Quantum Computation: Application in Finance

Group 2

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Core Problems in Finance

- **Risk and Volatility**: Finance deals with the uncertainty of future behavior and prices of assets. Risk quantifies the possibility of a difference between actual and expected returns. Volatility measures the degree of variation in trading price over time.
- **Optimal Portfolio**: An optimal portfolio maximizes return for a given risk, and minimizes risk for a given return. The challenge is constructing and modifying this portfolio based on market conditions.
- **Randomness**: Incomplete market knowledge means that assets and portfolios are intrinsically random systems, and this randomness is a source of risk that can be challenging to estimate.
- **Options**: Options are a type of derivative security whose payoffs depend on other assets. The problem of determining an option's value requires numerical simulation methods, such as Monte Carlo.

Why to Use Quantum Computation in finance?

- **Speed**: Quantum computing offers significant speedups over classical computing, which can be especially useful for solving complex optimization problems, such as portfolio optimization, risk management, and option pricing.
- **Scalability**: As the size of financial data continues to grow, classical computing becomes less efficient, while quantum computing can potentially offer scalable solutions to manage and process large amounts of data.
- **Risk management**: Quantum computing can be used to develop models for risk management, allowing financial institutions to better assess and manage risk exposure in real-time.
- **Financial modeling**: Quantum computing can help develop more accurate financial models, allowing for more precise predictions of future asset prices and returns, and enabling better decision-making by financial institutions.

Approaches to Problems in Quantitative Finance

Problem	Approach to solve	
Which assets should be included in an optimum portfolio, and how should one change its composition according to the market.	Optimization Models	Quantum Optimization
How to detect opportunities in the different assets in the market, and take profit by trading them?	Machine Learning	Quantum Machine Learning
How to estimate the risk of a portfolio, a company, or even the whole financial system?	Monte Carlo	Quantum Amplitude Estimation

What is Portfolio Optimization?

1. The concept of portfolio optimization essentially involves return maximization while ensuring minimization of risk with the given set of securities across or within different asset classes.
2. A method to realize the goal of portfolio optimization mentioned above was provided by **Modern Portfolio Theory**.

Modern Portfolio Theory

- **Modern Portfolio Theory(MPT)** is to maximise expected return for an amount of risk. It formalises and broadens the concept of diversification in investing, which holds that having a variety of financial assets reduces risk compared to holding a single type. Its main conclusion is that an asset's risk and return should not be evaluated on its own, but rather in the context of the risk and return of the entire portfolio.It uses the variance of asset prices as a proxy for risk
  1. Risk and expected return.
  2. Effective frontier with no risk-free asset

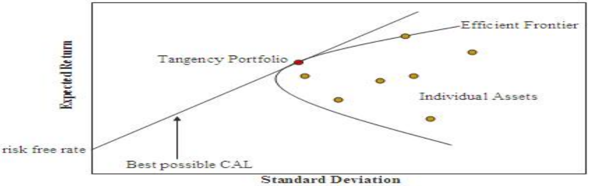
Modern Portfolio Theory

Risk and expected return: MPT assumes that investors are risk averse, meaning that given two portfolios that offer the same expected return, investors will prefer the less risky one. Thus, an investor will take on increased risk only if compensated by higher expected returns. Conversely, an investor who wants higher expected returns must accept more risk. The exact trade-off will not be the same for all investors. Different investors will evaluate the trade-off differently based on individual risk aversion characteristics.

- In General
  1. Expected return: $E(R_p) = \sum_i \omega_i E(R_i)$
  2. Portfolio return variance: $\sigma_p^2 = \sum_i \omega_i^2 \sigma_i^2 + \sum_i \sum_{j \neq i} \omega_i \omega_j \sigma_i \sigma_j \rho_{ij}$
  3. Portfolio return volatility: $\sigma_p = \sqrt{\sigma_p^2}$

Modern Portfolio Theory

Effective frontier with no risk-free asset: The image represents the risk-expected return space, where portfolios of risky assets can be plotted based on their expected return and risk (measured by standard deviation). The efficient frontier, which is the upper part of the hyperbolic boundary, represents portfolios with the lowest risk for a given level of expected return. The Capital Allocation Line (CAL) is a straight line that is tangent to the efficient frontier and represents portfolios that can be constructed by combining a risk-free asset and a portfolio of risky assets.Matrices are preferred for calculations of the efficient frontier.In matrix form, for a given "risk tolerance" $q \in [0, \infty)$, the efficient frontier is found by minimizing the following expression: $\omega^T \Sigma \omega - q \times R^T \omega$



Objectives

The main objectives of Portfolio Optimization are as follows:

1. Maximizing ROI
2. Managing risk appetite

Assumptions

The Modern Portfolio Theory, which forms the basis of the concept of portfolio optimisation, assumes the following assumptions:

1. Investors are rational: MPT assumes that investors are rational and make decisions based on expected returns and risks. Investors are assumed to prefer portfolios that offer higher expected returns for a given level of risk.
2. Investors are risk-averse: MPT assumes that investors are risk-averse, which means that they prefer less risky portfolios. Investors are assumed to be more concerned about downside risk than upside potential.
3. Friction-less Markets: One assumption made in portfolio theory is that markets are devoid of any obstacles, such as transaction costs, limitations, or other restraints.
4. Normal Distribution: MPT assumes that the returns of assets in the portfolio are normally distributed, which implies that the mean and variance of the returns can fully capture the behavior of the assets.

