Quantum Error Correction

Guided by: Prof. Jaideep Mulherkar
DA-IICT

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Name	Student ID
Meghaben Rathwa	202001128
Isha Shah	202001163
Bhavaina Kallakuri	202003046

Outline

Introduction

Classical - Redundancy (Repetition of codes)

Quantum - Not possible to create copies (due to no cloning theorem).

But it is possible to spread one qubit through highly entangled state.

Peter Shor first discovered this method of formulating a quantum error correcting code by storing the information of one qubit onto a highly entangled state of nine qubits.

Introduction

Quantum error correction protects information by encoding it across multiple physical qubits to form a “logical qubit,” and is believed to be the only way to produce a large-scale quantum computer with error rates low enough for useful calculations.Instead of computing on the individual qubits themselves, we will then compute on logical qubits. By encoding larger numbers of physical qubits on our quantum processor into one logical qubit, we hope to reduce the error rates to enable useful quantum algorithms.

Bits

Ancilla bits

- Ancilla bits are some extra bits being used to achieve some specific goals in computation (e.g. reversible computation).
- Such bits, whose values are known a priori, are known as ancilla bits in a quantum or reversible computing task.

Logical bits

- Qubits that obey these assumptions are often known as logical qubits.

Assumption:

They can be prepared in any state we desire, and be manipulated with complete precision

Physical bits

- Qubits which are not logical are called physical qubits.

Need of Quantum Error Correction

- In a quantum computer, the information is stored in a fragile quantum state that is highly sensitive to errors. Even small disturbances can cause the quantum state to collapse, leading to incorrect results. As the number of qubits in a quantum computer increases, so does the likelihood of errors occurring.
- Quantum error correction is therefore essential for quantum computing to be viable. It involves encoding the quantum information in a redundant way, such that errors can be detected and corrected without disturbing the quantum state itself. This allows quantum computers to perform reliable computations even in the presence of noise and other sources of errors.

Need of Quantum Error Correction

- Quantum error correction is necessary because quantum systems are inherently susceptible to errors due to various factors such as environmental noise, imperfect control of quantum operations, and hardware imperfections. Unlike classical bits, which can be copied and verified without changing their value, quantum bits (qubits) cannot be measured without disturbing their state.
- Quantum error correction is also important for quantum communication, which relies on the transmission of quantum states over long distances. Without error correction, the delicate quantum states can be easily disturbed by environmental factors such as electromagnetic radiation, making reliable communication impossible.

Quantum error correction is necessary to ensure the **reliability and scalability** of quantum computing and communication, and it is an active area of research in the field of quantum information science.

Classical Error Correction

- In the framework of quantum bits (qubits), classical error correction refers to techniques used to detect and correct faults that occur while storing and performing operations on the quantum information.
- Classical error correction techniques are used to protect classical information associated with or generated from qubits. These are the extension of classical error correction techniques which have been modified for quantum information.
- Number of errors is assumed to be countably many in the case of quantum.
- Repetition technique is one of the most frequently used classical error correction techniques. Here, the same information is repeated and a redundancy is created, creating multiple qubits with the same information, so that the qubits which are added can be used to check and correct the errors (if present).

Classical Error Correction

- If there is an error, this can also be used to detect the location of the error (not always accurate). This can't be applied directly for the quantum information as according to the no-cloning theorem, repeated quantum states are not possible, but can be applied by modifying this technique.
- Below is an example of repetition code,
  Example
  We are transmitting $X \in \{0, 1\}$ on a noisy channel and there's a possibility of occurrence of bit-flip error i.e., 0 can be measured as 1. Three bit repetition code can be used here (same state is sent thrice). 0 is transmitted as 000 and 1 is transmitted as 111. Let, the probability that a bit will be flipped is 1. So, the probability of obtaining the correct answer (no error) is $(1 - p)^3$.

Classical Error Correction

- Let, the received bits be $b_2b_1b_0$. XOR operations are done on the bits obtained and the error is corrected or can just look at the states and the one which is most repeated is the required state.
- Two bits are created, $b_2 \oplus b_1$ and $b_2 \oplus b_0$, which are known as **syndrome**. Error detection and correction can be done based on the following table,

$b_2 \oplus b_1$	$b_2 \oplus b_0$	Correction
0	0	No Error
0	1	Flip b_0
1	0	Flip b_1
1	1	Flip b_2

Limitations of Classical Error Correction

- But this technique can correct errors up to 1 bit. If more than 1 bit is changed, it is highly probable that the wrong bit will be updated.
- One of the limitations of classical error correction is that it can correct only a limited number of errors and fails in most cases where there are multiple errors.
- It is important to note that all errors which are possible in quantum computation cannot be addressed by classical error correction techniques.
- More complex and accurate measurements and codes are required to detect and correct errors which are specific states. However, these techniques are very useful in ensuring the stability and accuracy of quantum information, and are the backbone of Quantum error correction codes (QECC).

Quantum Error Correction

QEC techniques are designed to detect and correct errors by encoding the information redundantly in a larger space of qubits, so that errors can be detected and corrected without losing the information.

