# ECON 211 - HW 6

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## Problem 1

The unemployed worker Bellman equation is given by:

$$V_u = \max_{\lambda} b - c(\lambda) + \lambda \frac{\rho W}{(1+r)r} + \frac{1-\lambda}{1+r} V_u$$

Then if we fix $\lambda$, we get:

$$\begin{aligned}
V_u &= b - c(\lambda) + \lambda \frac{\rho W}{(1+r)r} + \frac{1-\lambda}{1+r} V_u \\
V_u &= \frac{1+r}{1+r-1+\lambda} \left( b - c(\lambda) + \lambda \frac{\rho W}{1+r} \right) \\
V_u &= \frac{1+r}{1+\lambda} \left( b - c(\lambda) + \lambda \frac{\rho W}{1+r} \right)
\end{aligned}$$

## Problem 2

Let the cost of effort be given by

$$c(\lambda) = \frac{a}{\gamma} \lambda^{1+\gamma}$$

Then we can take the first order conditions of the effort decision with respect to $\lambda$:

$$\begin{aligned}
\frac{\partial V_u}{\partial \lambda} = 0 \iff & -\frac{(1+r)}{(r+\lambda)^2} \left( b - c(\lambda) + \lambda \frac{\rho W}{r(1+r)} \right) + \frac{1+r}{r+\lambda} \left( -c'(\lambda) + \frac{\rho W}{r(1+r)} \right) = 0 \\
\iff & -\frac{(1+r)}{(r+\lambda)^2} \left( b - \frac{a}{\gamma} \lambda^{1+\gamma} + \lambda \frac{\rho W}{r(1+r)} \right) + \frac{1+r}{r+\lambda} \left( -(1+\gamma) \frac{a}{\gamma} \lambda^{\gamma} + \frac{\rho W}{r(1+r)} \right) = 0 \\
\iff & \frac{1}{r+\lambda} \left( \frac{a}{\gamma} \lambda^{1+\gamma} - b - \lambda \frac{\rho W}{r(1+r)} \right) + \frac{\rho W}{r(1+r)} - \frac{a}{\gamma} (1+\gamma) \lambda^{\gamma} = 0 \\
\iff & \left( 1 - \frac{\lambda}{r+\lambda} \right) \left( \frac{\rho W}{r(1+r)} \right) + \frac{a}{\gamma} \lambda^{\gamma} \left( \frac{\lambda}{r+\lambda} - (1+\gamma) \right) = \frac{b}{r+\lambda} \\
\iff & \frac{\rho W}{r(1+r)} \frac{r}{r+\lambda} - \frac{a}{\gamma} \lambda^{\gamma} \frac{r}{r+\lambda} = \frac{b}{\lambda} + a\lambda^{\gamma} \\
\iff & \frac{\rho W}{(r+\lambda)(1+r)} - \frac{a}{\gamma} \lambda^{\gamma} \frac{r}{r+\lambda} = \frac{b}{\lambda} + a\lambda^{\gamma}
\end{aligned}$$

Taking the limit as $r \to 0$, we find:

$$\frac{\rho W}{\lambda} = \frac{b}{\lambda} + a\lambda^{\gamma}$$

$$\lim_{r \to 0} \lambda^* = \left( \frac{\rho W - b}{a} \right)^{\frac{1}{1+\gamma}}$$

## Problem 4

For $D$ drawn from an exponential distribution parameterized by rate $\lambda$, it is easy to write the probability of observing $D$ for any realization of $\lambda$:

$$Pr(D|\lambda) = \lambda e^{-\lambda D}$$

So given that $\lambda$ is a function of wage $W$, the likelihood function for a set of observations for duration $D$ and wages $W$ $\{D_i, W_i\}_{i=1}^n$ is:

$$\begin{aligned} L(D_1, ..., D_n | W_1, ..., W_n) &= \prod_{i=1}^{n} Pr(D_i|W_i) \\ &= \prod_{i=1}^{n} \lambda e^{-\lambda D_i} \end{aligned}$$

Taking the logs of both sides, we find:

$$\log L(D_1, ..., D_n | W_1, ..., W_n) = l(D_1, ..., D_n | W_1, ..., W_n) = n \log \lambda - \lambda \sum_{i=1}^{n} D_i$$

To find the optimal $a$, we first differentiate with respect to $\lambda$ and then substitute in our expression for $\lambda$ from question 1, which is a function of $a$:

$$\begin{aligned} \frac{\partial l}{\partial \lambda} = 0 &\iff \frac{n}{\lambda} - \sum D_i = 0 \\ &\iff \lambda = \frac{n}{\sum D_i} \\ &\iff \left(\frac{\rho W - b}{a}\right)^{\frac{1}{1+\gamma}} = \frac{n}{\sum D_i} \\ &\iff a = (\rho W - b)\left(\frac{n}{\sum D_i}\right)^{1+\gamma} \end{aligned}$$