UNIVERSITEIT VAN AMSTERDAM

Eye-movement in Economic Games

Information search and decision rules in (non-)strategic choice

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Introduction

Experimental games

Game theory: The study of *strategic interactions* in situations where outcomes of *one's actions* are defined in respect to the *actions of others*

What it is amazing:

- Structural representation of social phenomena
- Complexity-friendly language
- Strong formal theory

Experimental games

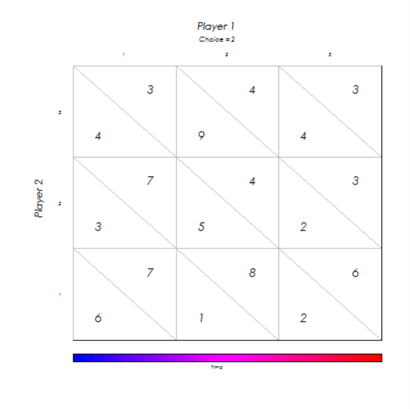
Experimental game theory: Relaxes the *rationality* assumption of game theory and introduces *empirical* knowledge to the models

What it is even more amazing:

- Behavioural game theory (Crawford, 2002)
- Empirically-friendly while opened to formalization
- Bridges disciplines (for example see Traulsen, Semmann, Sommerfeld, Krambeck and Milinski, 2010)

Matrix-form games

N × M games: Represent interactions between two players - one with N and the other with M available choices



Information search in matrix-form games

Polonio and Coricelli, 2019 tried to explain inconsistencies between choices and beliefs with information search

Focused on nine decision rules:

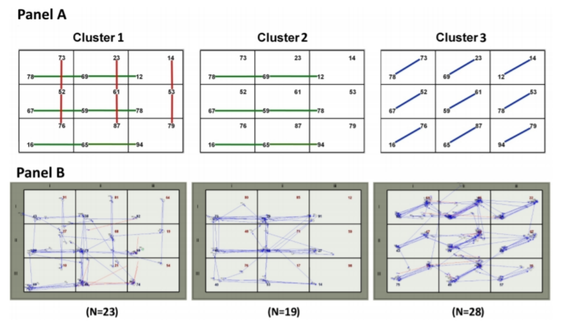
Non-strategic	Strategic
Optimist	K-1
Pessimist	K-2
Naive	D-1
Altruist	D-2
	Nash

Approach of Polonio and Coricelli, 2019

Look at transitions, movements between fixations, e.g. $A \to B$

Group transitions into five types by assumed function

Find clusters based on type counts across trials (games)



The plan

To do list

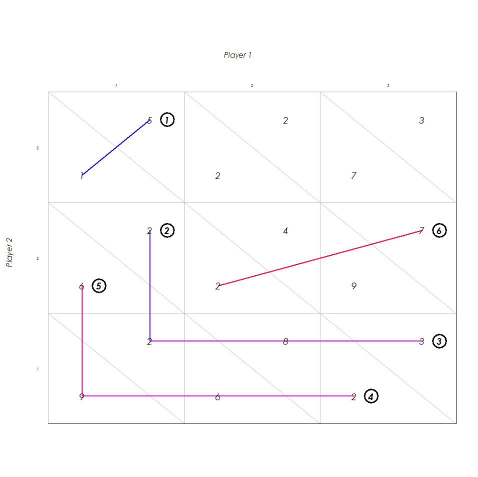
Our aim was to:

1. Revisit the *transition groups*
2. Use a more informative representation of data (*meta-transitions*)
3. Formulate likelihood functions for a model that could serve as a generative model
   3.1 Likelihoods of meta-transitions
   3.2 Error and chance meta-transitions
4. Allow the decision rules to vary over items (within individual)

Transition groups

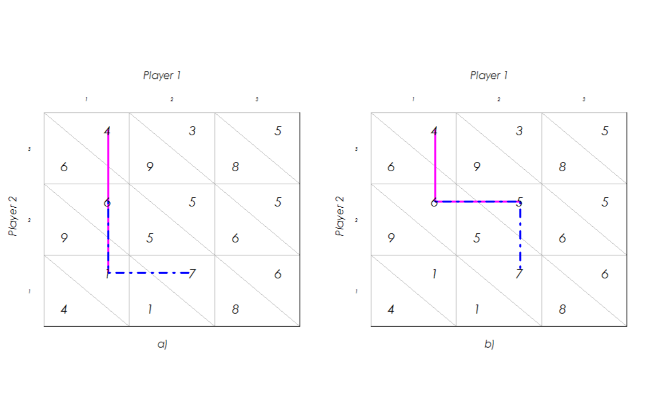
Transition types:

