

## 1. Mixed formulation

$$\begin{aligned}
& \Pi(\varepsilon_{\alpha\beta}, \kappa_{\alpha\beta}, N^{\alpha\beta}, M^{\alpha\beta}) \\
&= \int_{\Omega} \frac{h}{2} \varepsilon_{\alpha\beta} C^{\alpha\beta\gamma\eta} \varepsilon_{\gamma\eta} d\Omega + \int_{\Omega} \frac{h^3}{24} \kappa_{\alpha\beta} C^{\alpha\beta\gamma\eta} \kappa_{\gamma\eta} d\Omega \\
&+ \int_{\Omega} \varepsilon_{\alpha\beta} (N^{\alpha\beta} - h C^{\alpha\beta\gamma\eta} \varepsilon_{\gamma\eta}) d\Omega + \int_{\Omega} \kappa_{\alpha\beta} (M^{\alpha\beta} - \frac{h^3}{12} C^{\alpha\beta\gamma\eta} \kappa_{\gamma\eta}) d\Omega \\
&- \int_{\Gamma_v} \mathbf{t} \cdot \bar{\mathbf{v}} d\Gamma + \int_{\Gamma_{\theta}} M_{nn} \bar{\theta}_{\mathbf{n}} d\Gamma - (P \mathbf{a}_3 \cdot \bar{\mathbf{v}})_{\mathbf{x} \in C_w}
\end{aligned} \tag{1}$$

$$\begin{aligned}
& \bar{\Pi}(\varepsilon_{\alpha\beta}, \kappa_{\alpha\beta}, N^{\alpha\beta}, M^{\alpha\beta}, \mathbf{v}) \\
&= \Pi(\varepsilon_{\alpha\beta}, \kappa_{\alpha\beta}, N^{\alpha\beta}, M^{\alpha\beta}) \\
&+ \int_{\Omega} \mathbf{v} \cdot ((\mathbf{a}_{\alpha} N^{\alpha\beta})|_{\beta} + (\mathbf{a}_3 M^{\alpha\beta})|_{\alpha\beta} + \bar{\mathbf{b}}) d\Omega \\
&+ \int_{\Gamma_M} \mathbf{v}_{,\mathbf{n}} \cdot \mathbf{a}_3 (M_{nn} - \bar{M}_{nn}) d\Gamma \\
&- \int_{\Gamma_t} \mathbf{v} \cdot (\mathbf{t} - \bar{\mathbf{t}}) d\Gamma - \mathbf{v} \cdot \mathbf{a}_3 (P - \bar{P})_{\mathbf{x} \in C_P}
\end{aligned} \tag{2}$$

$$\delta \Pi(\varepsilon_{\alpha\beta}, \kappa_{\alpha\beta}, N^{\alpha\beta}, M^{\alpha\beta}, \mathbf{v}) = 0 \tag{3}$$

$$- \int_{\Omega} h \delta \varepsilon_{\alpha\beta} C^{\alpha\beta\gamma\eta} \varepsilon_{\gamma\eta} d\Omega + \int_{\Omega} \delta \varepsilon_{\alpha\beta} N^{\alpha\beta} d\Omega = 0 \tag{4a}$$

$$- \int_{\Omega} \frac{h^3}{12} \delta \kappa_{\alpha\beta} C^{\alpha\beta\gamma\eta} \kappa_{\gamma\eta} d\Omega + \int_{\Omega} \delta \kappa_{\alpha\beta} M^{\alpha\beta} d\Omega = 0 \tag{4b}$$

$$\begin{aligned}
& \int_{\Omega} \delta N^{\alpha\beta} \varepsilon_{\alpha\beta} d\Omega - \int_{\Gamma} \mathbf{a}_{\alpha} \delta N^{\alpha\beta} \cdot \mathbf{v} n_{\beta} d\Gamma + \int_{\Omega} \mathbf{v} \cdot (\mathbf{a}_{\alpha} \delta N^{\alpha\beta})|_{\beta} d\Omega \\
&+ \int_{\Gamma_v} \mathbf{a}_{\alpha} \delta N^{\alpha\beta} \cdot \mathbf{v} n_{\beta} d\Gamma = \int_{\Gamma_v} \mathbf{a}_{\alpha} \delta N^{\alpha\beta} \cdot \bar{\mathbf{v}} n_{\beta} d\Gamma
\end{aligned} \tag{4c}$$

