Quantum Optimization Algorithm

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Quantum Approximate Optimization Algorithm(QAOA)

CODE LINK

QAOA is a variational algorithm that uses unitary $U(\beta)$ and $U(\gamma)$, where (β, γ) are the parameters to prepare the quantum state $|\psi(\beta, \gamma)\rangle$. This algorithm aims to find $(\beta_{opti}, \gamma_{opti})$ such that the quantum state $|\psi(\beta_{opti}, \gamma_{opti})\rangle$ encodes the solution to the problem.

$$|\psi(\beta, \gamma)\rangle = U(\beta)U(\gamma) \cdots U(\beta)U(\gamma)|\psi_0\rangle$$

Max cut Problem

Problem Statement: A Max-Cut problem involves partitioning nodes of a graph into two sets, such that the number of edges between the sets is maximum.

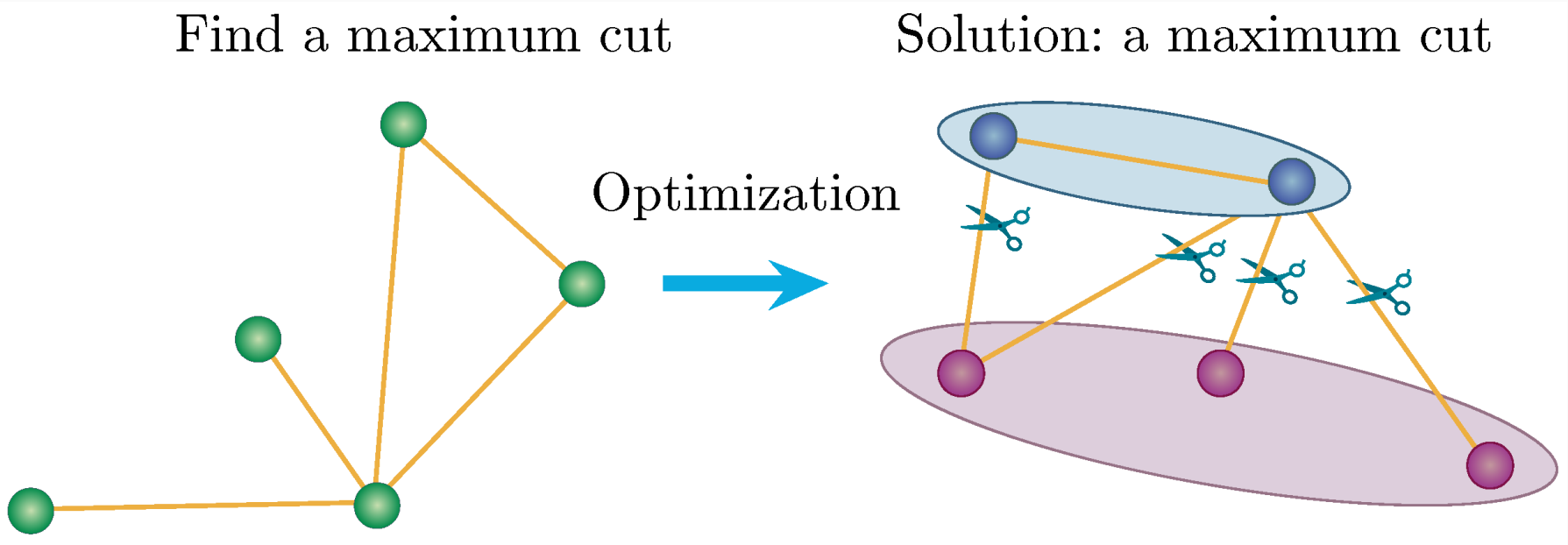


Figure 1: The maximum cut (Max-Cut) optimization problem.

