

Answer the following questions and submit your answers online:

1. [10 points] Transform and Conquer

Design a reasonably efficient algorithm for solving each of the following problems and determine its efficiency class.

- You are given n telephone bills and m checks sent to pay the bills ($n \geq m$). Assuming that telephone numbers are written on the checks, find out who failed to pay. (For simplicity, you may also assume that only one check is written for a particular bill and that it covers the bill in full.)
- You have a file of n student records indicating each student's number, name, home address, and date of birth. Find out the number of students from each of the 50 U.S. states.
- For a given positive integer n , find the pair of integers whose sum is n and whose product is as large as possible.
- How many one-digit additions are made by the pen-and-pencil algorithm in multiplying two n -digit integers? (You may disregard potential carries.)

2. [9 points] Input Enhancement and Pre-structuring

- There are ten Egyptian stone statues standing in a row in an art gallery hall. A new curator wants to move them so that the statues are ordered by their height. How should this be done to minimize the total distance that the statues are moved?
- The birthday paradox asks how many people should be in a room so that the chances are better than even that two of them will have the same birthday (month and day). Find the quite unexpected answer to this problem. What implication for hashing does this result have?
- Outline an algorithm for finding the largest key in a B-tree.

3. [5 points] Dynamic Programming

How would you modify the dynamic programming algorithm for the coin collecting problem if some cells on the board are inaccessible for the robot? Apply your algorithm to the board below, where the inaccessible cells are shown by X's. How many optimal paths are there for this board?

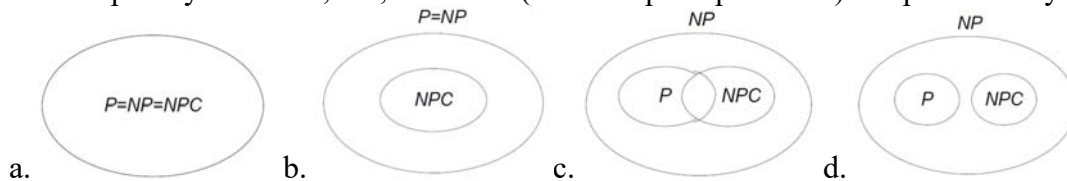
	1	2	3	4	5	6
1		X		•		
2	•			X	•	
3		•		X	•	
4				•		•
5	X	X	X		•	

4. [5 points] Greedy Algorithm

- a. Construct a Huffman code for the following data:
- | | | | | | | |
|-----------|---|-----|-----|-----|-----|-----|
| Symbol | : | A | L | I | Z | P |
| Frequency | : | 4.5 | 0.5 | 3.0 | 1.0 | 1.0 |
- b. Encode PLAZA using the code of part (a).
c. Decode 10011101110110 using the code of part (a).

5. [1 point] P vs. NP

Which of the following diagrams do not contradict the current state of our knowledge about the complexity classes P, NP, and NPC (NP- complete problems)? Explain briefly.



6. [5 points] Decision Trees

Draw a decision tree and find the number of key comparisons in the worst and average cases for the three-element enhanced bubble sort (which stops if no swaps have been made on its last pass).

7. [5 points] Backtracking

Apply backtracking to the problem of finding a Hamiltonian circuit in the graph below:

