Transportation Research Part C 137 (2022) 103553

Contents lists available at ScienceDirect

Transportation Research Part C

journal homepage: www.elsevier.com/locate/trc

Women-only parking spaces: Determinants of parking space choice, investment decision, and a case study of Wuhan, China

Zhi-Chun Li *, Wen-Jing Liu, Xiao-Yan Wang

School of Management, Huazhong University of Science and Technology, Wuhan 430074, China

A R T I C L E I N F O

Keywords:
Women-only parking space
Determinant
Parking time
Parking fee
Multiple regression
Investment
Profit maximization

A B S T R A C T

This paper investigates the effects of introducing women-only parking spaces on drivers' parking behavior and operator's investment behavior based on a field survey of a parking lot in Wuhan, China. An econometric model is presented to identify the major determinants of influencing drivers' time for completing the parking process at a parking space and their parking choice decisions towards the women-only parking spaces. Optimization models are developed to jointly determine the optimal proportion of the women-only parking spaces in a parking lot and/or the optimal parking fees so as to maximize the total net profit of operator, which is from the operations of car park and shopping mall. A case study from Wuhan China is conducted for illustrating the applications of the models. The findings show that driver's gender, driving age, vehicle length, and parking environment (obstacles on one or both sides of a parking space) have significant impacts on the time for completing the parking process at a parking space. Women-only parking spaces can help reduce such time. Driver's gender, driving age, and the past parking experience at the women-only parking spaces significantly influence driver's choice of the women-only parking spaces, whereas educational degree and the past parking experience have no statistically significant effects. However, investment in the women-only parking spaces may lead to a loss in the net profit of operator, and thus the investment decision on the women-only parking spaces should be carefully made for the operator in practice. These findings have important implications for planning of parking facilities and evaluation of parking policies.

1. Introduction

1.1. Background and motivation

The emergence of women-only parking spaces in some large Chinese cities, such as Beijing, Shanghai, and Wuhan, have recently received considerable attention. The concept of the women-only parking spaces may be traced back to the 1990s, in which Germany first introduced this type of parking facilities, referred to as "frauenparkplatz" in German.[1] Currently, several cities in Korea, Indonesia and Malaysia have also introduced such parking spaces designated for female drivers.[2] In the Chinese cities, the women-only parking spaces are currently prioritized for female drivers, but are also available for use by male drivers when the regular parking

* Corresponding author.
E-mail address: smzcli@hust.edu.cn (Z.-C. Li).

[1] https://stuckattheairport.com/2015/08/04/frankfurt-airport-has-pink-parking-spaces-for-women.
[2] https://en.wikipedia.org/wiki/Women%27s_parking_space.

https://doi.org/10.1016/j.trc.2022.103553
Received 15 June 2021; Received in revised form 14 November 2021; Accepted 2 January 2022
Available online 1 February 2022

spaces are insufficient.

Currently, the women-only parking spaces in Wuhan (a city located in Central China) are mainly distributed in the underground parking lots of some big shopping malls, such as Wanda Plaza at Han Street and K11 Art Mall at Guanshan Road.[3] Both the underground parking lot and the shopping mall above such a parking lot, which are called a building complex in this paper for ease of presentation, are usually operated by an identical operator. Total revenue of the operator is the sum of total parking fees and total sale revenue of the shopping mall. These women-only parking spaces are generally close to the lifts so as to enter the storeys of the malls conveniently. A women-only parking space usually has a bigger area than a regular parking space, and is marked by a color different from a regular parking space. Fig. 1 shows the profiles of typical women-only and regular parking spaces in the underground parking lot located at Han Street, Wuhan. The two types of parking spaces are marked in pink and green, respectively.