Quantum Approximate Optimization Algorithm(QAOA)

QAOA, or Quantum Approximate Optimization Algorithm, employs two types of gates to evolve the quantum state of the system and optimize the objective function:

1. Problem unitary: This unitary gate encodes the problem into the quantum state by implementing a sequence of operations that represent the problem constraints.
2. Mixer unitary: This unitary gate is used to mix the quantum states generated by the problem unitary, allowing the algorithm to explore a wider range of states and potentially converge to a better solution. It is typically a simple, local operation that is easy to implement on quantum hardware.

Problem Unitary

Problem Unitary: The problem unitary is a quantum gate that represents the objective function of an optimization problem (e.g., the Max-Cut problem) in a quantum circuit. Mathematically, it can be expressed as:

$$U(\gamma) = e^{-i\gamma H_P}$$

where γ is a real-valued parameter and H_P is the Hamiltonian that describes the problem being solved.

Here, H_P is the problem Hamiltonian.

$$where, H_P = \frac{1}{2} \sum_{(i,j) \in E} Z_i \otimes Z_j$$

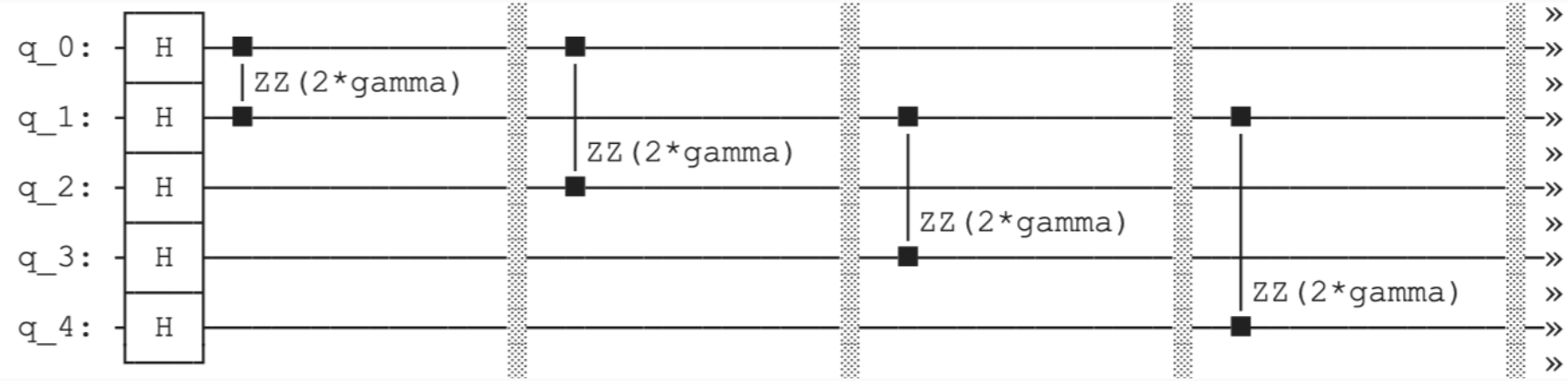
Problem Unitary

The problem Hamiltonian is a matrix that describes the objective function of an optimization problem using Pauli-Z operators. It is important to note that the problem Hamiltonian is Hermitian, which means that it is equal to its own conjugate transpose. This property is essential for the Hamiltonian to represent a physically meaningful observable in quantum mechanics.

Problem Unitary

When the problem unitary is applied to a quantum state, it creates a superposition of all possible solutions to the given optimization problem. This is achieved by repeatedly applying the problem unitary with different values of the parameter γ.

$$U(\gamma) = e^{-i\gamma H_P} = e^{-i\gamma(Z_0Z_1+Z_0Z_2+Z_1Z_3+Z_1Z_4)}$$
$$= e^{-i\gamma Z_0Z_1} e^{-i\gamma Z_0Z_2} e^{-i\gamma Z_1Z_3} e^{-i\gamma Z_1Z_4}$$



Mixer Unitary

Mixer Unitary: The mixer unitary is a quantum gate that is used to "mix" the quantum state of a system and disrupt any symmetries present in the Hamiltonian of the problem being studied.