1. Intracell
2. Own-within
3. Own-between
4. Other-within
5. Other-between
6. **Skip**



Meta-transitions

Level-n meta-transition is a sequence of $n+1$ transitions. For instance, level-2 meta-transition is $A \to B \to C$



Meta-transitions

Level-2 meta-transition replace the six transition types with 6×6 meta-transition types, some of them impossible:

$$
\begin{array}{r|cccccc|}
\text{Intracell} & 1 & 1 & 1 & 1 & 1 & 1 \\
\text{OwnW} & 1 & 1 & 1 & 0 & 0 & 1 \\
\text{OwnB} & 1 & 1 & 1 & 0 & 0 & 1 \\
\text{OtherW} & 1 & 0 & 0 & 1 & 1 & 1 \\
\text{OtherB} & 1 & 0 & 0 & 1 & 1 & 1 \\
\text{Skip} & 1 & 1 & 1 & 1 & 1 & 1
\end{array}
$$

Likelihood function

The probability of observing particular frequencies of meta-transition types given a decision rule. We have used multinomial likelihood function.

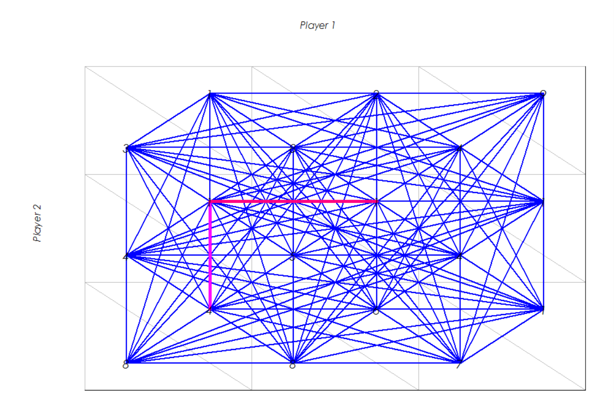
$$P(M|k,\varepsilon_k) = \prod_{a=1}^{I} P(T_a|k,\varepsilon_k)^{M_a} \quad (1)$$

$$P(T_a|k,\varepsilon_k)_t = (1-\varepsilon_k)P(T_a|k) + \varepsilon_k F_a \quad (2)$$

1. No mistake with probability $(1-\varepsilon_k)$ and a meta-transition with rule-implied probability.
2. Mistake with probability (ε_k) and meta-transition with chance probability F_a.

Chance meta-transitions

Chance probability of a meta-transition is a proportion of such meta-transition to all *n* long walks in a fully connected game network



Chance metatransition proportions can be computed for any level meta-transition.

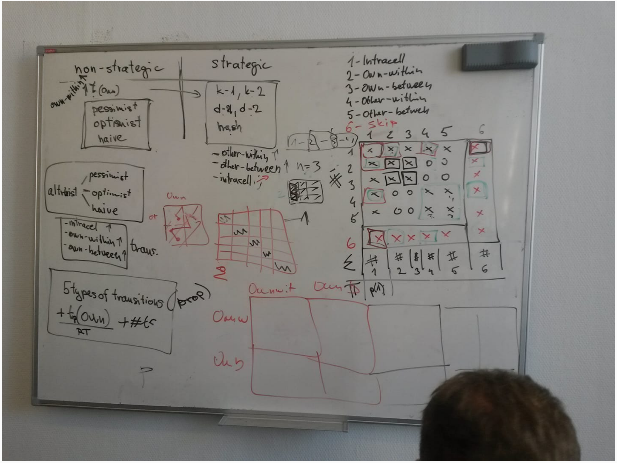
Chance meta-transitions

$$\tau_0 = \begin{pmatrix} C^2 & C^2 - C & C^4 - C^3 & C^2 - C & C^4 - C^3 & C^4 + C^3 - 3C^2 + C \end{pmatrix} \tag{3}$$

