Velocity Distribution in Smooth Rectangular Open Channels

By Kandula V. N. Sarma,[1] P. Lakshminarayana,[2] and N. S. Lakshmana Rao[3]

Abstract: Velocity distribution in a smooth rectangular channel is studied by dividing the channel into four regions. Region 1 comprises of the inner region of the bed and the outer region of the side wall. Region 2 belongs to the inner regions of both the bed and the side wall. Region 3 consists of the inner region of the side wall and the outer region of the bed. Region 4 forms the outer regions of both the bed and the side wall. The aspect ratio is varied from 2.0 to 8.0 and the Froude number from 0.2 to 0.7. Experiments included the aspect ratio of 1.0 also for Froude numbers of 0.2 and 0.3. The range of Reynolds number defined in terms of the hydraulic radius and the cross sectional mean velocity lies between 10^4 and 7×10^4.

Introduction

The theoretical investigations of Prandtl (16) and Von Karman (25,26) on flow through pipes, and the experimental studies of Nikuradse (15) have led to rational formulas for velocity distribution and hydraulic resistance for turbulent flows over flat plates and in circular pipes. These formulas have been extended to open channel flow. Although some general similarities exist between the flow through pipes and the flow through open channels, certain factors such as the presence of a free surface, three-dimensional nature of the flow due to noncircular cross section of the channel, and nonuniform distribution of shear along the wetted perimeter, distinguish the open channel flow from pipe flow.

However, in wide open channels the nature of flow is two-dimensional and the shear stress is constant along the bed. Thus, the formulas for pipes can be applied to those channels by changing the constants suitably to include the free surface effects (20). The velocity distribution in the middle region of such a channel (11,12) is not influenced by the presence of the sidewalls, and the flow in that region may be considered as two-dimensional in nature.

It is a customary practice to connect the cross-sectional mean velocity, V, and the mean shear stress, $\bar{\tau}_0$, over the entire wetted perimeter by

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formulas similar to those of flows through pipes by replacing the diameter, d_o, of the pipe with $4R$ in which R = the hydraulic radius of the rectangular open channel (4,7,8,11,18,21). But the mean turbulent velocity field over the entire cross section of such a channel has not been studied so far in a systematic way. There is a need for taking up this aspect for investigation.

Such an investigation requires the prior knowledge of shear stress and its distribution on the channel boundary. The direct measurement of shear stress at various locations of the wetted surface of the open channel is extremely difficult and one has to compromise and go for one or more of the indirect methods of shear measurements. One of the indirect methods commonly used is that due to Preston (17), which is based on a known velocity distribution. The velocity distribution contains some constants. If it is assumed that the values of these constants depend on the shape of the cross section of the channel in which the flow is occurring, then they have to be evaluated on the basis of experiments. The evaluation of these constants requires prior knowledge of the value of local shear stress. Thus, one gets into a vicious circle.

Another feature that is to be accounted for in the case of a turbulent flow through a rectangular open channel of small aspect ratio is that the maximum velocity does not occur at the free surface. The dip is the ratio of the depth below the free surface where maximum velocity occurs in the vertical section to the total depth of flow in the channel.

The present investigation is limited to subcritical smooth turbulent flows in rectangular open channels of finite aspect ratios. The range of Reynolds numbers lies between 10^4 and 7×10^4.

Review of Literature

Several regions can be distinguished in a shear flow (19).

Viscous Sublayer: In this layer the flow is laminar. The outer edge of the sublayer is approximately at a distance of $z/\delta_o \approx 0.001$ in which δ_o = the boundary layer thickness, and z = the distance from the boundary.

Inner Layer: Flow in this region is still dominated by the wall but the viscous stress is negligible. This layer approximately extends up to a distance of $z/\delta_o \approx 0.15$ corresponding to a shear Reynolds number of $zu_*/\nu = 750$. ν = the kinematic viscosity, and u_* = the shear velocity.

Outer Region: The flow in this region is influenced by the thickness of the boundary layer for flows over a flat plate, and the constraints of the entire periphery of the channel geometry in the case of flows through channels. This layer extends up to the end of the boundary layer in the case of flows over a flat plate, to the center of the pipe for flows through pipes and up to the free surface for flows through open channels.

Turbulent Flow Over Smooth Flat Plate.—Mixing length models forms the basis for determining the velocity profiles in shear flows. The Prandtl's mixing length theory (21) results in

$$\frac{u}{u_*} = \frac{1}{k} \ln (z) + \text{constant} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

in which $u_* = \sqrt{\tau_o/\rho}$; u = velocity in x direction at z; k = Von Karman's universal constant; τ_o = bed shear; and ρ = mass density.

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According to Batchelor (2), the mixing length expression represents the equilibrium between the production and dissipation of the turbulent kinetic energy.

Ludwieg, Tillman, Hama, and many others (10) measured the mean velocity distribution across a turbulent boundary layer along a smooth flat plate with zero pressure gradient. In the inner region, the data were found to fit the universal logarithmic velocity distribution given by the equation

$$\frac{u}{u_*} = A_o \ln \frac{zu_*}{\nu} + B_o \quad \text{(2)}$$

Clauser (5) suggested the values for A_o and B_o, respectively, as 2.44 and 4.9. A value of $A_o = 2.5$ or $k = 0.4$ is usually accepted (9). It may be noted that these two values of A_o are more or less equal. Townsend (22) remarked that many of the observed data seem to indicate a value of B_o nearer to 7 than 4.9. So Eq. 4 can be written as

$$\frac{u}{u_*} = 2.5 \ln \frac{zu_*}{\nu} + B_o \quad \text{(3)}$$

in which B_o has a value in the range of 4.9–7.0.

Hama (10) proposed the simple empirical formula for the mean velocity distribution in the outer region as

$$\frac{u_o - u}{u_*} = 9.6\left(1 - \frac{z}{\delta_o}\right)^2 \quad \text{(4)}$$

in which u_o = the free stream velocity. For the region between $u_*z/\nu = 100$ and 2,000, the simple power law given by

$$\frac{u}{u_*} = 8.3\left(\frac{zu_*}{\nu}\right)^{1/7} \quad \text{(5)}$$

was also found to hold good (10). The value 2,000 for zu_*/ν is a conservative estimate of the upper limit.