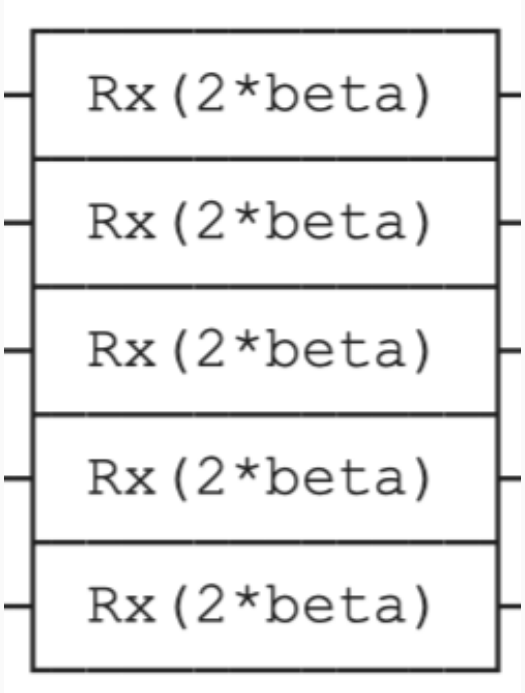
$$U(\beta) = e^{-i\beta H_B}$$

Mixer Hamiltonian is denoted by H_B.

This can be achieved by constructing a mixer Hamiltonian that consists of a sum of tensor products between the Pauli X gate of a particular node and identity operators of the remaining nodes, for all nodes in the system. By using this approach, the mixer unitary helps to introduce randomness and complexity into the system, making it easier to explore different possible solutions to the problem.

Mixer Unitary

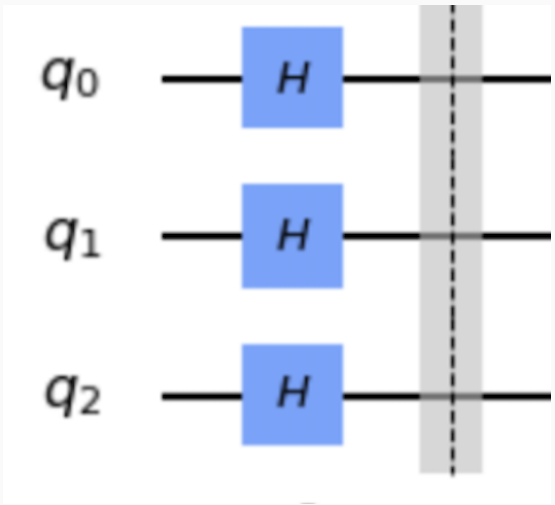
= The effect of the mixer unitary is to create a superposition of all possible states that differ by a single bit flip (i.e., a single qubit being flipped from 0 to 1 or vice versa)



