Quantum Project

Quantum Walks And Quantum Walk Based Algorithms

Group Members

202003005-Manan Parikh,

202003006-Vraj Chaudhari,

202003031-Shreyansh Kunjera

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Outline

Introduction

A Random Walk is a walk on the infinite integer line. In this case, we represent the walker's position with an integer, $|j\rangle : j \in \mathbb{Z}$, since the walker can walk all integers in $\mathbb{Z}$. A coin decides how the walker should move. In classical case if coin tosses Head then walker move to right else left. And in Quantum Walk the coin's computational basis is $[|0\rangle, |1\rangle]$, we move the walker in one direction if the coin is $|0\rangle$ and in the other direction if the coin is $|1\rangle$.

Classical Random Walk

- In a classical random walk, an object starts at a specific point and takes steps in a random direction with equal probabilities for each direction.
- These steps can be represented by a series of random variables, where each variable determines the direction and magnitude of the step.
- The object's position is updated after each step, and this process continues for a certain number of steps or until a specific condition is met.
- In a one-dimensional random walk, where the object can only move forward or backward along a line, the expected value of the object's position after a large number of steps is zero.

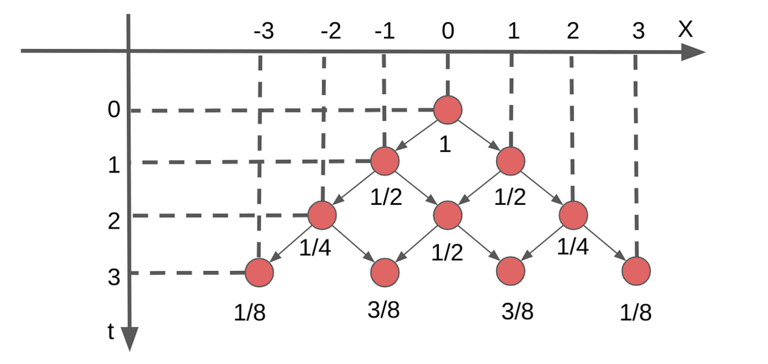
Formulation of Random Walk

- Suppose x denotes the position of the random walker and each step is of length L.

  $x_n = x_{n-1} + E_n L$ where E_n is +1 or -1 with probability 1/2
- Average position after n steps

  $< x_n - x_0 >= 0$

Probability In Classical Random Walk



Single Particle Random Walk

- Discrete-Time Quantum walk on a line for a single particle.
- The total Hilbert space in given by $H = H_p \otimes H_c$
- H_p represents the position space and H_c represents the coin state.
- H_p is spanned by orthonormal vectors $|i\rangle$ representing the position of the particle.
- H_c is the two-dimensional coin space spanned by two orthonormal vectors, denoted by $|\uparrow\rangle$ and $|\downarrow\rangle$.

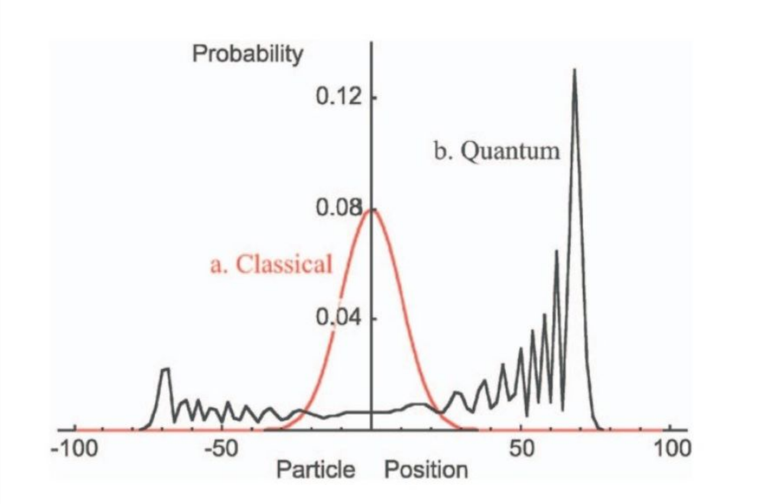
Single Particle Random Walk - 2

- Each step is based on the coin operator and shift operator
- $\hat{U} = \hat{S}(\hat{I_p} \otimes \hat{Uc})$ Here $\hat{Ip}$ is the identity operator on H_p.
- H_p is a Hadamard gate

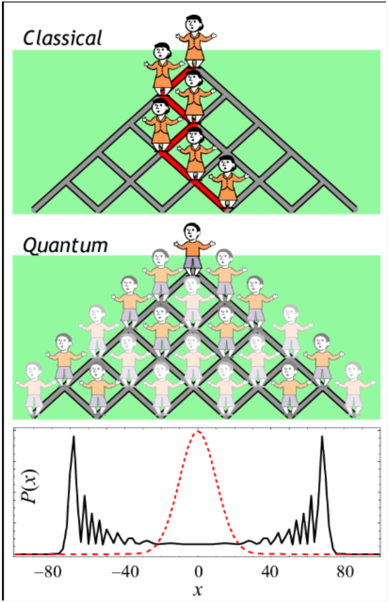
  Hadamard gate$=\frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$
- $$S |0\rangle |j\rangle = |0\rangle |j+1\rangle \tag{1}$$
- $$S |1\rangle |j\rangle = |1\rangle |j-1\rangle \tag{2}$$
- With the shift operator defined as above, we can represent one step of the coined quantum as the unitary operator U given by

$$U = SC, \tag{3}$$

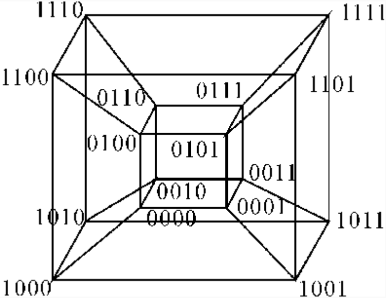
Probability Distribution Of Classical And Quantum Walk



Weird behaviour of quantum random walk



Quantum Random Walk On Hypercube



- The binary representation of the neighbors of a node will differ by only one binary number.
- A node is connected to all nodes to which the Hamming distance is 1. The edges are also labeled.

Quantum Random Walk On Hypercube -2

- The Hilbert space representing a coined quantum walk on the hypercube is $H = H^n \otimes H^{2^n}$, where H^n denotes the coin space and H^{2^n} the walker's position.
- The value of the coin computational basis , which is associated with edge , decides where the walker should move.
- Shift operator

$$S |a\rangle |\vec{v}\rangle = |a\rangle |\vec{v} \oplus \vec{e_a}\rangle . \tag{4}$$

- Let e_a be an n-tuple where all binary values, except the value with index a, are 0.

Quantum Random Walk On Hypercube - 3

- If coin is in state $|11\rangle$, we move the walker to direction where first node differs
- If coin is in state $|10\rangle$, we move the walker to direction where second node differs
- If coin is in state $|01\rangle$, we move the walker to direction where third qubit differs
- If coin is in state $|00\rangle$, we move the walker to direction where fourth qubit differs.

References

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- Research Gate https://www.researchgate.net/figure/A-4-dimensional-hypercube-interconnection-network_fig1_276037252
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Thank You