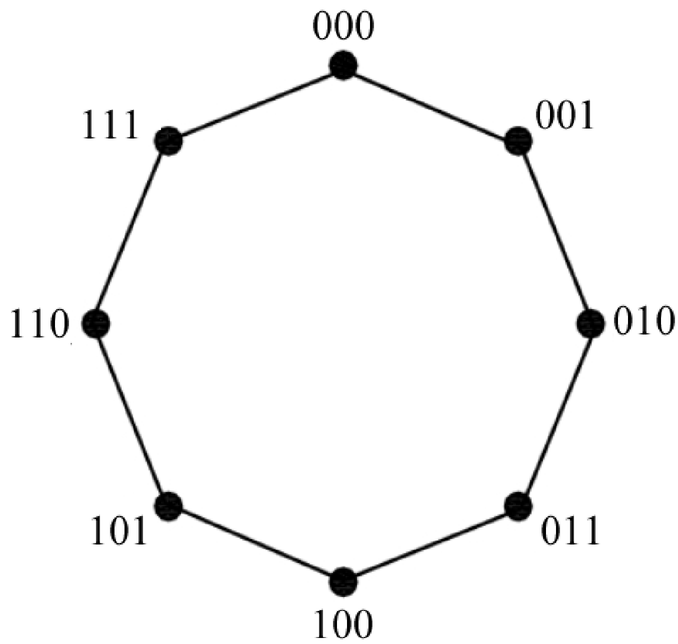

Figure 2.1: C8 Cyclic Graph

Chapter 3

Circuit Implementation and Analysis

3.1 Circuit Componenets

Hadamard Gate

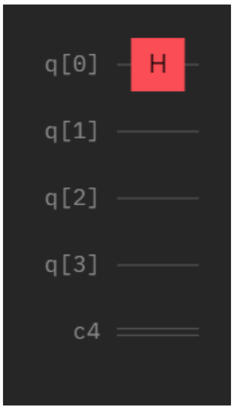

Figure 3.1: Hadamard Gate

Hadamard gate acts as the coin. It induces a superposition into the first qubit(the control bit) q[0].

Incremental Circuit

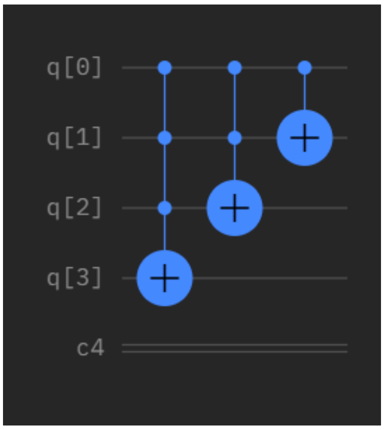

Figure 3.2: The incremental circuit

There can be two possibilities:

Case 1: if the control bit q[0] = 1; the qubits q[1], q[2], q[3] will get incremented.

Case 2: if the control bit q[0] = 0; the qubits will remain unchanged.

Decremental Circuit

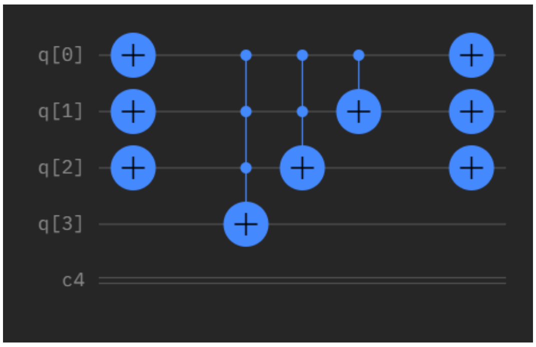

Figure 3.3: Decremental Circuit

There can be two possibilities:

Case 1: if the control bit q[0] = 1; the qubits q[1], q[2], q[3] will get decremented.

Case 2: if the control bit q[0] = 0; the qubits will remain unchanged.

3.2 Circuit Analysis

Using the above explaination of the circuit components and this circuit:

- Hadamard Gate will induce a superposition into the control bit q[0].
- In a coined quantum walk, the final output state is a superposition of the walker's position states, where each position state is incremented or decremented depending on the coin state.

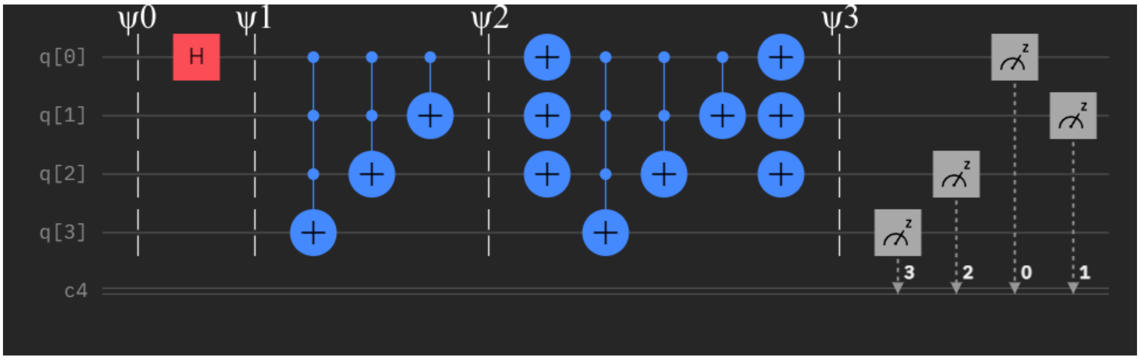

Figure 3.4: Circuit

Input: $|0011\rangle$

Node: $|110\rangle = 6$

$\psi_0 = |0\rangle \otimes |0\rangle \otimes |1\rangle \otimes |1\rangle$

$\psi_1 = |\frac{0+1}{\sqrt{2}}\rangle \otimes |0\rangle \otimes |1\rangle \otimes |1\rangle = \frac{|0\rangle\otimes|0\rangle\otimes|1\rangle\otimes|1\rangle}{\sqrt{2}} + \frac{|1\rangle\otimes|0\rangle\otimes|1\rangle\otimes|1\rangle}{\sqrt{2}}$

$\psi_2 = \frac{|0\rangle\otimes|0\rangle\otimes|1\rangle\otimes|1\rangle}{\sqrt{2}} + \frac{|1\rangle\otimes|1\rangle\otimes|1\rangle\otimes|1\rangle}{\sqrt{2}}$

$\psi_3 = \frac{|0\rangle\otimes|1\rangle\otimes|0\rangle\otimes|1\rangle}{\sqrt{2}} + \frac{|1\rangle\otimes|1\rangle\otimes|1\rangle\otimes|1\rangle}{\sqrt{2}}$

Output nodes are: $|101\rangle = 5$ and $|111\rangle = 7$

Bibliography

https://quantum-computing.ibm.com/composer/files/new

https://arxiv.org/pdf/0811.1795.pdf

Code Link: https://rb.gy/b7fgl