Turbulent Flow through Straight Circular Pipe.—In the fully turbulent part of the inner region, the experimental data of Nikuradse (10) indicated that the logarithmic velocity distribution given by Eq. 3 holds good. The constant B_o was evaluated as 5.5.

Schlichting (21) obtained a 1/7 power law with the constant 8.74 for velocity distribution in pipe flows based on the Blasius' resistance law. This power law fitted Nikuradse's data up to a Reynolds number of 10^5.

On the basis of the assumption of a constant eddy viscosity in the core region, and using Laufer's (14) velocity distribution data, a velocity distribution law for the outer region of the pipe flow similar to Eq. 4 was obtained as

$$\frac{u_m - u}{u_*} = 7.2\left(1 - \frac{2z}{d_o}\right)^2 \quad \text{(6)}$$

in which u_m = the maximum velocity occurring at the center of the pipe.

Zagustin and Zagustin (29) gave an analytical solution for turbulent flow in smooth pipes based on a new concept of 'balance of pulsation energy.' The velocity distribution was obtained by them as

$$\frac{u_m - u}{u_*} = \frac{2}{k} \tanh^{-1} \left(\frac{2r}{d_o}\right)^{3/2} \quad (7)$$

in which u = the velocity at radius r.

Turbulent Flow through Wide Open Channels.—From the measurements in the middle verticals in eight different channels. Bazin (13) obtained the velocity formula for wide channels in the form

$$\frac{u_m - u}{u_*} = 6.3\left(1 - \frac{z}{D}\right)^2 \quad (8)$$

in which D = the depth of flow. This equation resembles Eqs. 4 and 6 except for the numerical coefficient. From this it can be understood that Bazin's law is for the outer zone.

Keulegan's (11) arguments amount to saying that Eq. 3 which is valid for smooth pipes could also be extended to smooth, wide open channels. He adopted such a procedure where he could only verify the validity of the logarithmic form of the velocity distribution, but not simultaneously determine the constants of the logarithmic equation and u_*.

Vanoni (24) reported that in reference to two-dimensional channels, the logarithmic velocity distribution was justified by his data to be far below the free surface. He observed that the maximum velocities occurred below the free surface.

Tracy and Lester (23) studied the velocity distribution in wide rectangular open channels and drew the following general conclusions: (1) The flow is symmetrical about the vertical plane equidistant from the two sidewalls of the flume; (2) each half section may be divided into two regions, one in the central portion of the channel and the other in the vicinity of the wall; (3) each vertical curve in the central region of the flow is logarithmic from the surface to a point very close to the floor; (4) within the limits of experimental accuracy, the vertical velocity curves appear to be identical over the width of the central region, and are logarithmic; (5) the phenomenon of dip occurs near the vicinity of the sidewalls; and (6) there always exists a central region for channels of aspect ratio greater than 5. The aspect ratio, A_r, is defined as the ratio of the width B to the depth D.

Goncharov (8) analyzed the plane turbulent flow based on the principles of perturbations. The local longitudinal velocity, u, for plane flow at any distance, z, from the wall was derived as

$$\frac{u}{u_s} = \frac{\ln\left(\dfrac{z + c}{c}\right)}{\ln\left(\dfrac{D + c}{c}\right)} \quad (9)$$

in which u_s = the velocity at $z = D$, i.e., at the free surface; and c = a constant. According to this equation, the maximum velocity, u_m, occurs at the free surface and is equal to u_s, or, in other words, dip is zero.

The standard logarithmic law can be obtained if the following model is assumed for the kinematic eddy viscosity in wide open channels:

$$\epsilon = k\, z\, u_* \left(1 - \frac{Z}{K}\right)^{1/2} \qquad (10)$$

in which ϵ = the kinematic eddy viscosity.

Willis (28) developed a model for the velocity distribution for uniform flow in wide open channels using an error function approximation (27) to the distribution of eddy viscosity.

Willis's model, like that of Zagustin and Zagustin, gives velocity distribution over the entire boundary layer. However, the value for k suggested by him was slightly different from 0.4.

Chandrasekharan and Sakthivadivel (3) extended the use of the analytical solution for turbulent flow in pipes suggested by Zagustin and Zagustin (29) to flow in wide rivers. Their equation was verified by them for rough turbulent flows in wide rivers.

Turbulent Flow in Open Channels of Finite Widths.—For channels of polygonal cross sections, Keulegan (11) suggested an approach for obtaining the velocity distribution. He chose a trapezoidal channel section to illustrate the method and drew internal bisectors of the base angles. He considered that each of the three zones in which the cross section of the channel was divided by the bisectors was influenced by the wall of that zone only. The velocity distribution at any point in any particular zone was assumed to follow logarithmic distribution involving the shear velocity at the foot of the normal drawn from the point to the corresponding wall. This method, in effect, amounts to considering that there is no momentum exchange across the planes passing through the bisectors of the base angles and parallel to the flow direction.

Schlichting (21) observed on the basis of Nikuradse's data that the flow at the free surface was not two-dimensional for a narrow rectangular channel and that the maximum velocity occurred below the free surface.

Allen (1) conducted experiments in a rectangular open channel with smooth sides and bed. He observed that for the range of measurements made in his studies, a close approximation to the mean velocity in a vertical was obtained by averaging the velocities at 0.2 and 0.8 of the depth. This implies the validity of logarithmic law up to the surface from a point very close to the bed.

Enger (6) conducted experiments in a trapezoidal channel having rough boundaries and concluded that the velocity distribution was logarithmic and was unaffected by any secondary currents of appreciable magnitude.

Goncharov (7) extended the application of the equations derived by him for the plane flow to the flow in a rectangular open channel. The equation for the velocity distribution was given by him as

$$\frac{u}{V} = \left(\frac{\log \dfrac{\left(\dfrac{B}{2} - y\right) + C_b}{C_b}}{\log \dfrac{D}{2.7C_b}} \right) \left(\frac{\log \dfrac{z + C_w}{C_w}}{\log \dfrac{B}{5.4C_w}} \right) \qquad (11)$$

in which u = the velocity at any point (y, z) in the cross section; y = the lateral distance from the sidewall; and C_b and C_w = constants. Measured profiles, however, were not in close agreement. The dip was also not zero as implied by the model.