$$\tau_1 = \begin{array}{c} \textit{Intracell} \\ \textit{OwnW} \\ \textit{OwnB} \\ \textit{OtherW} \\ \textit{OtherB} \\ \textit{Skip} \end{array} \begin{pmatrix} 2 & C-1 & C^2-C & C-1 & C^2-C & 2C^2-2 \\ 1 & C-1 & C^2-C & 0 & 0 & C^2-1 \\ 1 & C-1 & C^2-C & 0 & 0 & C^2-1 \\ 1 & 0 & 0 & C-1 & C^2-C & C^2-1 \\ 1 & 0 & 0 & C-1 & C^2-C & C^2-1 \\ 2 & C-1 & C^2-C & C-1 & C^2-C & 2C^2-2 \end{pmatrix} \tag{4}$$

First model

Procedure


non-strategic
strategic
pessimist
optimist
naive
k-1, k-2
1 - Intracell
2 - Own-within
3 - Own-between
4 - Other-within
5 - Other-between
6 - skip
- other-within
- other-between
- intracell
n=3
altruist
pessimist
optimist
naive
- intracel
- own-within
- own-between
trans.
5 types of transitions

Meta-transition groups

Two classes: Non-strategic and strategic

1. A: only NS
2. B: NS and S
3. C: only S
4. D: neither

$$\Phi = \begin{array}{c} \mathit{Intracell} \\ \mathit{OwnW} \\ \mathit{OwnB} \\ \mathit{OtherW} \\ \mathit{OtherB} \\ \mathit{Skip} \end{array} \begin{pmatrix} A & B & B & B & B & A \\ B & B & B & 0 & 0 & C \\ B & B & B & 0 & 0 & C \\ B & 0 & 0 & C & C & C \\ B & 0 & 0 & C & C & C \\ A & C & C & C & C & D \end{pmatrix} \tag{5}$$

Likelihoods

$$P(A|k,\epsilon_k) = \epsilon_k f_a \tag{6}$$

$$P(B|k,\epsilon_k) = (1-\epsilon_k)\left(\frac{e^{\beta_0+\beta_1}}{1+e^{\beta_0+\beta_1}}\right) + \epsilon_k f_b \tag{7}$$

$$P(C|k,\epsilon_k) = (1-\epsilon_k)\left(\frac{1}{1+e^{\beta_0+\beta_1}}\right) + \epsilon_k f_c \tag{8}$$

$$P(D|k,\epsilon_k) = \epsilon_k f_d \tag{9}$$

$$P(A|\neg k) = (1-\epsilon_{\neg k})\left(\frac{1}{1+e^{\beta_0}}\right) + \epsilon_{\neg k} f_a \tag{10}$$

$$P(B|\neg k) = (1-\epsilon_{\neg k})\left(\frac{e^{\beta_0}}{1+e^{\beta_0}}\right) + \epsilon_{\neg k} f_b \tag{11}$$

$$P(C|\neg k) = \epsilon_{\neg k} f_c \tag{12}$$

$$P(D|\neg k) = \epsilon_{\neg k} f_d \tag{13}$$

First product

We ended up with:

1. Guesstimated groupings of meta-transitions
2. A messy array of likelihood formulae
3. Only two detectable classes (strategic and non-strategic
4. Potential identification problems

Second model

General aim

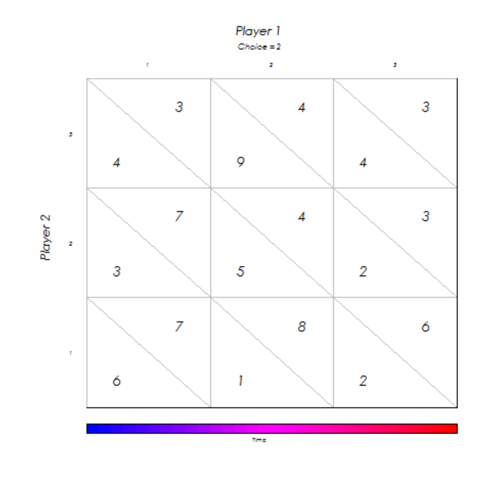
The logic of falsification stands on the following proposition:

$$(t \rightarrow o) \rightarrow (\neg o \rightarrow \neg t)$$

In empirical research we test theoretical statements by assessing the truth value of o.

In this project we explore the premise that $(t \rightarrow o)$.

