TSP Problem With Simulated Annealing

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What is the TSP problem?

- Traveling Salesman Problem - finding the shortest route for a salesman to visit cities once and return to the starting city.
- TSP is NP-hard problem As its sequential time complexity is exponential $O(n^2\ 2^n)$.
- The optimization is done using bitmask dynamic programming in sequential algorithm.
- But here we will discuss the Quantum approximation for this problem.

Quantum Annealing

- Quantum annealing is a type of quantum computing that focuses on optimization problems.
- It involves using a quantum device, such as a quantum annealer, to find the lowest energy state of a given optimization problem.
- The optimization problem is mapped onto a physical system, where the energy of the system corresponds to the objective function value of the problem.
- The quantum annealer slowly changes the quantum state of the system towards the solution, using a process known as annealing.

TSP using Quantum Annealing

- TSP is formulated as a quadratic unconstrained binary optimization (QUBO) problem in the QUBO formulation.
- The QUBO problem is mapped onto a quantum annealing device to find the lowest energy state of the corresponding Ising model.
- The quantum annealing device slowly cools the system from a high-energy state to a low-energy state using an annealing schedule.
- The success of quantum annealing in solving TSP depends on the quality of the QUBO formulation, the annealing schedule, and the performance of the quantum annealing device.

TSP using Quantum Annealing

Suppose we have N cities labeled as $1, 2, ..., N$. We can represent each possible route as a binary string x is a path matrix, where $x_{ij} = 1$ if the cities i and j are directly connected and $x_{ij} = 0$ otherwise.
To formulate TSP as a QUBO problem,
the objective function as:

$$L(x) = \frac{1}{2} \sum_{i} \sum_{j} d_{ij} x_{ij} \tag{1}$$

Note that the factor of $1/2$ is included to avoid double-counting the distances.

TSP using Quantum Annealing

Next, we impose constraints on the binary variables x. The conditions ensure that each city is visited exactly once:

$$\sum_{i}^{N} x_{ij} = 1 \forall j. \tag{2}$$

$$\sum_{j}^{N} x_{ij} = 1 \forall i. \tag{3}$$

TSP using Quantum Annealing

We can then write the QUBO problem as:

$$\text{minimize } L(x) = \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N} d_{ij}y_{ij}$$

subject to equations (3) and (4).

TSP using Quantum Annealing

This QUBO problem can be mapped onto a quantum annealing device by defining the corresponding Ising model. The Ising model is given by:

$$H = \sum_{i} h_i \sigma_{zi} + \sum_{i<j} J_{ij} \sigma_{zi} \sigma_{zj} \tag{4}$$

where σ_{zi} is the Pauli-Z matrix acting on the ith qubit (spin), h_i and J_{ij} are the biases and couplings, respectively, and we use the spin representation $\sigma_{zi} = 1$ if $x_i = 1$ and $\sigma_{zi} = -1$ if $x_i = 0$.

TSP using Quantum Annealing

To map the QUBO problem to the Ising model, we use the following mappings:

$$h_i = -\frac{1}{2}\sum_j d_{ij}, \tag{5}$$

$$J_{ij} = -\frac{1}{2}d_{ij}, \tag{6}$$

if $i < j$

$$J_{ij} = 0, \tag{7}$$

if$i = j$

TSP using Quantum Annealing

Finally, we apply a quantum annealing algorithm to the Ising model to find the ground state, which corresponds to the optimal solution of the TSP problem. The annealing schedule controls the rate at which the system is cooled from a high-energy initial state to a low-energy ground state. The annealing schedule is typically optimized empirically to balance the trade-off between speed and success probability.

Quantum Annealing pseudocode

1. Initialize the system in a high-energy state.
2. Choose an annealing schedule, which specifies how to change the Hamiltonian from the initial Hamiltonian to the final Hamiltonian over time.
3. Evolve the Hamiltonian according to the annealing schedule, using a unitary transformation.
4. Measure the final state of the system and record the energy.
5. Repeat steps 1-4 multiple times to sample from the probability distribution of the final states.
6. Select the state with the lowest energy as the solution.

Results

- Quantum annealing has shown promise in solving small to medium-sized TSP instances, but scalability and practicality for large-scale TSP instances are still an active area of research.
- Quantum annealing showed promising results for the TSP, with a 30% improvement in solution quality over classical algorithms for 50-city instances.
- Despite the promising theoretical advantages of quantum annealing, Experiments showed no significant improvement in solution quality or speed over classical algorithms for the TSP.

Applications

1. Logistics and Transportation: optimizing delivery routes for packages, goods, or services.
2. Network Design: optimizing the routing of data packets in a computer network.
3. Manufacturing: optimizing ordering and scheduling of production processes.
4. Genome Sequencing: determining the order of genes to reconstruct a full genome.

Reference

1. Quantum annealing of the Traveling Salesman Problem
2. Simulated annealing
3. Quantum annealing