The measurements of Rajaratnam and Muralidhar (18) on the velocity profiles which were made over almost the entire width of the channel suggested the validity of the logarithmic law in the vertical direction. The phenomenon of dip could be observed nearer the side walls. Their flows, however, were super-critical.

Facts Established So Far:

1. When the width of a rectangular channel is larger than a certain value depending on the depth of flow, there exists a central region where the flow can be considered two-dimensional.
2. The flow in a rectangular channel is symmetrical about the vertical passing through the midpoint of the channel.
3. The vertical velocity profiles can generally be described by a logarithmic law in the inner region of the bed. This law seems to hold good for vertical velocity profiles even close to the sidewalls. It may be noted here that a logarithmic law can be closely approximated by a power law.
4. There exists an outer zone away from the wall where the velocity defect is found to follow a parabolic law.
5. The maximum velocity in a vertical does not always occur at the free surface, but sometimes occurs at a certain depth below the free surface. The dip is more pronounced near the sidewalls.

Constants of Velocity Laws

Numerical Constants: Logarithmic Law.—The law contains two constants: one is the reciprocal of Von Karman's constant, k, and the other an additive constant, B_o.

In the literature one finds different values for k and B_o. Attention is drawn here to the investigations of Huffman and Bradshaw (10) in which the bulk of the available data on turbulent flows was analyzed and the constancy of k was established. Based on this information and earlier studies presented, the following statements can be made:

1. k is a universal constant.
2. B_o can vary for different flow geometries.

The constancy of k implies that the slopes of the velocity distribution curves in the inner region of the wall for flat plates, pipes, and wide and narrow open channels should be parallel to one another in a semilog plot. Fig. 1 compares the theoretical solutions of Willis and Zagustin and Zagustin with the logarithmic law of Prandtl. While the slope of the Zagustins' curve is parallel to that of a flat plate solution in the inner region, the slope of Willis' curve is not. Perhaps Willis' solution does not hold good for the inner region.

Power Law.—When the logarithmic law is approximated by a power law the numerical values of the constants are as given in Table 1.

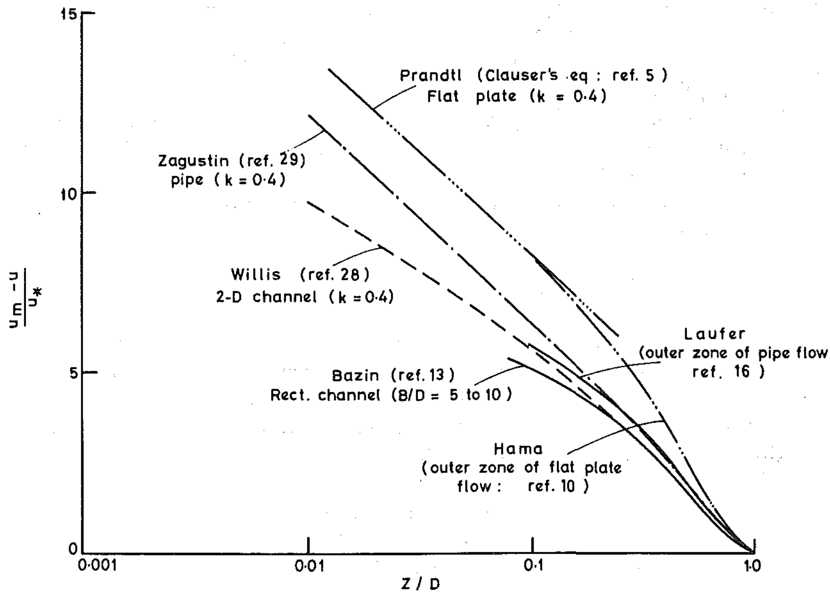

FIG. 1.—Velocity Defect Laws

Velocity Defect Law for Outer Region.—From Fig. 1 it is interesting to note that the solution of Zagustin and Zagustin obtained for the pipe flows agrees well with the velocity defect law proposed for the outer region of a pipe by Laufer (14), and Willis' solution obtained for wide open channels agrees well with the velocity defect law proposed by Bazin for the same type of channels. The numerical constants obtained for the velocity defect laws in the outer region are shown in Table 2.

Formulation of Velocity Model

The facts established at the end of the literature review have been taken into account in formulating the model for velocity distribution in narrow rectangular open channels. The definition sketch shown in Fig. 2 gives the notation used for describing the velocity field.

The flow in the rectangular open channel cross section can be divided into four regions, as shown in Fig. 3.

Region 1. This region is close to the bed and away from the sidewall and thus belongs to the inner region of the bed and outer region of the sidewall.

TABLE 1.—Values of Coefficient for Inner Region

Case (1)	Coefficient, C (2)	Exponent, n (3)	Upper limit of validity (4)
Smooth flat plate	8.3	1/7	$zu_*/\nu = 2{,}000$
Smooth circular pipe	8.74	1/7	$Vd_o/\nu = 10^5$

TABLE 2.—Value of Coefficient for Outer Region

Case (1)	Constant, K_b (2)
Flat plate	9.6
Circular pipe	7.2
Rough wide open channel	6.3

Region 2. This region is close to both the bed and sidewall and thus forms a part of the inner regions of both walls.

Region 3. This region is close to the sidewall but away from the bed. It belongs to the outer region of the bed and inner region of the sidewall.

Region 4. This region is away from both bed and sidewalls and thus belongs to the outer region of both bed and sidewall.

Velocity Distribution

Wall Region.—The velocity distribution in the inner regions of the bed and sidewalls follows the logarithmic law which can be approximated by a power law as

$$\frac{u}{u_*} = C\left(\frac{zu_*}{\nu}\right)^n \quad (12)$$

for the inner region of the bed and

$$\frac{u}{u'_*} = C\left(\frac{yu'_*}{\nu}\right)^n \quad (13)$$

for the inner region of the sidewall, in which u_* and u'_* = shear velocities on the bed and sidewalls, respectively; C and n = the coefficient and exponent of the power law to be determined from experiments; and y = the lateral distance from the sidewall.