The main motivation of introducing the women-only parking spaces for the park operator (or the operator of the building complex in this paper) is to attract more women customers to patronize the shopping mall so as to increase commercial revenue of the shopping mall. However, the land value in the shopping mall area becomes increasingly expensive with rapid urbanization and economic development. As a result, the rents of parking spaces in this area and thus the parking operating cost dramatically increase. Therefore, it is very important for a profit-driven operator to carefully make decision of investing in the women-only parking spaces based on a trade-off between the increased revenue due to increased parking demand and increased number of shoppers and the increased parking operating cost due to rising land rents. The women-only parking spaces are currently a new thing in China, and the operators usually determine the number and/or parking fees of the women-only parking spaces only according to their intuition and perception, and no scientific approach can be used as a reference of decision-making. Accordingly, an appropriate approach is urgently needed, particularly at an initial development stage of the women-only parking spaces. This study just serves this purpose.

In addition, the emergence of the women-only parking spaces in Chinese cities has stirred up widespread debates. Some respondents think that introduction of the women-only parking spaces is a kind of humanistic care towards women because they make ladies park their cars easier and enter the shopping mall more conveniently. However, some respondents think that the provision of the women-only parking spaces is a kind of discrimination towards women because it implicitly assumes that female drivers are less skilled in car driving than male drivers, and thus need big parking spaces to park easier. Such distinct attitudes towards the women-only parking spaces raise some important and intriguing issues. For example, is it necessary to provide the women-only parking spaces? Can the women-only parking spaces really reduce the time for completing the parking process at a parking space? What factors affect drivers' choice of women-only parking spaces? Given that the women-only parking spaces are introduced in a parking lot, what is the optimal ratio, and how much the optimal parking fee is? This paper will attempt to address these intriguing issues.

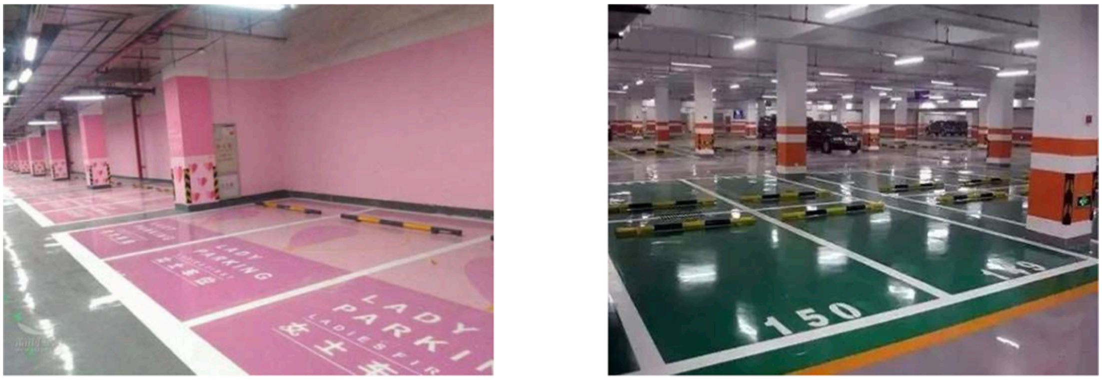
1.2. Review of related literature

In literature, there are a number of studies involving parking issues, including parking behavior modeling, parking demand forecasting, and parking pricing etc. The existing parking behavior modeling studies can generally be classified into two categories: one concerns the choice of parking location or lot (ignoring travel choice behavior on the road network), and the other considers the combined parking and travel choices. The former determines the parking location / lot choice based on various influencing factors, including the characteristics of parking spots and drivers (Brooke, 2015; Shaaban and Pande, 2016). Discrete choice modeling approaches are usually adopted, such as multinomial logit models (Van Der Goot, 1982; Axhausen and Polak, 1991; Thompson and Richardson, 1998), nested logit models (Hunt and Teply, 1993; Hensher and King, 2001), mixed logit models (Hess and Polak, 2004), and multinomial probit models (Qin et al., 2020). The latter attempts to reveal the interaction between parking congestion and travel choices on road network (e.g., route, departure time, and mode choices). Such integrated models usually adopt a network equilibrium modeling approach, which determines the demand-supply equilibrium of parking services and travel choices of drivers in the network (Gur and Beimborn, 1984; Bifulco, 1993; Lam et al., 1999, 2006; Tong et al., 2004a; Li et al., 2007, 2008; Leurent and Boujnah, 2014; Liu, 2018; Du et al., 2019). For a detailed review of the topic, we refer readers to Young et al. (1991) and Parmar et al. (2020).

