Flume Parameter Calculations

Justin Nghiem

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1 Preparing scripts and data

The required scripts may be accessed in the Google Drive folder "Biogeomorphic Flume." In the folder, navigate to "R code" for the scripts. Download the scripts

- bin_visualization.R
- k_calculation.R
- settling_velocity_kc_estimates.R
- peristaltic_pump_k_estimation.R
- mass_balance.R

You will also need the processed LISST data for the experiment (.asc file), sediment trap data, and peristaltic pump data. The readme files may be useful when working with the scripts. Refer to the readme file if there are any unclear steps in the explanations below. The scripts themselves also contain comments next to lines of code, which may be useful for understanding them.

2 collector Reynolds number

The collector Reynolds number Re_c is defined as

$$Re_c = \frac{ud_c}{\nu}$$

where u is flow velocity, d_c is collector diameter, and ν is the kinematic viscosity of water. Computing Re_c comes down to measuring the flow velocity (e.g. with Vectrino) since all other parameters are well-known. However, the interpretation of the value is much less clear because of the ill-defined transition between turbulent and laminar flow. Espinosa-Gayosso et al. (2013) suggest that Reynolds numbers for circular cylindrical collectors behave according to the following:

Re_c	behavior
0 to 47	steady two-dimensional flow regime
47 to 180	two-dimensional vortex shedding
180 to 260	three-dimensional vortex shedding
260 to 1000	"another kind" of three-dimensional vortex shedding
>1000	instability of the separating shear layers from cylinder side

As of the time of writing, all experiments with dowels in the flume have had $Re_c = 179.09$.

3 Particle capture rate estimation

3.1 Selecting an appropriate bin

Estimating the particle capture rate k relies on the assumption that particle concentration follows an exponential decay. Specifically, it is parametrized by equation 10 in Fauria et al. (2015)

$$\phi(t) = \phi_0 e^{-kt}$$

where ϕ is the particle volumetric concentration (μL/L), ϕ_0 is the initial particle volumetric concentration, k is the particle capture rate (s^{-1}), and t is time (s). This applies for a specific particle size, so we want to select a LISST bin for which the calculation will be performed. This equation is also independent of height, so we take the reference height to be that of the LISST sensor during the experiment.

- Open the script bin_visualization.R.
- Replace the variable values in lines 5-8 with appropriate values and run the script.
- Inspect the resulting plot for a pattern that matches exponential decay over time. Call the corresponding bin number B.

3.2 Computing k

These next steps work on estimating the two unknown parameters in the previous equation, ϕ_0 and k, from LISST data. ϕ_0 is not directly useful, but may be able to serve as a check if initial particle concentrations for a particular size is known. The script uses nonlinear least squares to estimate these parameters.

- Open the script k_calculation.R in R.
- Replace the variable values in lines 4-10 with appropriate values and run the script (most importantly, set the variable "bin_k" to B).
- The final k estimate is contained in the variable "k_exp." An error may occur if the data poorly fit an exponential decay (then there's not much you can do, except possibly choose a different bin).
- The script produces a plot of the actual data points vs fit, which is useful for qualitatively determining the goodness-of-fit.

3.3 Computing effective capture efficiency

Combining equations 3 and 9 from Fauria et al. (2015) and rearranging yield a useful expression for calculating the effective capture efficiency η'. This expression is

$$\eta' = \frac{k}{u d_c l_c}$$

where u is the flow velocity (m/s), d_c is the collector diameter (m), and l_c is the stem density(m^{-2}).

- Provided the script was able to compute k successfully, it will also compute η' (last two lines of code).
- Make sure to save all your changes to the script k_calculation.R. You will need this later.

4 Separating settling and capture effects

The previous steps only considered total particle capture, not accounting for capture due to settling vs capture onto collectors. We define k_s to be the particle capture rate due to settling and k_c to be the particle capture rate onto collectors. From equation 8 in Palmer et al. (2004), we have that

$$k_s = \frac{w_s}{h}$$

where w_s is the particle settling velocity and h is the depth of water in the flume. In general, $k = k_s + k_c$ but we need to correct for the periodic exposure of the particles to the collectors (e.g. k_c should only occur in the vegetated test section). In equation 11, Fauria et al. introduce a scaling coefficient such that

$$k_c = \frac{V_T}{Lwh}(k - k_s)$$

where V_T is the volume of water in the flume, L is the length of the vegetated region, w is the flume width, and h is the water depth. Note that $\frac{V_T}{Lwh}$ will be larger than 1 so if k_s is small compared to k, k_c may be larger than k. These two equations will be used to compute k_s and k_c.