Corner Region (Region 2).—It is assumed in this model that Eqs. 12–13 are simultaneously valid in this region. If the assumption is valid then the shear distributions in the wall regions of the bed and sidewall can be shown to follow a power law with the exponent $n/(n + 1)$. It was borne out by experiments (12,13).

Outer Region of Bed.—In this region, the phenomenon of dip occurs in the vertical velocity profiles. The velocity distribution in the outer regions of the bed (Regions 3 and 4) is assumed to take the form

$$\frac{u_m - u}{u_*} = K_b\left(1 - \alpha - \frac{z}{D}\right)^2 \quad (14)$$

in which K_b = the coefficient in the law for the outer region of the bed, to be determined from experiments.

Outer Region of Sidewall.—The dip is not a constant over the width of the open channel. It is known to increase as the sidewalls are approached. This feature makes the formulation of a model for lateral velocity distribution in the outer region of the sidewall difficult.

However, in the outer region of the sidewall very close to the bed,

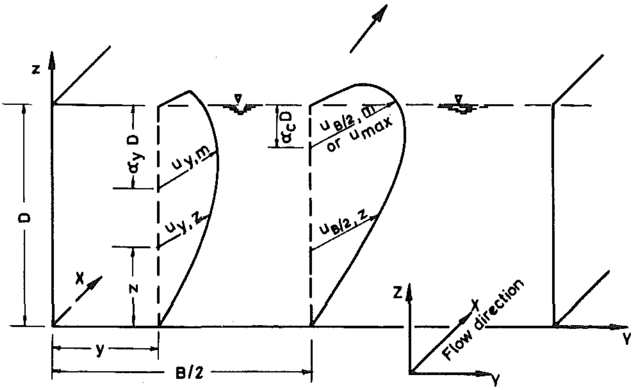

FIG. 2.—Definition Sketch Showing Notation for Velocity Distribution

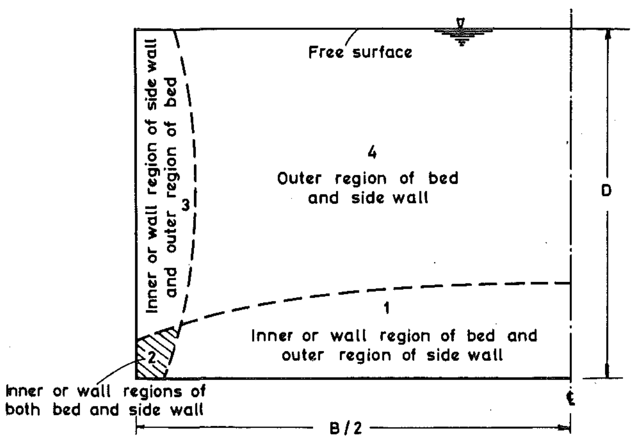

FIG. 3.—Various Regions in Proposed Model

where the dip effects are not felt, a parabolic law of the following form for the velocity distribution is again assumed:

$$\frac{u_c - u}{u'_*} = K_w\left(1 - \frac{2y}{B}\right)^2 \quad \text{(15)}$$

in which K_w = the coefficient in the law for the outer region of the sidewall very close to the bed; and u_c = the maximum velocity in the horizontal, to be determined from experiments.

The value of α can be expected to vary with A_r, F, and y/B. Study of this variation is beyond the scope of this paper.

Experimental Setup

The experimental setup mainly consisted of a smooth, straight, horizontal rectangular flume of smooth walls. It has the internal dimensions

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TABLE 3.—Scheme of Experiments

Aspect ratio (1)	Froude Numbers at 0.1 Intervals: B = 30.5 cm (2)	Froude Numbers at 0.1 Intervals: B = 61.0 cm (3)
1.0	0.2, 0.3	Nil
2.0	0.2 to 0.7	Nil
3.0	0.2 to 0.7	Nil
4.0	0.3 to 0.7	0.2
5.0	0.6, 0.7	0.2 to 0.5
6.0	0.7	0.2 to 0.6
7.0	Nil	0.2 to 0.7
8.0	Nil	0.2 to 0.7

of length 15.25 m, width 61 cm, and height 30 cm. The flume is connected at its upstream end to a head supply tank with a smooth bellmouth entry.

The water was supplied to the flume through the head tank by a high head, constant speed centrifugal pump capable of pumping a maximum discharge of about 50 L/sec from the main sump in the laboratory.

The water flowing through the flume was allowed to fall into a tail channel which is provided at its end with a standard right angled sharp crested triangular notch designed as per B.S.3680. The water after flowing over the triangular notch was allowed to fall back into the main sump.

Scheme

The experiments have been conducted in two stages. The first stage of experiments was carried out in the 61 cm wide flume covering certain aspect ratios and Froude numbers. The second stage of experiments was done in a 30.5 cm wide flume covering certain other Froude numbers and aspect ratios. Table 3 gives the details of the scheme of experiments.

Experiments

The test section was chosen in the region of fully developed boundary layer flow in the channel and, in order to ensure this within the length of the flume, the channel bed was kept very nearly horizontal. Fully developed turbulent flow reach was indicated by the similarity of the velocity profiles in that reach.

For a particular aspect ratio, A_r, and a Froude number, F, Q and D could be determined. The point gage in the tail channel was set for the head which gives the required discharge. The pump was started and the sluice valve in the delivery line supplying water to the head tank of the flume was adjusted until the water level in the tail channel just touched the tip of the point gage. The point gage for depth measurement in the experimental flume at the predetermined test cross section was set to the reading calculated to provide the necessary depth of flow at the test section for the particular aspect ratio chosen. The tail gate at the downstream end of the channel was adjusted till the necessary water level was reached in the channel. The flow in the channel was allowed to stabilize and actual readings corresponding to the depth of flow in the channel and the head above the sill of the triangular notch were noted to compute the actual values of the aspect ratio, Froude number, and discharge. These now differed only slightly from the values originally chosen.

In both stages of experiments, the test cross section was divided into a rectangular grid and the velocity measurements were taken at the nodes of the grid. Depending on the width of the channel experimented with and the depth of flow in it, the grid was formed by choosing 11–13 vertical sections across the width, and 7–8 horizontal sections along the depth in the flow cross section. The limiting point in a vertical or a horizontal was reached when the pitot-static tube was just touching the wall. The upper limiting point was chosen as near the free surface as practicable.