General Approach for Portfolio Optimization

Calculate the annualized daily rate of return for each asset using the acquired daily price data of assets

→

Calculate the annualized co-variances between the daily rate of return values of all assets. [1]

→

Formulate optimization objective and constraints as QUBO/ISING models

→

Run optimization solvers (for example: VQE)

[1] The variance of the portfolio is referred to as portfolio risk and the standard deviation of the portfolio is referred to as portfolio volatility.

Modeling Portfolio Optimization Problem I

Step 1 Getting the optimization problem using MPT as follows:
$\max_{\omega i} \quad E(R_p) - \frac{\gamma}{2} Var(R_p)$
s.t. $\sum_{i=1}^{n} \omega_i = 1$ and $\omega_i \geq 0, \forall i = 1, \ldots, n$

Step 2 Mapping to QUBO/Ising Models: The MVO problem can be mapped to a QUBO or Ising Hamiltonian using the following steps:

- Define binary variables x_i that represent whether asset i is included in the portfolio or not.
- Define the expected return and covariance matrix for each asset in the portfolio.
- Define the objective function that represents the expected return and variance of the portfolio as a function of the binary variables.
- Use the QUBO or Ising formulation to map the objective function to a Hamiltonian.

Modeling Portfolio Optimization Problem II

The QUBO formulation is:

$$H = \sum_{i=1}^{n} E(R_i)x_i - \gamma(\sum_{i=1}^{n}\sum_{j=1}^{n} C_{ij}x_ix_j - (\sum_{i=1}^{n}\omega_i)^2$$

Step 3 Solving the QUBO Model with VQE Algorithm:

- Encode the QUBO or Ising Hamiltonian as a quantum circuit.
- Choose a parameterized ansatz circuit that represents a trial solution.
- Define a cost function that measures the expectation value of the Hamiltonian.
- Use a classical optimization algorithm to find the optimal parameters that minimize the cost function.
- Extract the optimal solution from the optimized parameters.

Modeling Portfolio Optimization Problem III

The VQE algorithm iteratively adjusts the parameters of the ansatz circuit using classical optimization techniques and evaluates the cost function on a quantum computer until a satisfactory solution is found.

Results I

We ran code for the Portfolio Optimization on data of four assets' generated randomly over a time period of 30 years, whose time-series graph can be visualised as:

Figure: Time Series Plot of Assets

Results II

Setting budget equal to 2, we get the optimized portfolio as follows:

```
fval=-0.00023285626449450188, x_0=1.0, x_1=0.0, x_2=1.0, x_3=0.0, status=SUCCESS
```

Figure: Output

The above output represents that the person should invest in x_0 i.e Stock0 and x_2 i.e Stock2 to obtain maximum return with minimum risk.

Why Quantum for Portfolio Optimization? I

Quantum computing can revolutionize portfolio optimization by solving currently intractable optimization problems. Here are a few reasons why it's advantageous:

- Portfolio optimization is a classic optimization problem that involves selecting a combination of assets that maximizes expected returns while minimizing risk.
- Classical optimization algorithms have limitations in solving portfolio optimization problems, especially when dealing with a large number of assets, and they may also get stuck in sub-optimal solutions.
- Quantum computing can provide a more efficient way to solve portfolio optimization problems using quantum algorithms such as quantum annealing or quantum-inspired algorithms like Variational Quantum Eigensolver (VQE).

Why Quantum for Portfolio Optimization? II

- Quantum algorithms can solve complex optimization problems much faster than classical algorithms, thereby enabling investors to generate better portfolios and react to market changes in real-time.
- Quantum optimization algorithms also have the potential to incorporate complex constraints such as transaction costs, liquidity requirements, and other regulatory or financial constraints.
- Moreover, quantum computing could lead to the development of new financial instruments and investment strategies that exploit quantum computing's ability to efficiently solve optimization problems.
- Quantum optimization algorithms can also facilitate faster and more accurate risk analysis, allowing investors to better understand the risks associated with different investment options.

Why Quantum for Portfolio Optimization? III

- Quantum computing shows significant promise in solving optimization problems for portfolio optimization, and as it continues to evolve, we can expect to see more sophisticated applications in finance and investment management.

Thank You!

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Link to Code