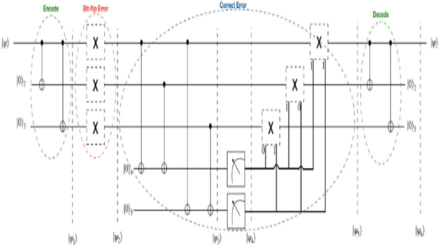
There are several QEC techniques, but the most common ones are:

QEC Techniques

- The Repetition Code
- The Shor Code
- The Surface Code

The Repetition Code

In this technique, a single qubit is encoded by repeating it multiple times. **For example**, a logical qubit can be encoded as three physical qubits, with state 0 represented as 000 and state 1 represented as 111. If one of the physical qubits flips due to an error, the other two can be used to detect and correct the error.
In repetition code we can do QEC with Bit-Flip and Phase-flip code.
Bit-Flip Code:
Example: The Bit-flip error in three bit code Algorithm



Bit-Flip Code

- Step - 1: Take the Qubit We want to transfer to be in the state.
  $|\psi >= \alpha|0> + \beta|1>$
- Step-2: we encode this qubit in a three-qubit entangled state
  $|\psi 1 >= \alpha|000> + \beta|111>$
- Step-3: Then three qubits are transmitted via the unreliable communication channel and retrieved on the receiving end.
  Any one of the three qubits could have experienced a bit-flip error during the communication, resulting in one of the following states.
  $|\psi 2 >= \alpha|000> + \beta|111>$
  $|\psi 2 >= \alpha|100> + \beta|011>$
  $|\psi 2 >= \alpha|010> + \beta|101>$
  $|\psi 2 >= \alpha|001> + \beta|110>$

Bit-flip Code

- Step-4: In order to correct the state received at the other end of the quantum communication channel from possible errors, two more ancilla qubits in the state $|0>$ are added to the system.
  $|\psi 3>= \alpha|000>|00>+\beta|111>|00>$
  $|\psi 3>= \alpha|001>|01>+\beta|110>|01>$
  $|\psi 3>= \alpha|010>|10>+\beta|101>|10>$
  $|\psi 3>= \alpha|100>|11>+\beta|011>|11>$
- Step-5: The newly added two ancilla qubits are then measured. They provide an "error syndrome" that may be used to identify and then fix any errors that may have affected the three qubits. The error syndrome can have four possible outcomes:

Bit-flip Code

- 00 : $|\psi 4 >= \alpha|000 > +\beta|111 >$
  01 : $|\psi 4 >= \alpha|001 > +\beta|110 >$
  10 : $|\psi 4 >= \alpha|010 > +\beta|101 >$
  11 : $|\psi 4 >= \alpha|100 > +\beta|011 >$
- Step-6: Three qubits can be corrected by applying the corresponding gate.

Corresponding gate pattern

00: No correction is needed
01: Apply X gate to the third qubit
10: Apply X gate to the second qubit
11: Apply X gate to the first qubit

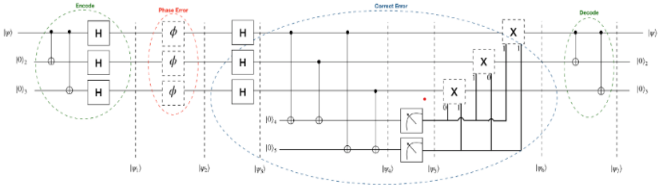
- After which the state of the system will be:
  $|\psi 5 >= \alpha|000 > +\beta|111 >$

Bit-flip Code

- Step-7: we decode it from the three-qubit state by disentangling the qubit with the two ancillas.
  $|\psi 6 >= \alpha|0 > +\beta|1 >= |\psi >$
- We have successfully communicated the state of the qubit $|\psi >$ through a noisy channel, protecting it from any single bit-flip error.

Phase-flip Code

- The phase flip code works the same way as the bit flip code.
- It transfers the state of the main qubit to the secondary qubit using CNOT gates.
- All qubits are brought into superposition with the help of a Hadamard gate. Then, a phase inversion error occurs on the main qubit, affecting its phase.



Phase-flip Code

- The Hadamard gates are again applied to all qubits, which brings them out of superposition, since two applied Hadamard gates leave the state of the qubits unchanged. After the phase of the main qubit is changed, it is no longer in its previous state.
- Since this has changed the computational state of the qubit, we can correct this with CNOT gates and a Toffoli gate, where the main qubit is the target and the control qubits are the auxiliary qubits.

The Shor's Code

This is a more sophisticated technique in which one logical qubit is encoded in nine physical qubits. The Shor code uses a combination of two-qubit gates and one-qubit twists to protect the logical qubit from errors.

- Shor's code is used when there is both bit-flip and phase flip errors
- Bit-flip is one bases may be represented by phase flip in other bases.
- In Shor's code we use 9-qubits to send information over quantum channel

Encoding

- $|0{>}as|0{>} = | + + + >$
- $|1{>}as|1{>} = | - - - >$

- In encoding phase , we encode $|0>$ state as $| + + + >$ and $|1>$ state as $| - - - >$

The Shor's Code

Error

- When this information is passed through a faulty channel , it introduces bit and phase both flip errors randomly.

Error Detection and Correction

- In this phase , we detect errors using measurements.
- For bit flip errors we check parities using,
- $m1 = z1z2, m2 = z1z3, m3 = z4z5, m4 = z4z6, m5 = z7z8, m6 = z7z9$
- m1 , m2 represents parity check of first block. If m4 $=$ 1 then flip 1st qubit. If m2 $=$ 1 then flip second qubit and if both m1 , m2 are 1 then flip first qubit.
- Similarly we can correct bit flip errors using X-gate.

The Shor's Code

- For phase flip errors , we check
  $m7 = x1x2x3x4x5x6$
  $m8 = x4x5x6x7x8x9$
- m7 is parity check of blocks 1,2 and m8 is parity checks of block 2,3.

Qubits which are affected

If m7$= +1$ then we conclude that error is in first 3 qubits , if m8 $= +1$ then error is in last three qubits . If both are $+1$ then error is in middle three qubits.

- We can correct this error by applying Z-gate to the respective qubit from corresponding block.

Decoding

- Here we apply reverse operation of encoding and get back the state that was sent over the channel.
- After the entire process we can check that if sent and received state is matching or not i.e. error is detected and corrected or not.

The END !!

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Bhavajna Kallakuri	202003046

Thank You!