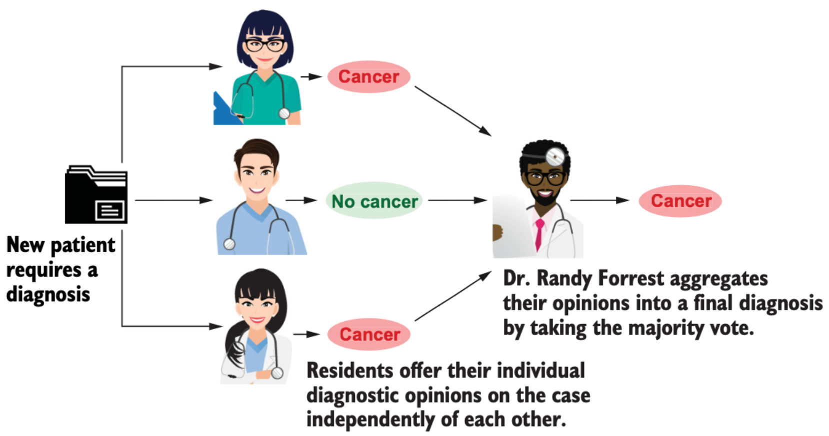
Ensemble Methods: Rationale

This section presents techniques for improving classification accuracy by **aggregating** the predictions of **multiple classifiers**. These techniques are known as **ensemble** methods. An ensemble method constructs a set of base classifiers from training data and performs a classification. There are a few ensemble methods, many of which **outperform** a single classifier.

We will present the rationale of the ensemble methods using a majority vote and later move on to the topics of **bagging**, **random forests**, and **boosting** – more practical and powerful ensemble learning techniques to construct classification models. Most of the techniques presented herein are also applicable to regression problems.

1. Motivation: Majority Vote

Let us walk through an example by taking a **majority vote** on the predictions made by each base classifier.



[1]

Example: Consider an ensemble of 3 binary classifiers, each with an error rate[2] of $e = 0.35$. All the possible combinations of the ensemble are:

A	B	C	Error Rate
o (0.35)	o (0.35)	o (0.35)	0.042875
o (0.35)	o (0.35)	x (0.65)	0.079625
o (0.35)	x (0.65)	o (0.35)	0.079625
x (0.65)	o (0.35)	o (0.35)	0.079625

[1] G. Kunapuli (2023) Ensemble Methods for Machine Learning, Manning.

[2] Error rate means how often it is wrong. For a binary classifier, error rate is defined as $e = \frac{\text{Number of wrong prediction}}{\text{Total Number of prediction}}$.

o (0.35)	x (0.65)	x (0.65)	0.147875
x (0.65)	o (0.35)	x (0.65)	0.147875
x (0.65)	x (0.65)	o (0.35)	0.147875
x (0.65)	x (0.65)	x (0.65)	0.274625

Formally, the error rate for each combination can be computed by:

$\varepsilon_i = \binom{3}{i} e^i (1-e)^{3-i}$

in which i is the number of base classifiers that predict the right answer.

$\binom{n}{k}$ is read ***n* choose *k***. $\binom{n}{k}$ is called binomial coefficient and can be computed as $\binom{n}{k} = \frac{n!}{k!(n-k)!}$.

The ensemble classifier predicts the class label of a test example by taking a majority vote on the prediction made by the base classifiers. The ensemble error rate through majority vote is:

$\varepsilon_{\text{ensemble}} = 0.079625 + 0.079625 + 0.079625 + 0.042875 = 0.28175$

Or mathematically,

$\varepsilon_{\text{ensemble}} = \sum_{i=2}^{3} \binom{3}{i} e^i (1-e)^{3-i} = 0.28175$

which is slightly better than the error rate of a base learner $e = 0.35$.

Example: Consider an ensemble of 25 binary classifiers with an error rate of $e = 0.35$. The ensemble classifier predicts the class label of a test example by taking a majority vote on the prediction made by the base classifiers. Assume that the base classifiers are independent, and the ensemble makes a wrong prediction only if more than half of the base classifiers predict incorrectly.

(1) What is the error rate of the ensemble for the case with 13 base classifiers predicting a correct answer?

(Ans)

$$\varepsilon_{13} = \binom{25}{13} e^{13} (1-e)^{12}$$

$\binom{n}{k}$ is read ***n* choose *k***. $\binom{n}{k}$ is called binomial coefficient and can be computed as $\binom{n}{k} = \frac{n!}{k!(n-k)!}$.

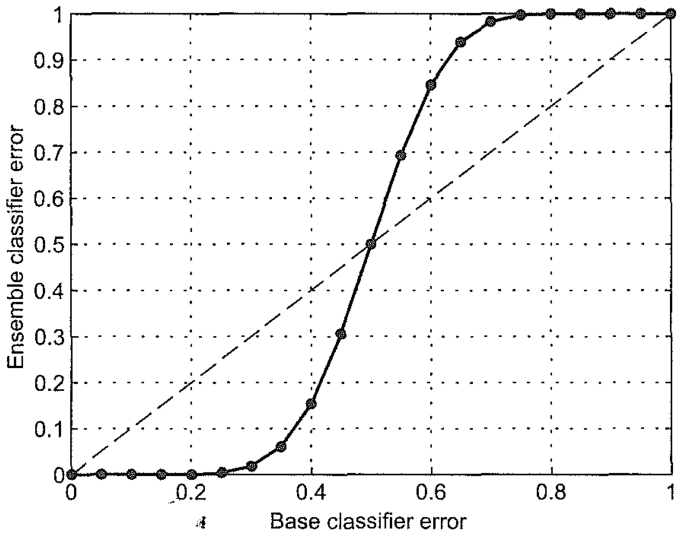
(2) What is the error rate of the ensemble method?