Parking demand forecasting, which is a basis of efficient planning and management of parking facilities, aims to forecast the temporal and spatial distributions of parking demand in urban areas. The prevailing parking demand forecasting models can be classified into parking (and trip) generation rate models, regression models, and data-driven models. For example, Steiner (1998) estimated the level of required parking to support the level of trip generation for each of shopping areas in the Oakland-Berkeley subarea of the San Francisco Bay Area. Al-Sahili and Hamadneh (2016) presented parking generation rate models for forecasting the parking demand of three major types of land uses (residential, office, and retail) in some Palestinian cities. The regression approach was used to estimate the demand for each parking activity in each zone of a city concerned, and was successfully applied in the study of Hong Kong's parking demand (Lam et al., 1998; Wong et al., 2000; Tong et al., 2004b; Lau et al., 2005). Dave et al. (2019) proposed a method to assess the on-street parking demand for commercial and hospital land-use types in central business districts (CBD) of developing countries using data monitoring techniques. Guan et al. (2020) proposed a big data-driven framework for parking demand forecasting. It consists of two steps: parking zone division based on the statistical information grid and multi-density clustering algorithms, and parking demand estimation based on support vector machines. For a comprehensive review of parking demand forecasting, readers can refer to Lim et al. (2017) and Wong et al. (2021).

Parking pricing has been widely recognized as an effective measure to alleviate parking congestion in busy parking lots during rush

[3] https://www.sohu.com/a/205959684_100000914.



(a) (b)

Fig. 1. Two types of parking spaces at Han Street of Wuhan, China: (a) women-only parking spaces; (b) regular parking spaces.

hours. The basic idea behind the parking pricing policy is to control the spatial and temporal distributions of parking demand by properly setting parking rates to maintain an ideal level of parking occupancy such that any newly arriving drivers can always find a suitable open space without circling around. Shoup (1999) argued that parking can generate a large external cost, cities could price on-street parking rather than require off-street parking, and the market prices can allocate parking spaces fairly and efficiently compared to the minimum parking requirements. Hensher and King (2001) investigated the role of parking pricing and supply by time of day in whether to drive and park in the CBD. The result showed that the change in CBD parking share attributable to supply by time of day is less than 3%, compared to 97% attributable to parking prices. Anderson and de Palma (2004) examined the benefits of pricing parking, and found that when cruising for parking congests both parkers and through traffic, the benefits from pricing are substantially reduced. Qian and Rajagopal (2014) investigated how recurrent parking demand can be managed by dynamic parking pricing and information provision in the morning commute. Nourinejad and Roorda (2017) looked at the impact of hourly parking pricing on travel demand, and found that hourly parking pricing can reduce or induce demand depending on the parking dwell time elasticity (to the hourly parking price). Balac et al. (2017) examined the impact of parking price policy on free-floating carsharing with Zurich of Switzerland as a case study. Alemi et al. (2018) studied the effect of a demand-response parking pricing program in San Francisco on both the time and distance of cruising for parking. It showed that average parking search time and distance decline by approximately 15% and 12%, respectively, from the control to the treatment areas. Khordagui (2019) explored the impact of parking prices on the decision to drive to work using a California household travel survey dataset. Mo et al. (2021) examined the impacts of on-street parking price adjustment on parking demand and user satisfaction with an application to Nanning, China.