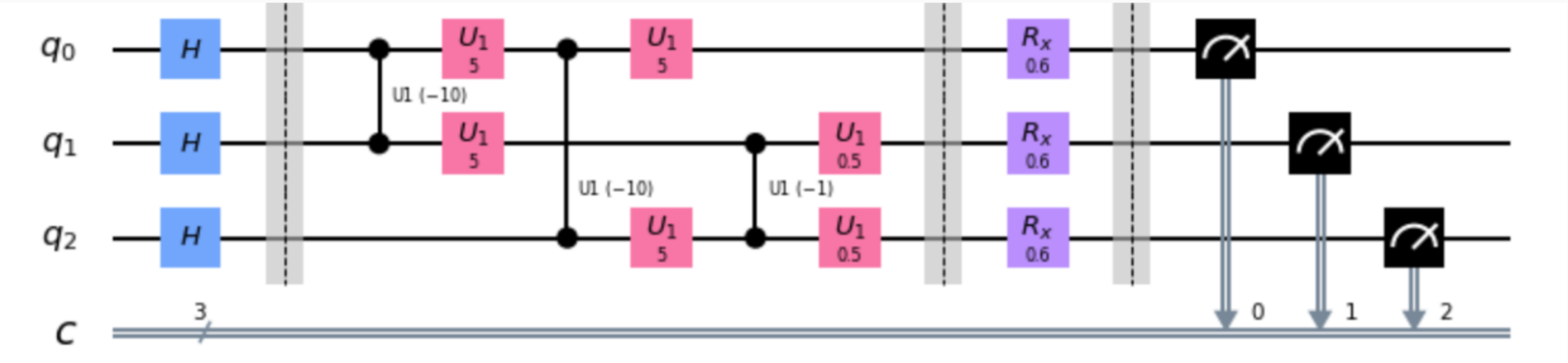
Initial state

The initial state used during QAOA is usually an equal superposition of all the basis states i.e.

$$|\psi_0\rangle = \left(\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)\right)^{\otimes n}$$



QAOA circuit



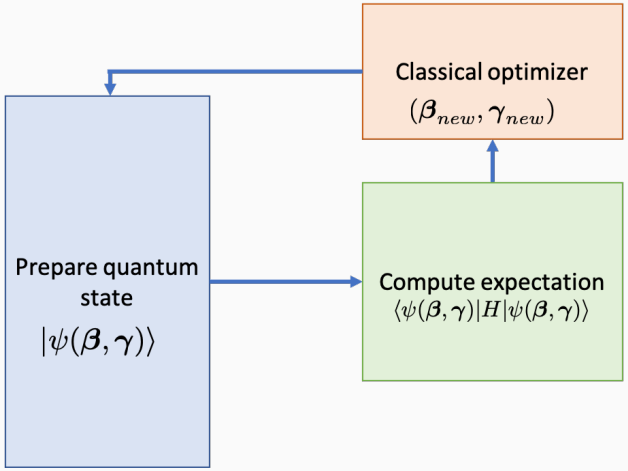
The next step is to find the optimal parameters $(\beta_{opti}, \gamma_{opti})$ such that the expectation value is minimum.

Algorithm

To run the Quantum Approximate Optimization Algorithm (QAOA), we first set initial real values for two parameters β and γ. We then repeat the following steps until a convergence criteria is met:

Using the QAOA circuit,

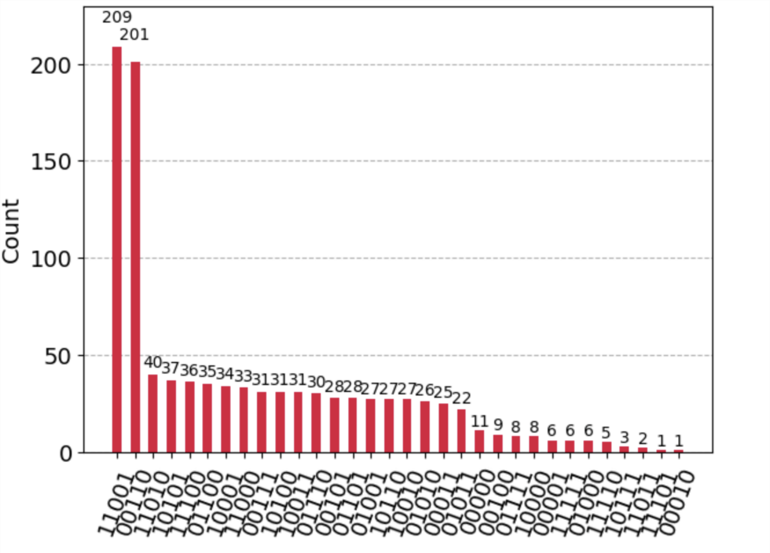
we prepare a quantum state $|\psi(\beta,\gamma)\rangle$. We measure the state in the standard basis.



1. We calculate the expectation value of the Hamiltonian H_P with respect to the state: $\langle\psi(\beta,\gamma)|H_P|\psi(\beta,\gamma)\rangle$.

2. We find a new set of parameters (βnew , γnew) using a classical optimization algorithm.

3. We update the current parameters (β,γ) to be equal to the new parameters (βnew , γnew).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References

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Thank you!