(Ans)

$$\varepsilon_{\text{ensemble}} = \sum_{i=13}^{25} \binom{25}{i} \varepsilon^{i} (1-\varepsilon)^{25-i} = 0.06$$

which is **considerably lower** than the error rate of the base classifiers.

The figure below shows the error rate of an ensemble of 25 binary classifiers through the majority vote ($\varepsilon_{\text{ensemble}}$) for <u>different base classifier error rates</u> (e).



for <u>25</u> binary classifiers

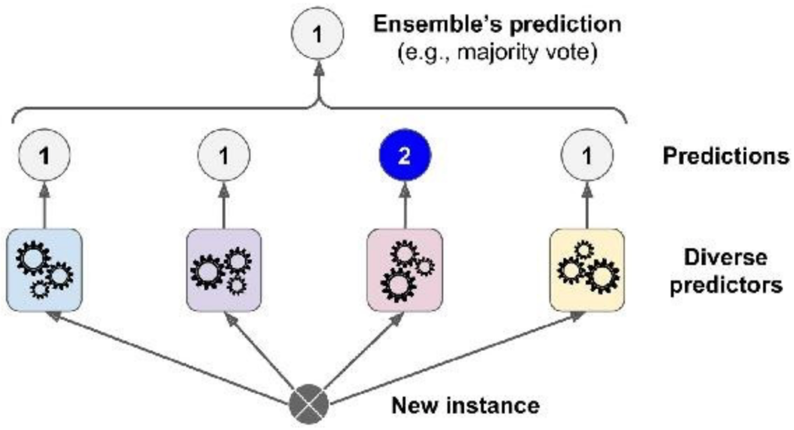
Remark: The error rate for random guessing is $e = 0.5$.

Remark: The majority vote explains the statistical rationale behind the election. 民主制度投票的合理性。However, there are **two necessary conditions** for an ensemble classifier to perform better than a single classifier:

1. The base classifiers should be independent of each other.
2. The base classifier should do better than a classifier that performs random guessing.

In practice, ensuring **total independence** among the base classifiers is difficult. Nevertheless, even though the base classifiers are correlated, improvements in classification accuracy have been observed in the ensemble methods. The winning solutions in machine learning competitions often involve several ensemble methods.

2. Python example: majority vote



Let us use `makes_moon` to make two interleaving half circles and creates and trains a voting classifier in `Scikit-Learn`, composed of three diverse classifiers. Let us first create and plot the data:

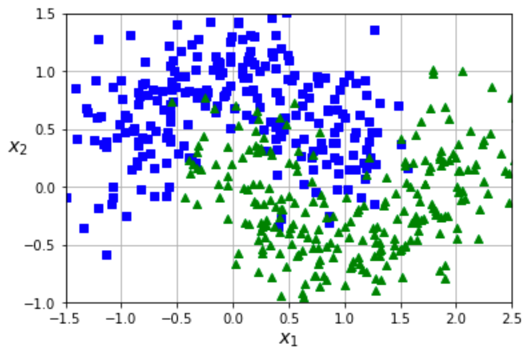
```
%matplotlib inline
import matplotlib.pyplot as plt
import numpy as np

from sklearn.model_selection import train_test_split
from sklearn.datasets import make_moons

X, y = make_moons(n_samples=500, noise=0.30, random_state=42)

def plot_dataset(X, y, axes):
    plt.plot(X[:, 0][y==0], X[:, 1][y==0], "bs")
    plt.plot(X[:, 0][y==1], X[:, 1][y==1], "g^")
    plt.axis(axes)
    plt.grid(True, which='both')
    plt.xlabel(r"$x_1$", fontsize=14)
    plt.ylabel(r"$x_2$", fontsize=14, rotation=0)

plot_dataset(X, y, [-1.5, 2.5, -1, 1.5])
plt.show()
```

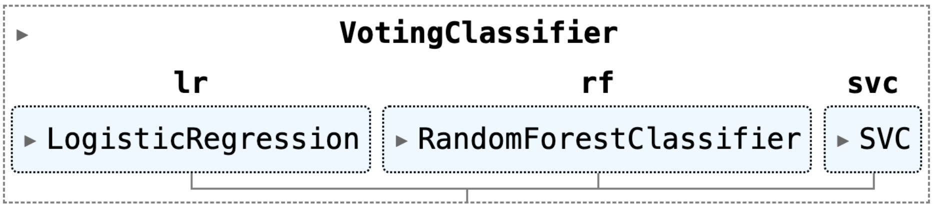


Now, the training and testing:

```
X_train, X_test, y_train, y_test = train_test_split(X, y,
random_state=42)
from sklearn.ensemble import VotingClassifier
from sklearn.ensemble import RandomForestClassifier
from sklearn.linear_model import LogisticRegression
from sklearn.svm import SVC
from sklearn.metrics import accuracy_score

log_clf = LogisticRegression(random_state=42)
rnd_clf = RandomForestClassifier(random_state=42)
svm_clf = SVC(random_state=42)

voting_clf = VotingClassifier(
    estimators=[('lr', log_clf), ('rf', rnd_clf), ('svc', svm_clf)],
    voting='hard')
```




```
for clf in (log_clf, rnd_clf, svm_clf, voting_clf):
    clf.fit(X_train, y_train)
    y_pred = clf.predict(X_test)
    print(clf.__class__.__name__, accuracy_score(y_test, y_pred))
```

LogisticRegression 0.864

RandomForestClassifier 0.896

SVC 0.896

VotingClassifier 0.912

There you have it! The voting classifier slightly outperforms all the individual classifiers. You can download the above source code `MajorityVote.ipynb` from the course website.