

1. Basic Concept:

(1) ADT

- a. Concept: encapsulate data & operations on the data, and hide them from users.
- b. What ADTs did we learn in this class? Set, Bag, Stack, Queue, Priority Queue, Tree (General tree, binary tree, BST, AVL, heap), and Graph.

(2) Big O

- a. Basic rules: drop all but the fastest-growing term; drop any constant coefficient on that remaining term
- b. Be able to analyze big O run time

2. Searching

(1) Sequential search

- a. How does it work
- b. big O analysis

(2) Binary search

- a. How does it work
- b. What is the pre-condition? A sorted sequence is required
- c. Big O analysis

(3) Hash

- a. Concept: key to address mapping process
- b. Concept of Hash table; hash function; perfect hash function; collision; load factor
- c. Hash function
 - i. Perfect hash function– direct hashing & subtraction hashing
 - ii. Other - modulo arithmetic (a prime number is often required)
- d. Collision resolution strategy - probing
 - i. Linear probing
 - 1. General form: $h(k,i) = (h(k) + c*i) \% m$. When given certain $h(k)$ and c , know how to insert an element into the correct position.
 - 2. Problem – primary clustering
 - ii. Quadratic probing
 - 1. General form $h(k,i) = (h(k) + c1*i + c2*i^2) \% m$. understand how it works.
 - 2. Problem – secondary clustering.

- iii. Double hashing – general form $h(k,i) = (h_1(k) + h_2(k)*i)\%m$.
 - e. Hash operations: insertion, searching, min/max, deletion; understand how to conduct these operations; Big O run time analysis
- 3. List: implementation – array & linked list
 - (1) Linear list – each element has only one predecessor and one successor
 - a. Stack – LIFO.
 - i. Given an example, know whether it's a queue or a stack.
 - ii. When using an array to implement a stack, how to conduct push and pop? Big O?
 - b. Queue – FIFO
 - i. Given an example, know whether it's a queue or a stack
 - ii. When using an array to implement a queue, how to conduct enqueue and dequeue? Two different implementations: always use index 0 to place the first element; use a pointer to indicate the first element.
 - iii. Big O for enqueue and dequeue when use different array implementations.
 - c. Implementation: linked-list
 - i. Differences between linked list and array
 - ii. Basic types of linked list: singly linked list, doubly linked list, circularly linked list, doubly linked circular list
 - iii. Basic operations: create a list, insertion, deletion, search, traversal, destroy a list
 - iv. Basic coding and big O analysis for different operations on sorted/unsorted linked list; on linked list with head pointer only or head pointer & tail pointer.
 - (2) Non-linear list – each element can have multiple successors
 - a. Tree – each element can have multiple successors but only one predecessor
 - i. Concepts:
 - 1. Degree, in-degree, out-degree
 - 2. Root node, leaf node, internal node
 - 3. Parent, children, sibling, ancestor, Descendent
 - 4. Level, height
 - 5. Subtree – note that a single node is also a subtree.
 - 6. Balance factor (only exists for binary trees): it can be positive, negative or zero.
 - 7. Balanced tree; complete tree

- ii. Logical architecture: understand the concept for general tree, binary tree, BST, AVL and Heap as well as their relationship

General tree

- Binary tree

- BST – nodes are sorted (left < root < right); each subtree is also a BST.
 - AVL – a balanced BST
 - Not balanced BST
- Heap – root is the largest node (for max-heap); each subtree is also a heap.
- Others...

- Trees in which one node can have more than two children

- iii. General Tree Operations: (how to conduct operations in preorder & postorder)

Note that, these operations are all done in a recursive way.

1. Traverse a tree: e.g. Print out all the nodes. What orders? Both preorder & postorder
2. Count tree node: what order? Doesn't matter
3. Count tree height
4. Destroy a tree: what order? Only postorder.
5. Note that, the general tree is not sorted, so we have to traverse the whole tree to search a specific node or the min/max node. It's very similar to traversal operation. Therefore, we did not explicitly mention search and Max/min.