These parking modeling studies mentioned above mainly focused on homogeneous parking services. Although one can find some studies involving heterogeneous parking services in the literature (e.g., some studies considered the competition of two parking groups: reserved and non-reserved parking spaces, such as Liu et al. (2014), Shao et al. (2016), Lei and Ouyang (2017), Wang and Wang (2019), they did not involve the issues of the women-only parking spaces. Some exceptions are as follows. For example, Zainon (2013) investigated the perception of women drivers towards the women-only parking spaces based on a case study of Suria KLCC (Kuala Lumpur City Centre), which is one of the most popular shopping complexes in Malaysia. Zulhasni (2015) designed a gender identification system for the women-only parking lot of the Suria KLCC. Such a system aims to ensure the safety of all the women who park their cars in the parking zones which are often quiet and dark. Iqbal (2018) evaluated the environment of women-only parks in Karachi Pakistan, in terms of crime prevention, users' and non-users' perceptions of safety, and the relationship between the environment of the women-only parks and the women's perceptions. Dunckel-Graglia (2013) examined and evaluated the women-only transportation services in Mexico City, focusing on the roles of culture and public opinion. She found that the violence that women face in public transit leads them to always opt for women-only services, encouraging local policy-makers to increase their amounts. Shahrokni (2014) analyzed the development of women-only parks as a major site of gender segregation in Iran, and explored how Iran now promotes women's outdoor exercise as a solution to women's health problems. Rivadeneyra et al. (2015) conducted a survey to identify the attitudes of different stakeholders (transit passengers, transit operators, and the government) in the system towards the issue of whether the gender-exclusive public transport services can reduce the gender-based violence and sexual harassment. Arjmand (2017) analyzed both technical and social aspects of women-only parks through addressing the relationships between ideology, urban planning and gender, and interpreted power relations and how to define and plan public and semi-public urban spaces. Mejia-Dorantes and Soto Villagran (2020) explored the link between urban planning and transport from a gender perspective, and discussed the effects of female-only public transit services on gender in equality based on the case study of Mexico City. These previous related studies mainly focused on the evaluation of the effects of the women-only parking spaces on the female travel safety, violence, and sexual harassment from a perspective of sociology. They did not answer the determinants of influencing the drivers' choices of the women-only parking spaces and the investment decision of the profit-driven park operator from an angle of economics. The answers to the influencing factors and investment decisions are especially important for more properly understanding the parking behavior of drivers and for more effective planning and management of parking spaces in a car park, particularly at an initial development stage of the women-only parking spaces.

1.3. Problem statement and contributions

In view of the above discussions, this paper aims to: (i) find out the major factors of influencing drivers' parking space choices, and (ii) optimize the investment plan of the women-only parking spaces for a profit-driven private investor. The influence factors identified in item (i) are closely related to the parking demand in a parking lot, which significantly affects the revenue of the private operator and thus its investment decision presented in item (ii). The main contributions of this paper are twofold. First, an econometric model is presented for identifying the major determinants of influencing the time for completing the parking process at a parking space and the women-only parking space choice, in terms of the characteristics of drivers, parking spaces, and vehicles. Second, optimization models are developed for jointly determining the optimal proportion of the women-only parking spaces in a parking lot and/or the optimal parking fees of the women-only and regular parking spaces so as to maximize the total net profit of the operator, which is from the parking fees and the sale revenue of the shopping mall. A case study from Wuhan China is implemented for model illustration purpose. The results show that driver's gender, driving age, vehicle length, and parking environment (obstacles on one or both sides of a parking space) have significant impacts on the time required to complete the parking process at a parking space. Women-only parking spaces can help reduce such time. Driver's gender, driving age, and the past parking experience at the women-only parking spaces significantly influence driver's choice of the women-only parking spaces, whereas educational degree and the past parking experience have no statistically significant effects. However, investment in the women-only parking spaces may lead to a loss in the net profit of park operator, and thus the investment decision on the women-only parking spaces should be carefully made for the park operator in practice. The proposed methodology in this paper can serve as a useful tool for evaluating the feasibility and efficiency of introducing the women-only parking spaces and for optimizing the quota and parking fees of parking spaces for a given parking lot.

The remainder of this paper is organized as follows. In the next section, some basic information about survey and data collection, such as study area and questionnaire design, is described. Section 3 identifies the major factors of influencing driver's time of parking process at a parking space. Section 4 ascertains the major determinants of affecting driver's choice of the women-only parking spaces. In Section 5, the optimal proportions of the women-only parking spaces and the optimal parking fees for a parking lot under various scenarios are determined. In Section 6, the properties of the optimization models with a linear parking search time function are analytically explored. In Section 7, a case study is conducted based on the data of a parking lot located at Han Street, Wuhan China. Section 8 concludes this paper, discusses policy implications, and provides recommendations for further studies.