Instrumentation

The velocities were measured with a Prandtl-type pitot-static tube made of stainless steel. The design of the tube was based on the standard N.P.L. pitot-static tube. The tube was calibrated for the velocity measurement in a wind tunnel and also in a pipe carrying water. The coefficient, C_v, of the pitot-static tube was found to be 0.97. The static head and the total head points of the tube were connected to a simple inclined water manometer with a magnification factor of 16 for runs with higher discharges. For other runs, the differential head was read by a U-tube differential manometer using benzaldehyde of specific gravity 1.044 as the manometric liquid. Under ordinary differential pressures and temperatures, benzaldehyde was found to be immiscible with water, and the nearness of its density to that of the water yields a magnification factor of 22.7 with the manometer kept in a vertical position, and much more if kept in any inclined position. The manometer scale could be read up to 1 mm.

The depth of flow in the channel and the position of the pitot-static tube at any point in the flow depth of the test cross section were obtained by means of a point gage, which could read up to 0.035 cm. The uncertainty in the point velocity value was found to be 1.18% (12).

Observation

The flow pattern in the channel was noted to be very nearly symmetrical about the vertical plane passing through the longitudinal axis of the channel. The deviations were found to be less than the accuracy of the experiments.

Velocity Distributions in Inner Regions of Bed and Sidewall

Value of Exponent n in Power Law.—For a particular flow and a particular vertical, C, D, u_*, and ν of Eq. 12 are all constants. Preliminary plots of measured point velocity, u, vs. relative height, z/D, were made

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on a log-log scale for each of the verticals in the cross section of the channel from the center to the sidewall of the channel for each run. From the experiments, the value of the exponent, n, in the power law of the inner region of the bed was obtained as 1/7 for all values of $2y/B$.

Similar plots were made between the lateral distance relative to half width of the channel, i.e., $2y/B$ and the point velocities for each of the horizontal sections at various distances above the bed of the channel for each run. These plots also indicated a value of 1/7 for the exponent in the power law for the inner region of the side wall for all values of z/D.

Deviations from the power law were observed at large values of z/D and $2y/B$, i.e., for the outer region of the bed and the sidewall.

Value of Constant C in Power Law.—To determine the value of C from experiments, the local u_* values should be known. One method of determining the values of u_* is by using a probe calibrated in a wind tunnel and use it in the studies on open channels. It is as good as assuming C to be 8.3 for open channels. Instead, if the choice is to calibrate the probe in a pipe, it is as good as assuming C to be 8.74.

No such attempt was made in the present studies to determine shears through a calibrated probe and then obtain the value of C using the values of shear so determined. Instead it was assumed that the flow over the bed and the sidewalls of an open channel of rectangular cross section is similar to the flow over a flat plate. Thus the value of C was taken to be 8.3.

Determination of Local Shear.—Making use of the power law with C and n values as 8.3 and 1/7, respectively, and measuring the differential head when the velocity probe was in contact with the boundary,

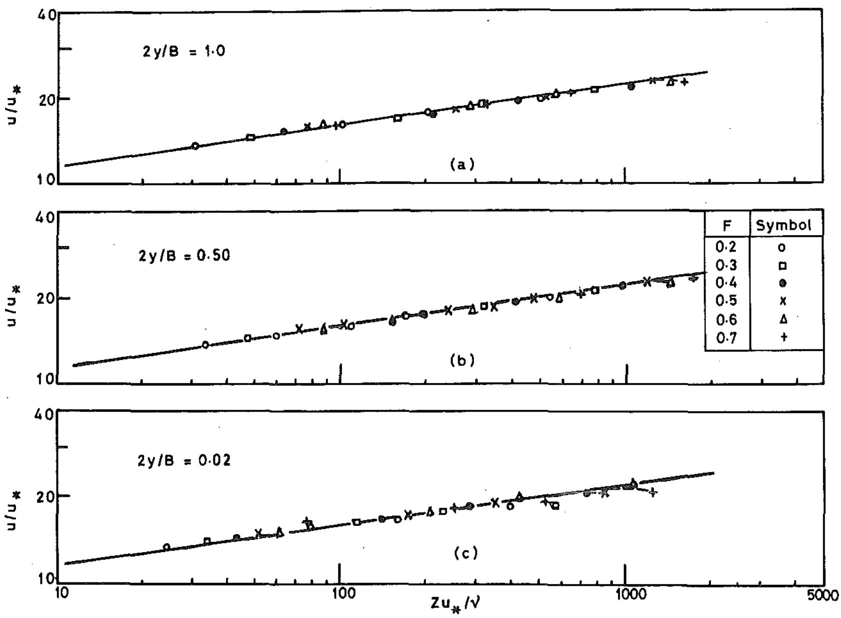

FIG. 4.—Velocity Profiles in Wall Region of Bed (A_r = 3.0)

the shear velocities were calculated at various lateral distances on the bed and at various heights on the sidewall.

It was felt desirable not to depend on a single δh (differential head) value in the preceding manner for the determination of shear velocity since an error made in the observation of δh would lead to a wrong value of u_* or u'_*. Thus the value of u_* or u'_* at a point on the bed or sidewall was calculated using again the same power law, with $C = 8.3$ and $n = 1/7$, but taking the entire velocity profile in the wall region into account. From the intercept value of the log-log plot between u and z/D at $z/D = 1$, the values of u_* and u'_* were calculated.

The values of u_* and u'_* determined in this fashion are given in Ref. 12 for all the runs. These values agreed well with those obtained by measuring the δh values with the pitot-static tube in contact with the surface at corresponding locations along the bed and the sidewall.

The assumption of the value 8.3 for C is shown to be justified as the values of the average shear obtained on the basis of this assumption agreed with those obtained by other methods (12,13).

Effects of Froude Number and Aspect Ratio.—In order to study the effect of a Froude number on the velocity distribution in the inner region of the bed, plots of u/u_* and zu_*/ν were made on a log-log scale for various Froude numbers at a particular lateral distance from the sidewall relative to half width of the channel, $2y/B$, and aspect ratio, A_r. Similar plots were made for all other pairs of values of $2y/B$ and A_r. Typical plots are presented in Figs. 4(*a–c*). It can be seen from these figures that there is no significant effect of a Froude number on the velocity distribution in the inner region of the bed.