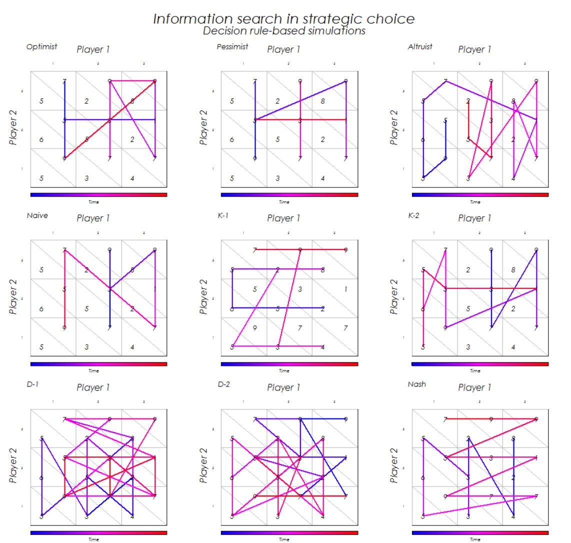
My beautiful robots (K-2)



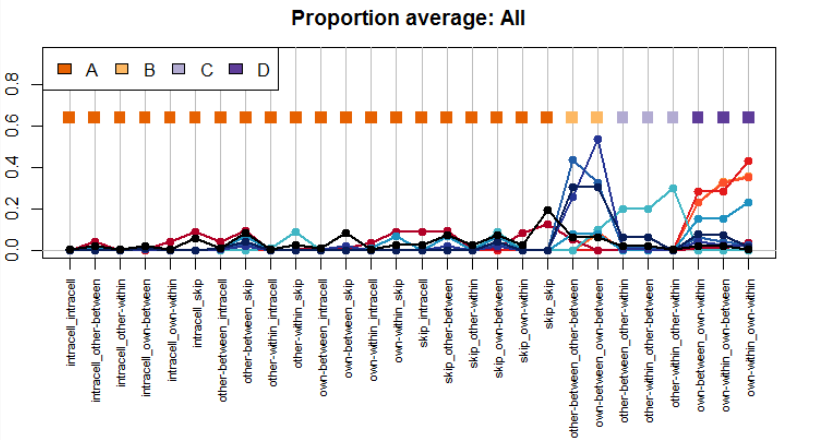
My beautiful robots (all of them)

```
c_own_own<-all_c_own_own[eye_retain_0[[2]]]
c_own_other<-all_c_own_other[eye_retain_0[[2]]]
c_other<-all_c_other[eye_retain_0[[2]]]

search_seq_bet<-sample(c_own_own,length(c_own_own),replace = FALSE)
possible_space<-search_seq_bet

retain<-eye_retain_0[[2]]
eye_movement<-list()

ind<-0
for(i in search_seq_bet){
  if(!i%in%possible_space){
    next
  }
  possible_space<-possible_space[-1]
  if(length(possible_space)==0){
    break
  }
  for(j in search_seq_bet){
    if(!j%in%possible_space){
      next
    }
    search_seq_in<-sample(c_other,length(c_other),replace = FALSE)
    memory<-c()
    equal<-c()
    lower<-c()
    counter<-0
    for(k in search_seq_in){
      loc<-c(k,i)
      loc1<-c(k,j)

      counter<-counter+1
      memory[counter]<-game[loc[1],loc[2]]>game[loc1[1],loc1[2]]
```

My beautiful robots (all of them)



Robot meta-transition simulations



Robot meta-transitions

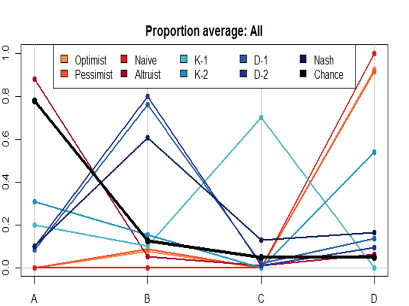
To find new groups of meta-transitions, we ran a k-means clustering on a transposed data frame with decision rules as variables and meta-transitions as observations.

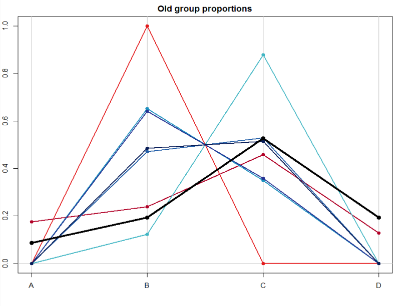
	Optimist	Pessimist	...
Intracell intracell	.0	.0	
Intracell own-within	.0	.0	
...	...	...	