2. Data collection

2.1. Study area and parking lot

In this paper, an underground parking lot at Han Street, Wuhan is selected as the object of study, as shown in Fig. 2. There is a river

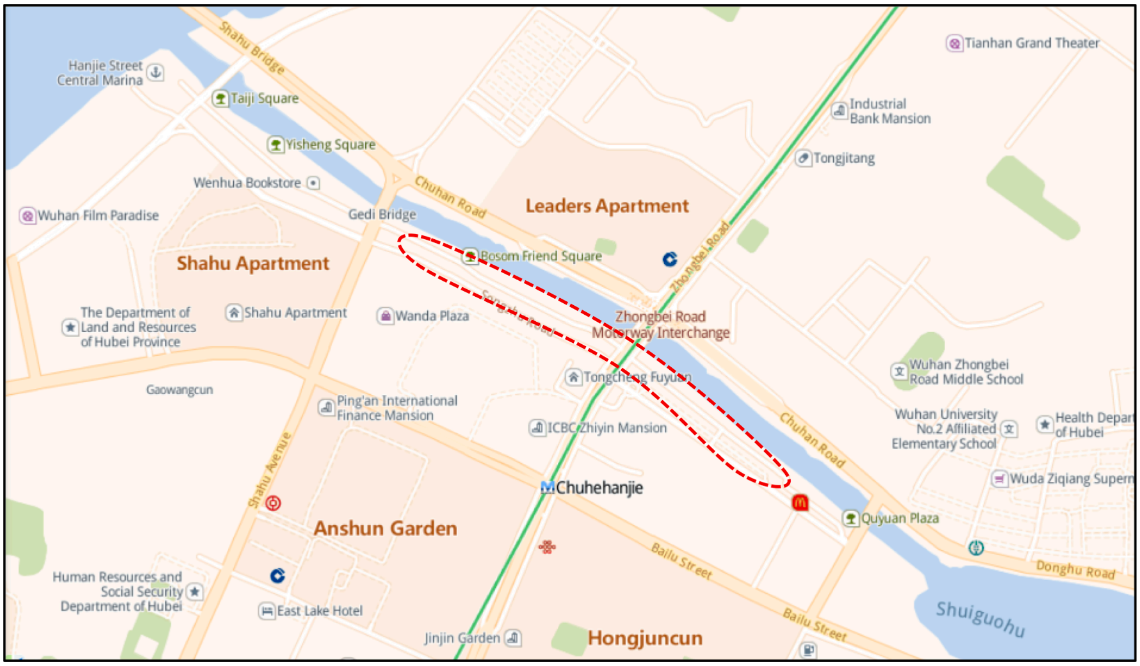


Fig. 2. The geographical location of Han Street in Wuhan, China.

with a length of 2.2 km, called Chu river, connecting Sha lake and East lake. Han Street and the underground parking lot are distributed along the Chu river (i.e., the area in the red dotted line in Fig. 2). Han Street was operated since 2011 as a commercial pedestrian street with a length of 1.5 km, and is a CBD of Wuhan, in which there are more than 200 business firms, including Wanda Plaza shopping mall, brand clothing stores, luxury stores, senior/five-star hotels, Starbucks, theater, cinema, and banks. The general office of the government of Hubei province is close to Han Street, together with Wuhan University. Most of the pedestrians and customers at this street are thus young people. Thereby, Han Street is also called "paradise of young people".

Since 2018, 23 women-only parking spaces were introduced into the underground parking lot of Han Street. There are currently a total of 1408 parking spaces. The women-only parking spaces are wider and longer than the regular ones. Table 1 shows the lengths and widths of the two types of parking spaces. Specifically, the length and width of a regular parking space are, respectively, 5.19 m and 2.48 m, implying an area of 12.8712 square meters. The length and width of a women-only parking space are 0.39 m and 0.13 m larger than those of a regular parking space, respectively. As a result, the area of a women-only parking space is 1.6926 square meters larger than that of a regular parking space, and their area proportion is 1.13.