A similar analysis is made for the sidewall region. Typical plots are presented in Figs. 5(*a–c*). Again, from these figures it can be concluded

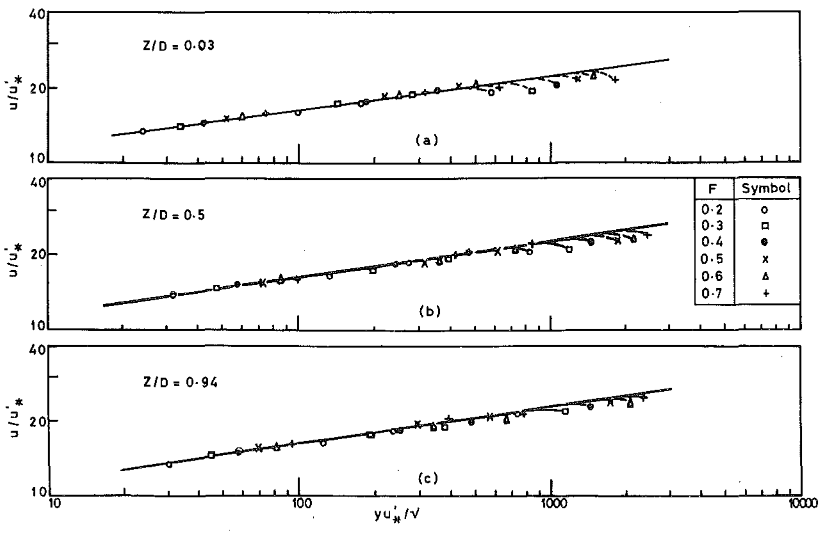

FIG. 5.—Effect of Froude Number on Velocity Distribution in Inner Region of Sidewall (A_r = 3.0)

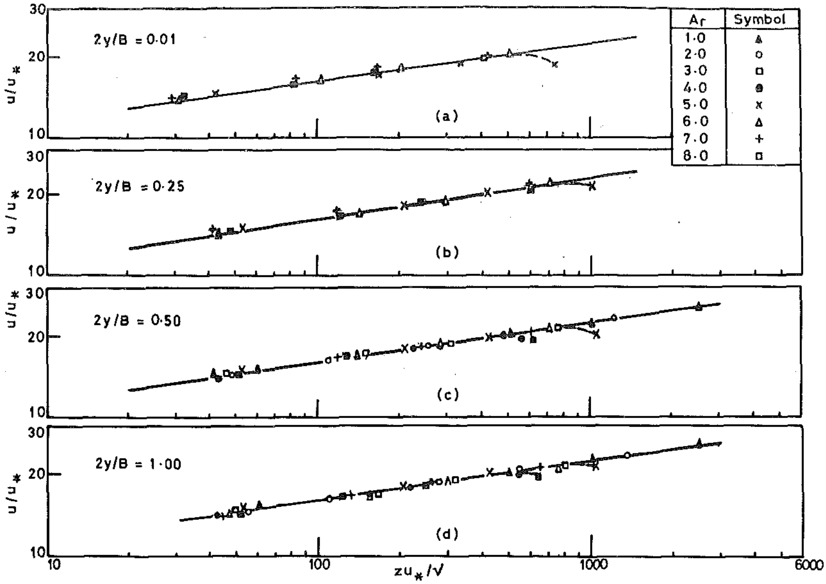

FIG. 6.—Velocity Profiles in Wall Region of Bed (F = 0.3)

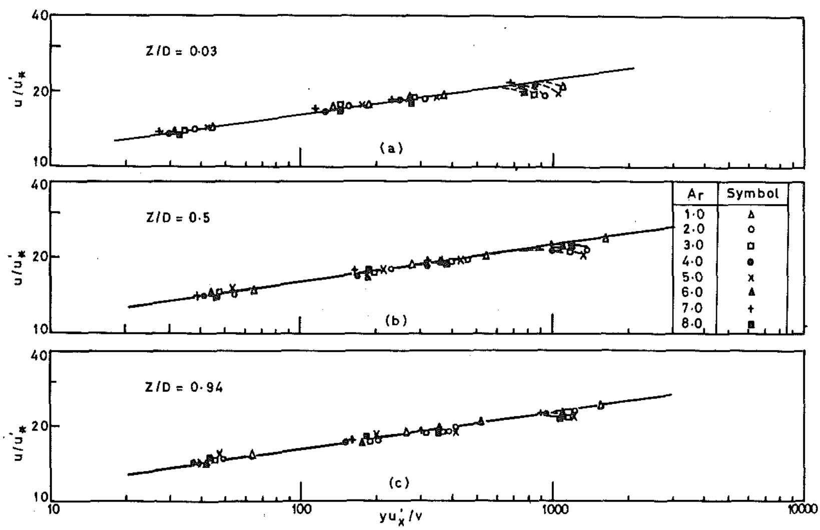

FIG. 7.—Effect of Aspect Ratio on Velocity Distribution in Inner Region of Sidewall (F = 0.3)

that there is no significant effect of a Froude number on the velocity distribution of the inner region of the sidewall.

Similarly, the effect of the aspect ratio is studied. Typical plots are shown in Figs. 6(*a*–*d*) and 7(*a*–*c*). From these plots one can see that the aspect ratio is not a parameter affecting the velocity distribution in the

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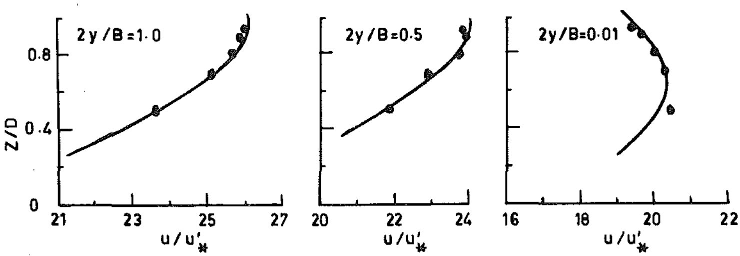

FIG. 8.—Velocity Profiles in Outer Region of Bed (A_r = 6.0, F = 0.5)

inner regions of the bed and sidewall.