Meta-transition groups

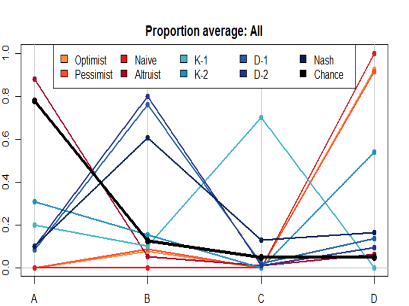
$$\Phi = \begin{array}{c} \mathit{Intracell} \\ \mathit{OwnW} \\ \mathit{OwnB} \\ \mathit{OtherW} \\ \mathit{OtherB} \\ \mathit{Skip} \end{array} \begin{pmatrix} A & A & A & A & A & A \\ A & D & D & 0 & 0 & A \\ A & D & B & 0 & 0 & A \\ A & 0 & 0 & C & C & A \\ A & 0 & 0 & C & B & A \\ A & A & A & A & A & A \end{pmatrix} \tag{14}$$

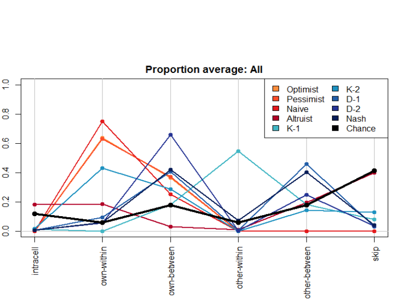
New expected proportions: new vs old groups





New expected proportions: meta-transitions vs transitions





New likelihoods

$$
\begin{array}{c|cc}
k & P(A|k,\varepsilon_k) & P(B|k,\varepsilon_k) \\
\hline
Alt & F_A & F_B \\
OPN & \varepsilon_{OPN} F_A & F_B \\
ND & \varepsilon_{ND} F_A & (1-\varepsilon_{ND}) + \varepsilon_{ND} F_B \\
K1 & \varepsilon_{K1} F_A & F_B \\
K2 & (1-\varepsilon_{K2}) \frac{e^{\beta_0}}{1+e^{\beta_0}} + \varepsilon_{K2} F_A & F_B
\end{array}
\tag{15}
$$

New likelihoods

$$
\begin{array}{c|cc}
k & P(C|k,\varepsilon_k) & P(D|k,\varepsilon_k) \\
\hline
Alt & F_C & F_D \\
OPN & F_C & (1-\varepsilon_{OPN}) + \varepsilon_{OPN} F_D \\
ND & F_C & F_D \\
K1 & (1-\varepsilon_{K1}) + \varepsilon_{K1} F_C & F_D \\
K2 & F_C & (1-\varepsilon_{K2}) \frac{1}{1+e^{\beta_0}} + \varepsilon_{K2} F_D
\end{array}
\tag{16}
$$

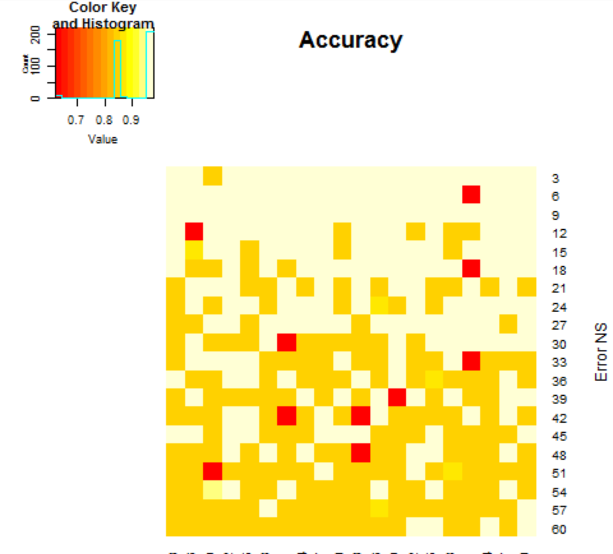
Fitting the data

Model classifying simulated data

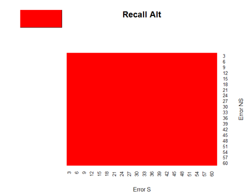
Simulation process

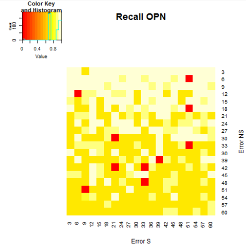
- Use robots to generate small sample of data (100 trials per decision rules)
- Draw error probability separately for strategic and non-strategic decision rules from beta distributions $Beta(\alpha_k, 100)$
- Draw number of errors in each trial from the probability and number of meta-transitions and generate mistake meta-transition counts.
- Fit model at various α_S $\alpha_{\neg S}$ parameters.