$$\begin{aligned}
& \int_{\Omega} \delta M^{\alpha\beta} \kappa_{\alpha\beta} d\Omega - \int_{\Gamma} \delta M_{nn} \theta_{\mathbf{n}} d\Gamma + \int_{\Gamma} \delta \mathbf{t} \cdot \mathbf{v} d\Gamma + (\delta P \mathbf{a}_3 \cdot \mathbf{v})_{\mathbf{x} \in c} - \int_{\Omega} (\mathbf{a}_3 \delta M^{\alpha\beta})|_{\alpha\beta} \cdot \mathbf{v} d\Omega \\
&+ \int_{\Gamma_{\theta}} \delta M_{nn} \theta_{\mathbf{n}} d\Gamma - \int_{\Gamma_t} \delta \mathbf{t} \cdot \mathbf{v} d\Gamma - (\delta P \mathbf{a}_3 \cdot \mathbf{v})_{\mathbf{x} \in c_w} \\
&= \int_{\Gamma_{\theta}} \delta M_{nn} \bar{\theta}_{\mathbf{n}} d\Gamma - \int_{\Gamma_t} \delta \mathbf{t} \cdot \bar{\mathbf{v}} d\Gamma - (\delta P \mathbf{a}_3 \cdot \bar{\mathbf{v}})_{\mathbf{x} \in c_w}
\end{aligned} \tag{4d}$$

$$- \int_{\Gamma} \delta \mathbf{v} \cdot \mathbf{t} d\Gamma + \int_{\Omega} \delta \mathbf{v} \cdot \mathbf{b} d\Omega + \int_{\Gamma_t} \delta v \cdot \mathbf{t} d\Gamma = \int_{\Gamma_t} \delta \cdot \bar{\mathbf{t}} d\Gamma + \int_{\Omega} \delta \mathbf{v} \cdot \bar{\mathbf{b}} d\Omega \quad (4e)$$

$$\mathbf{v}_{,\alpha}(\mathbf{x}) = \mathbf{p}^T(\mathbf{x}) \mathbf{d}_{\alpha}^{\varepsilon}, \quad \varepsilon_{\alpha\beta}(\mathbf{x}) = \mathbf{p}^T(\mathbf{x}) \frac{1}{2} (\mathbf{a}_{\alpha} \cdot \mathbf{d}_{\beta}^{\varepsilon} + \mathbf{a}_{\beta} \cdot \mathbf{d}_{\alpha}^{\varepsilon}) \quad (5)$$

$$(\mathbf{v}_{,\alpha})|_{\beta}(\mathbf{x}) = \mathbf{p}^T(\mathbf{x}) \mathbf{d}_{\alpha\beta}^{\kappa}, \quad \kappa^{\alpha\beta}(\mathbf{x}) = -\mathbf{p}^T(\mathbf{x}) \mathbf{a}_3 \cdot \mathbf{d}_{\alpha\beta}^{\kappa} \quad (6)$$

$$N^{\alpha\beta}(\mathbf{x}) = \mathbf{p}^T(\mathbf{x}) \mathbf{a}^{\alpha} \cdot \mathbf{d}_{\beta}^N, \quad \mathbf{a}_{\alpha} N^{\alpha\beta} = \mathbf{p}^T(\mathbf{x}) \mathbf{d}_{\beta}^N \quad (7)$$

$$M^{\alpha\beta}(\mathbf{x}) = \mathbf{p}^T(\mathbf{x}) \mathbf{a}_3 \cdot \mathbf{d}_{\alpha\beta}^M, \quad \mathbf{a}_3 M^{\alpha\beta} = \mathbf{p}^T(\mathbf{x}) \mathbf{d}_{\alpha\beta}^M \quad (8)$$

$$\mathbf{v}(\mathbf{x}) = \sum_{I=1}^{n_p} \Psi_I(\mathbf{x}) \mathbf{d}_I \quad (9)$$

<sup>12</sup> **2. Numerical examples**

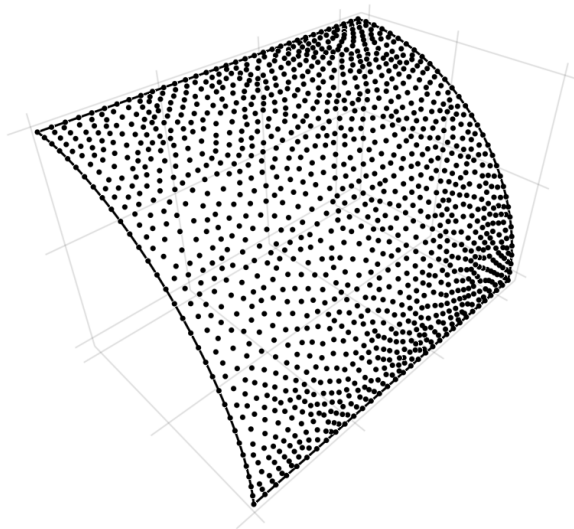


Figure 1: Meshfree discretization for patch test