- iv. Binary Search Tree Operations: (Basic coding & big O analysis & illustration of the insertion & deletion process)

1. Traverse a tree: similar to general tree case. Preorder & postorder & inorder. can only be done recursively.
2. Count tree height: similar to general tree case.
3. Search a tree: can be done both recursively and iteratively
4. Max/Min: can be done both recursively and iteratively
5. Insert a node
 - a. BST – search + add; All inserts take place at a leaf or at a leaflike node
 - b. AVL – search + add + rebalance. (understand how to conduct rebalance and be able to illustrate.)
6. Delete a node – the deleted node has no children; one child; or two children.
 - a. BST – search + remove
 - b. AVL – search + remove + rebalance

- v. Heap operations (bigO analysis & illustration of insertion and deletion process)
 - 1. Insert a node – add + reheap; the new node can only be inserted at a fixed position.
 - 2. Delete a node – delete + reheap; we only care about delete the root node.
 - 3. Know how to store a heap in an array without any pointers.
- b. Graph – each element can have multiple successors and multiple predecessors
 - i. Concepts: directed graph; undirected graph; vertex; edges; cycle; acyclic; dag; planar; clique;
 - ii. Relationship between vertex and edges – given n vertices, maximum number of edges (directed/undirected graph)?
 - iii. Graph representation:
 - 1. Operations: Given a graph, know how to represent it with adjacency matrix & adjacency list. Given an adjacency matrix or adjacency list, know how to draw the graph.
 - 2. Understand when to use adjacency matrix and when to use adjacency list.
 - a. When you care more about a given vertex's neighbors – adjacency list
 - b. When you care more about whether an edge exists or not – adjacency matrix
 - iv. Graph traversal – Worklist algorithm
 - 1. Be able to do depth-first traversal – stack based worklist
 - 2. Be able to do width-first traversal – queue based worklist
- 4. Sorting – understand the sorting category; be able to illustrate the sorting process; big O and stability analysis.
 - (1) Selection based sorting
 - a. Selection sort (basic coding)
 - b. Heap sort
 - (2) Insertion based sorting
 - a. Insertion sort (basic coding)
 - b. Shell sort
 - (3) Exchange based sorting
 - a. Bubble sort (basic coding)
 - b. Quick sort

	<u>Best case</u>	<u>Average case</u>	<u>Worst case</u>	<u>Stability</u>
<u>Selection sort</u>	$O(n^2)$	$O(n^2)$	$O(n^2)$	Depends
<u>Heap sort</u>	$O(n \log n)$	$O(n \log n)$	$O(n \log n)$	Not stable
<u>Insertion sort</u>	$O(n)$	$O(n^2)$	$O(n^2)$	Stable
<u>Shell sort</u>	$O(n \log n)$	Quadratic	Quadratic	Not stable
<u>Bubble sort</u>	$O(n)$	$O(n^2)$	$O(n^2)$	Stable
<u>Quick sort</u>	$O(n \log n)$	$O(n \log n)$	$O(n^2)$	Not stable

<u>SET</u>	<u>Unsorted Array</u>	<u>Sorted Array</u>	<u>Hash Table</u>	<u>Unsorted Linked List</u>	<u>Sorted Linked List</u>	<u>BST</u>	<u>AVL tree</u>
<u>Search</u>	$O(n)$	$O(\log n)$	$O(n)$ (expected $O(1)$)	$O(n)$	$O(n)$	$O(h)$ ($\log(n) \leq h \leq n$)	$\log(n)$
<u>Add</u>	$O(n) + O(1) = O(n)$	$O(\log n) + O(n) = O(n)$	$O(n)$ (expected $O(1)$)	$O(n) + O(1) = O(n)$	$O(n) + O(1) = O(n)$	$O(h) + O(1) = O(h)$	$\log(n)$
<u>Remove</u>	$O(n) + O(1) = O(n)$	$O(\log n) + O(n) = O(n)$	$O(n)$ (expected $O(1)$)	$O(n) + O(1) = O(n)$	$O(n) + O(1) = O(n)$	$O(h) + O(1) = O(h)$	$\log(n)$
<u>Min/Max</u>	$O(n)$	$O(1)$	$O(m)$	$O(n)$	$O(1)$ (assuming fast access to tail)	$O(h)$ ($\log(n) \leq h \leq n$)	$\log(n)$