The velocity distributions in the inner regions of the bed and side wall may, therefore, be given as

$$\frac{u}{u_*} = 8.3\left(\frac{zu_*}{\nu}\right)^{1/7} \quad (16)$$

for the inner region of the bed, and for the inner region of the sidewall it is given by

$$\frac{u}{u'_*} = 8.3\left(\frac{yu'_*}{\nu}\right)^{1/7} \quad (17)$$

Velocity Distribution in Outer Regions

Outer Region of Bed.—The model for velocity distribution is taken to be a parabola. Preliminary plots of u/u_* vs. z/D are drawn for various runs. By reasonably fitting a parabola, the dip, α, and the maximum velocity, u_m, in the vertical from the curve are noted. Then it was found that the constant, K_b, in Eq. 14 for the model is 9.6. A few typical plots for the run with A_r = 6.0 and F = 0.5 are shown in Figs. 8(*a–c*). The values of α and u_m/u_* for various verticals in various runs are presented in Ref. 12.

It may be noted that when C is taken equal to 8.3 to determine u_*, the outer law constant also is found to be equal to 9.6, the flat plate constant.

Effect of Froude Number and Aspect Ratio.—For a particular value of A_r, graphs of u/u_* vs. z/D were plotted for various verticals, i.e., for different $2y/B$ values. The procedure was repeated for other aspect ratios. Typical graphs are presented in Figs. 9(*a–d*) for an aspect ratio of 2.0 for various Froude numbers. It is found that the graphs in all the cases follow the parabolic law of the outer region.

A similar analysis was made to study the effect of A_r. A few typical graphs are shown in Figs. 10(*a–d*) for a Froude number of 0.3. It may be observed that, in this case also, the graphs again follow the parabolic law of the outer region.

Although the Froude number and the aspect ratio do not have any significant effect on the parabolic form of the outer law equation, it is possible that F and A_r affect the values of α and u_m. In other words, F

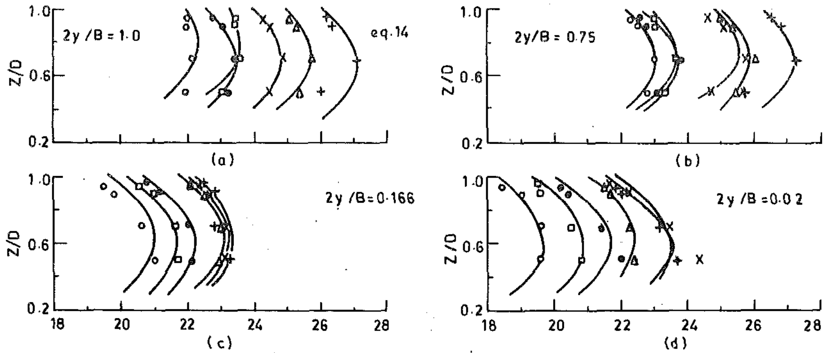

FIG. 9.—Velocity Profiles in Outer Region of Bed (A_r = 2.0)

and A_r can affect the velocity distribution in as much as they determine α and u_m in the following equation:

$$\frac{u_m - u}{u_*} = 9.6\left(1 - \alpha - \frac{z}{D}\right)^2 \quad \text{.................................} \quad (18)$$

Outer Regions of Sidewalls

Away from Bed.—Curves between u/u'_* and $2y/B$ were drawn for different runs in the outer region of the sidewall away from the bed. No systematic trend was observed as α varied from vertical to vertical. It was, therefore, not possible to fit any single equation for velocity distribution in this region.

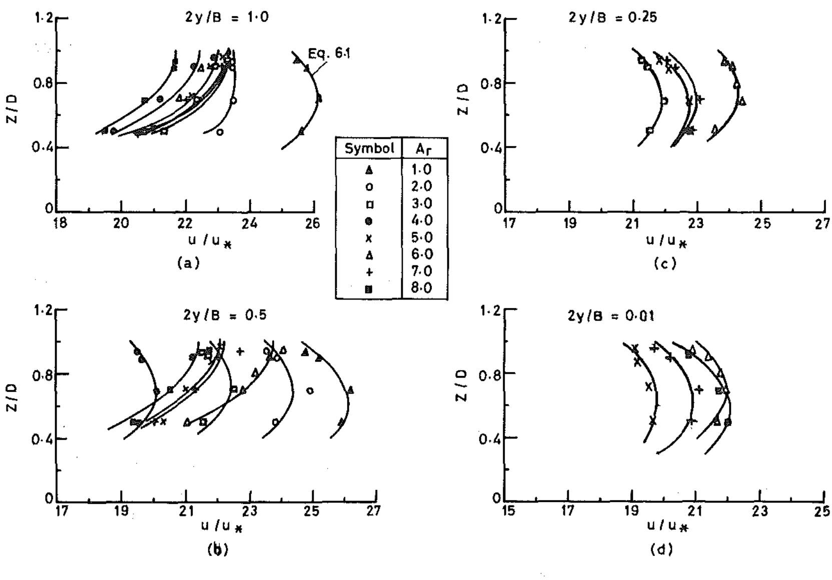

FIG. 10.—Velocity Profiles in Outer Region of Bed (F = 0.3)

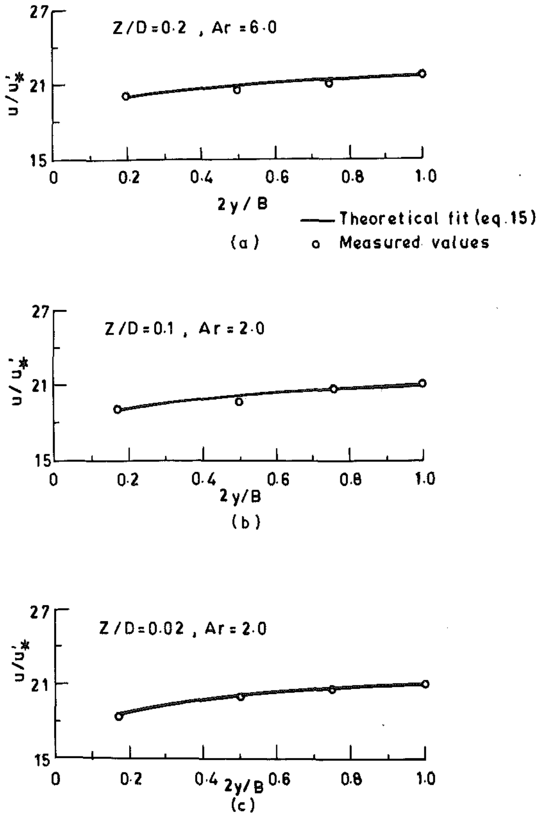

FIG. 11.—Velocity Distribution in Outer Region of Sidewall and Inner Region of Bed

