Portfolio Optimization

CS-405 Quantum Computation

Contents

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Introduction

Portfolio optimization is a technique used to construct investment portfolios that aim to maximize returns while minimizing risk. It involves analyzing historical data, expected returns, volatilities, and correlations of different assets to find the optimal asset allocation. The goal is to achieve the best trade-off between risk and reward. Common methods include Markowitz Mean-Variance Optimization, the Capital Asset Pricing Model (CAPM), and the Black-Litterman Model. Portfolio optimization helps investors make informed decisions to achieve their financial objectives.

Traditional Portfolio Optimization

The Markowitz mean-variance optimization framework maximizes returns for a given level of risk or minimizes risk for a given level of returns.

- It involves selecting a set of assets, estimating their returns and risks, and quantifying their correlations. By constructing an efficient frontier of portfolios, it identifies the optimal portfolio that balances risk and return based on an investor's preferences.
- This framework provides a systematic approach to portfolio construction, considering the trade-off between expected returns and risks.

However, accurate estimates of returns and risks are crucial, and it assumes that historical performance is indicative of future outcomes.

Modern Portfolio Theory (MPT)

Modern Portfolio Theory (MPT) is a framework developed by Harry Markowitz in 1952 which is based on the principle of diversification and aims to maximize expected returns while minimizing portfolio risk. Here's a short explanation of the algorithm of Modern Portfolio Theory:

- Define the Assets: Identify the set of available assets for investment, such as stocks, bonds, or other financial instruments.
- Gather Historical Data: Collect data for the returns and risks of each asset. This data typically includes expected returns, standard deviations (or variances), and correlation coefficients between pairs of assets.
- Calculate Asset Statistics: Compute key statistics for each asset, including the expected returns and the risk (variance or standard deviation).

Modern Portfolio Theory (MPT)

- Build the Efficient Frontier: The efficient frontier represents a set of optimal portfolios with varying risk levels. It is created by generating a range of portfolio allocations that combine different weights for each asset in the portfolio.
- Optimize the Portfolio: Using mathematical optimization techniques, such as the mean-variance optimization, find the optimal portfolio allocation that maximizes the expected return for a given level of risk or minimizes the risk for a given level of expected return.
- Assess the Portfolio: Evaluate the characteristics of the optimized portfolio, such as the expected return, risk, and asset weights.
- Monitor and Rebalance: Periodically review and adjust the portfolio allocation to maintain its desired risk-return characteristics. Rebalancing may be necessary to accommodate changes in asset prices, market conditions, or the investor's investment goals.

Quantum Optimization Problem

We need to minimize the given below mean-variance portfolio optimization problem for n assets:

$$\min_{x \in \{0,1\}^n} qx^T \sum x - \mu^T x$$

$$\text{subject to: } 1^T x = B$$

where we use the following notation:

- $x \in \{0,1\}^n$ denotes the binary decision variables, which indicates which variables to pick($x[i] = 1$) and which not to pick($x[i] = 0$),
- $\mu \in \mathbb{R}^n$ defines the expected returns of the assets,
- $\sum \in \mathbb{R}^{n \times n}$ specifies the covariances between the assets,
- $q \geq 0$ controls the risk appetite of the decision maker,
- **B** denotes the budget, i.e. no. of assets to be selected out of n

The algorithm for portfolio optimization

- Define the portfolio optimization problem, including the objective function, constraints, and any other relevant factors.
- Formulate the problem as a QUBO or Ising model, representing it as a matrix
- Instantiate a NumPyMinimumEigensolver object (exactmes) to find the minimum eigenvalue and eigenvector of the problem.
- Create a MinimumEigenOptimizer object (exact eigensolver) with exact mes as the input.

Continue...

- Solve the portfolio optimization problem by calling the solve() method of exact eigensolver, passing in the QuadraticProgram object (qp) representing the problem.
- Retrieve the results from the optimization process, which likely include the minimum eigenvalue, the corresponding eigenvector (optimized portfolio weights), and any other relevant information.
- Perform post-processing and analysis on the obtained results, such as mapping the eigenvector back to meaningful asset weights, evaluating the performance of the optimized portfolio, and making investment decisions based on the results.

Quantum Portfolio Optimization Techniques

Quantum optimization techniques leverage the principles of quantum mechanics to solve complex optimization problems. They exploit quantum phenomena such as superposition and entanglement to explore multiple potential solutions simultaneously.

There are majorly two methods which we will be using to minimize our loss.

- SamplingVQE (Sampling Variational Quantum Eigensolver)
- QAOA (Quantum Approximate Optimization Algorithm)

SamplingVQE

- The SamplingVQE is an algorithm that aims to find the minimum eigenvalue and corresponding eigenvector of a given problem. In the context of portfolio optimization, the problem is formulated as a Quadratic Unconstrained Binary Optimization (QUBO) or Ising model.
- The objective function of the portfolio optimization problem is typically represented as a quadratic function involving the asset weights. The goal is to find the minimum eigenvalue of the associated matrix, where the eigenvector corresponds to the optimized asset weights.

SamplingVQE utilizes numerical linear algebra techniques, specifically the NumPy library, to calculate the minimum eigenvalue and eigenvector of the problem's matrix representation.

Quantum Approximate Optimization Algorithm (QAOA)

QAOA leverages the principles of quantum computing to search for an optimal solution. It combines classical optimization techniques with the power of quantum superposition and entanglement to explore a larger solution space efficiently.

QAOA consists of:

- Variational Optimization: The parameters in the quantum circuit are optimized using classical optimization algorithms to minimize the cost function. This optimization process iteratively adjusts the parameters to find the optimal values that yield the lowest possible cost.

In the mathematical sense, QAOA aims to solve an optimization problem by mapping it onto a quantum system. For portfolio optimization, the problem is typically formulated as a Quadratic Unconstrained Binary Optimization (QUBO) problem or an Ising model.

QAOA algorithm

This algorithm can be summarized as follows:

- Formulate the portfolio optimization problem as a QUBO or Ising model. This involves defining the objective function, constraints, and binary decision variables representing the portfolio weights.
- Define the cost Hamiltonian: The cost Hamiltonian represents the objective function of the portfolio optimization problem. It is formulated as a sum of terms, with each term corresponding to a specific aspect of the problem, such as maximizing returns or minimizing risk.
- Prepare an initial quantum state: Construct the quantum circuit using a combination of mixing Hamiltonian and cost Hamiltonian gates. The mixing Hamiltonian gate prepares an initial quantum state, while the cost Hamiltonian gate encodes the objective function into the quantum circuit.

QAOA algorithm

- Optimize the parameters: Use classical optimization algorithms, such as COBYLA, to find the optimal values of the parameters in the quantum circuit. This is done by iteratively adjusting the parameters to minimize the cost function.
- Measure the quantum state: Measure the final state of the quantum circuit to obtain a solution. The measurement outcome provides an approximation of the optimal portfolio weights.
- Post-processing and analysis: Perform post-processing on the obtained results, such as mapping the binary variables to meaningful asset weights and evaluating the performance of the optimized portfolio.

By leveraging quantum properties such as superposition and entanglement, QAOA explores a larger solution space compared to classical optimization algorithms, potentially leading to better approximations of the optimal solution for combinatorial optimization problems like portfolio optimization.

Summary and Further Discussion

In conclusion, quantum portfolio optimization holds promise for improving traditional portfolio allocation techniques by leveraging quantum computing principles. However, it is still an evolving field facing challenges related to limited quantum computing power, qubit stability, and scalability. Further research and technological advancements are needed to overcome these limitations and realize the full potential of quantum portfolio optimization in practice.

Thank you!