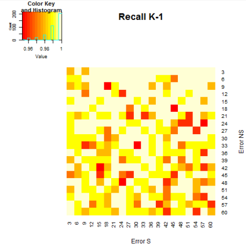
Accuracy

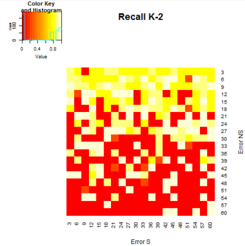

Color Key and Histogram
Count
0
100
200
0.7 0.8 0.9
Value
Accuracy
3
6
9
12
15
18
21
24
27
30
33
36
39
42
45
48
51
54
57
60
Error NS

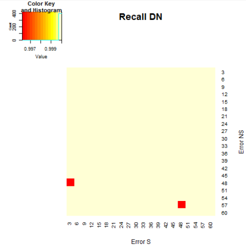
Recall

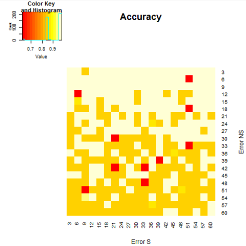

Recall Alt
Error S
Error NS


Color Key and Histogram
Value
Recall OPN
Error S
Error NS

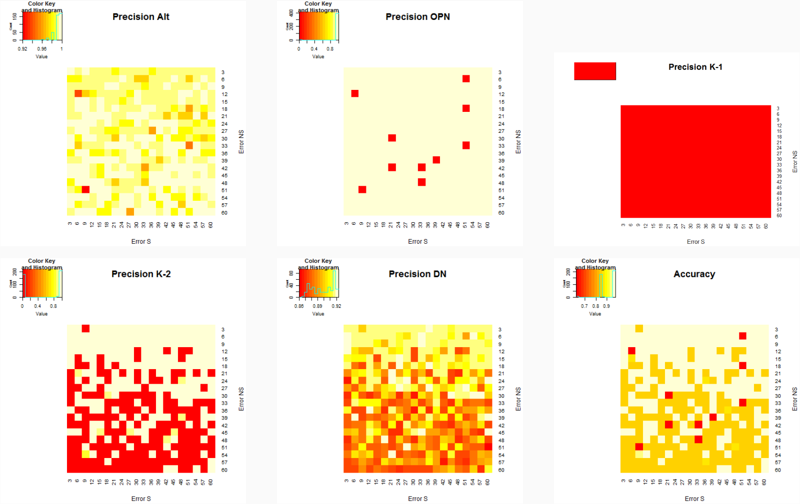

Color Key and Histogram
Value
Recall K-1
Error S
Error NS


Color Key and Histogram
Value
Recall K-2
Error S
Error NS


Color Key and Histogram
Value
Recall DN
Error S
Error NS


Color Key and Histogram
Value
Accuracy
Error S
Error NS

Precision


Precision Alt
Precision OPN
Precision K-1
Precision K-2
Precision DN
Accuracy
Error S
Error NS

Future plans

What comes next

To do:

1. Resolve the OPN - K misclassifications
   1.1 Number of transitions
   1.2 Player's choice
2. Nesting

$$P(k = z)_{it} = \frac{1}{1 + \sum_{z=1}^{Z-1} e^{\beta_{z0} + \beta_{zi} + \beta_{zt}}} \qquad (17)$$

3. Explore new ways to work with/generate error
   3.1 Weights for the chance matrix
   3.2 Inside the robots
4. Try with empirical data
5. Modify the robots
   5.1 Adjust the algorithms
   5.2 Combine the decision rules

Conclusion

To recap:

1. Develop a model for identification of decision rules in economic games from information search
2. Fail at reasoning because it is hard
3. Emulate theoretical behaviour with robots
4. Use robots to reason about a better model
5. Does it work? We will see

References

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