Inner Region of Bed.—Curves between u/u'_* and $2y/B$ were drawn for various values of z/D in the inner region of the bed. Typical plots are shown in Fig. 11 for aspect ratios of 6.0 and 2.0, with Froude number equal to 0.3. It is observed that, in most of the plots, the maximum value of the velocity, u_c, occurs at the center of the channel, and the distribution is as given by Eq. 15. When the experimentally determined constant is substituted, Eq. 15 becomes

$$\frac{u_c - u}{u'_*} = 2.4\left(1 - \frac{2y}{B}\right)^2 \qquad (19)$$

In the horizontal velocity profile close to the bed, the velocity decreases from the center of the channel towards the sidewall. But as the sidewall is approached, the bed is relatively closer to the point of maximum velocity in the vertical because of the larger values of dip nearer the sidewall region. This might have resulted in homogenizing in the lateral direction so that the outer law constant for the sidewall is far less than 9.6 for the bed.

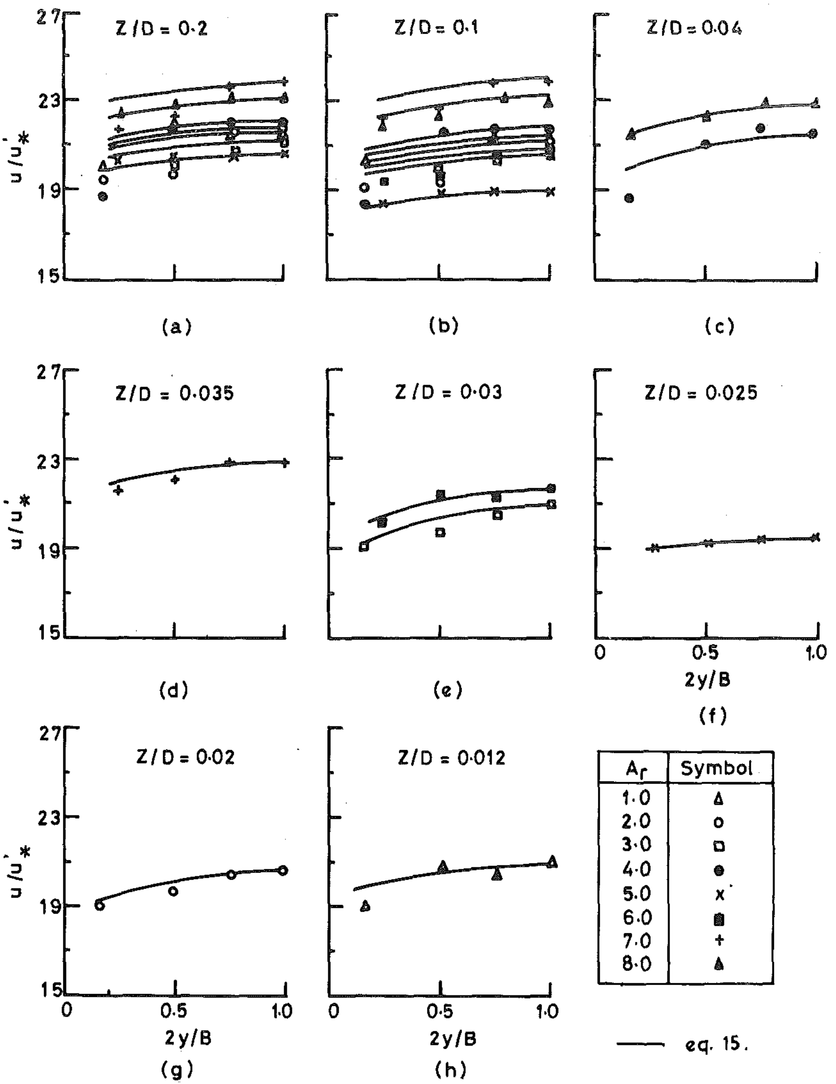

FIG. 12.—Horizontal Velocity Distribution in Outer Region of Sidewall but within Inner Region of Bed (F = 0.3)

Effect of Aspect Ratio and Froude Number.—For a particular value of the Froude number, plots were made between u/u'_* and $2y/B$ for various aspect ratios. The plots were repeated for other Froude numbers also. Typical curves are presented in Figs. 12(*a–h*). It is observed that the parabolic law with constant equal to 2.4 was found to fit reasonably well. It may therefore be stated that the aspect ratio has no effect on the horizontal velocity distribution given by Eq. 15.

CONCLUSIONS

1. The velocity distribution for the inner regions of the bed and sidewalls is given by 1/7th power law. This law is valid at all elevations for the sidewall and at all distances from the sidewall to the bed.

2. The velocity distribution of the inner region is unaffected by a Froude number and aspect ratios.

3. The velocity distribution in the outer region of the bed is parabolic and the constant is found to be 9.6, the same as the outer law constant for a flat plate.

4. The form of the equation obtained for the outer region of the bed is unaffected by the variation either in the Froude number or in the aspect ratio. The Froude number and aspect ratio may affect the magnitude of the dip, α, and the maximum velocity, u_m, in the vertical velocity profile.

5. For the lateral velocity distribution in the outer region of the sidewall away from the bed, it is not possible to formulate any equation as the dip in the vertical velocity distribution affects the lateral distribution in a very irregular manner.

6. The lateral velocity distribution in the outer region of the sidewalls, but close to the bed, is parabolic and the constant is obtained as 2.4.

7. The Froude number and aspect ratio are found to have no significant effect on the form of the equation for the velocity distribution in the outer region of the sidewall close to the bed.

8. There is no significant dip in the horizontal velocity profiles close to the bed.

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