4.1 Computing k_s and k_c

Computing k_s relies on estimating the settling velocity since water depth is known. There are two methods for this: sediment trap data and Stokes' law.

4.1.1 Sediment trap method

For a given sediment trap, the mass at any time during an experiment is

$$m(t) = \rho A_c w_s t$$

where m is the mass of the captured particles, ρ is the particle density, A_c is the sediment trap cross-sectional area, w_s is settling velocity, and t is time. Taking a time derivative gives

$$\frac{dm}{dt} = \rho A_c w_s$$
$$w_s = \frac{1}{\rho A_c}\frac{dm}{dt}$$

in which we can approximate $\frac{dm}{dt}$ as $\frac{\Delta m}{\Delta t}$ (accumulation of mass in sediment trap divided by experiment time).

4.1.2 Stokes' law method

The Stokes' law estimate for settling velocity is given by the equation

$$w_s = \frac{2}{9}\frac{(\rho - \rho_f)}{\mu} g R^2$$

where ρ_f is the fluid density, μ is the fluid dynamic viscosity, g is gravitational acceleration, and R is particle radius (all in mks units). The key assumptions for Stokes' law are laminar flow and spherical particles (which may not hold in the flume).

- Open the script settling_velocity_kc_estimates.R. This script refers back to k_calculation.R, which is why you need to have saved your changes earlier.
- Replace the variable values in lines 5-12 with appropriate values (importantly, set "bin" to the same as earlier, B) and run the script.

- The data frame "ST_estimates" contains the estimated settling velocity and k_c for each sediment trap (using sediment trap method).
- The data frame "stokes_estimates" contains the estimated settling velocity for the median particle size for each bin. The variable "kc_stokes" contains the estimated k_c using Stokes' law for the selected bin.
- The script does not explicitly return k_s. Probably the easiest way to compute k_s is to divide settling velocities by water depth.

5 Estimating k from peristaltic pump data

Estimating k from the peristaltic pump data is useful because it provides a check against LISST data. The same nonlinear least squares procedure is performed on the mass concentrations from the peristaltic pumps to fit an exponential decay model.

- Open the script peristaltic_pump_k_estimation.R.
- Replace the variable values in lines 5-8 with appropriate values and run the script.
- The script produces an estimate of k, a plot with the fitted exponential and the data, and the mean-squared error.

6 Mass balance

The mass balance incorporates the sediment trap and peristaltic pump data. The basic steps are to estimate the sediment mass that settled on the test section using the sediment traps and to estimate the total mass that was removed from flow using the peristaltic pump data. The difference will be the mass that was captured on the collectors.

The mass that settled on the test section may be estimated by interpolating the sediment trap data points over the test section. Spatial interpolation by inverse distance weighting (with inverse distance power = 2) gives the settled mass per area for each 10 by 10 cm square cell in the test section. Multiplying each cell by the area and summing the cells in the interpolated surface yield the estimated settled mass over the test section.

The total mass removed may be estimated from the peristaltic pump data. Each time point has an upstream and downstream observation at three heights from the bed. For a given time point and height, the upstream concentration minus the downstream concentration represents a volumetric removal rate (mass/volume). This volumetric removal rate may be multiplied by the discharge to obtain a general removal rate (mass/time). We assume concentration and height are independent, so we average across heights to obtain a single removal rate for a given time point. However, we may also assume that the removal rate is constant since the particle capture rates (and thus rates of decay) of the upstream and downstream sections are identical. Thus, we average across time to obtain a constant removal rate, which is multiplied by the total time to obtain a total removed mass.

- Open the script mass_balance.R
- Replace the variable values in lines 6-9 with appropriate values and run the script.
- The script produces estimates of settled, collected, and total mass.