2.2. *Field and questionnaire surveys*

The population considered in this study is all the parkers using the underground parking lot of Han Street. In order to collect relevant data, field survey was conducted in this parking lot from August 2018 to March 2019. The information collected includes:

- Respondent's personal information, such as gender, age, driving experience, education, income, and occupation;
- Past parking experience of drivers, including the difficulty in finding a parking space, high parking fee, narrow parking slot, number of parking accidents (if any);
- The time for completing the parking process, counting from the time that vehicle starts to enter the parking trajectory to the time that driver gets off;
- Parking environment, i.e., whether there exist **obstacles** on one side or both sides of the parking space selected;
- Vehicle information, including length, width, and parking navigation equipment;
- Awareness and attitudes towards the women-only parking spaces (some relevant questions designed in the questionnaire are shown in Table 2).

In this field survey, some information was collected through a face-to-face questionnaire, including drivers' personal information, past parking experience, and their awareness and attitudes towards the women-only parking spaces. The interviewers asked the respondents questions according to the questionnaire, and then the respondents answered them one by one. There are also some parameters, the values of which are obtained through on-site measurement by our interviewers. For example, the lengths and widths of vehicles are obtained by the interviewers using rulers, and the drivers' time for completing the parking process at a parking space was counted by the interviewers using a stopwatch timer. In addition, the information about parking environment was collected through on-site observation.

In this questionnaire survey, a total of 950 questionnaires were handed out, and 907 valid questionnaires were collected after getting rid of some invalid samples with incomplete and/or logically inconsistent responses. In order to evaluate the validity of samples, we attempted to make a comparison between the sample distribution and the population distribution. However, unfortunately, the detailed information of the respondents, such as the distributions of age, gender, and education of all parkers, is not available. This is because the respondents do not need to tell the park operator such information when parking their cars at the parking lot. To this end, we extracted randomly multi-week gender data from the samples, and found that the gender distributions by week are basically consistent and robust. We also checked the monthly gender / age distributions that were randomly extracted from the samples. It showed that the consistency and robustness still hold, which justifies the validity of the samples.

After making a thorough analysis of samples, we obtain the demographic characteristics of the respondents, as shown in Fig. 3. It can be seen that in the sample data, the male drivers account for 64.7%, and the female drivers account for 35.3%. The age distribution is approximately normal over interval [18, 65], and most of drivers are between 21 and 40 years old. The distribution of driving age is also approximate to a normal distribution. The proportion of the highly educated (bachelor degree or above) respondents reaches 85.3%. This is because the parking lot under study is located at a high-end commercial street of Wuhan, surrounding by governmental departments and university, in which most of visitors or customers are young people with good educational experience.

Fig. 4 shows the statistical distribution of the respondents for the questions of the awareness and attitudes towards the women-only parking spaces. Fig. 4a indicates that only 13% of the respondents have ever used the women-only parking spaces, and 60% of the respondents have only heard of this type of parking spaces but never used them. This may be due to limited development of the women-

Table 1
The characteristics of two types of parking spaces.

Type of parking space	Length (m)	Width (m)	Area (m^2)
Regular parking space	5.19	2.48	12.8712
Women-only parking space	5.58	2.61	14.5638
Difference	0.39	0.13	1.6926
Ratio	1.08	1.05	1.13

Table 2
A survey of awareness and attitudes towards women-only parking spaces.

Q1: Did you hear of the women-only parking spaces before? • Have already used • Never hear • Hear, but never used	**Q2: What is your attitude towards women-only parking space?** • A show-off • A care for women • Discrimination for women
Q3: Do you think it is necessary to set up exclusive parking space for female drivers? • Yes • No • Don’t care	**Q4: Is it currently necessary to increase the number of women-only parking space?** • Yes • No • Don’t care

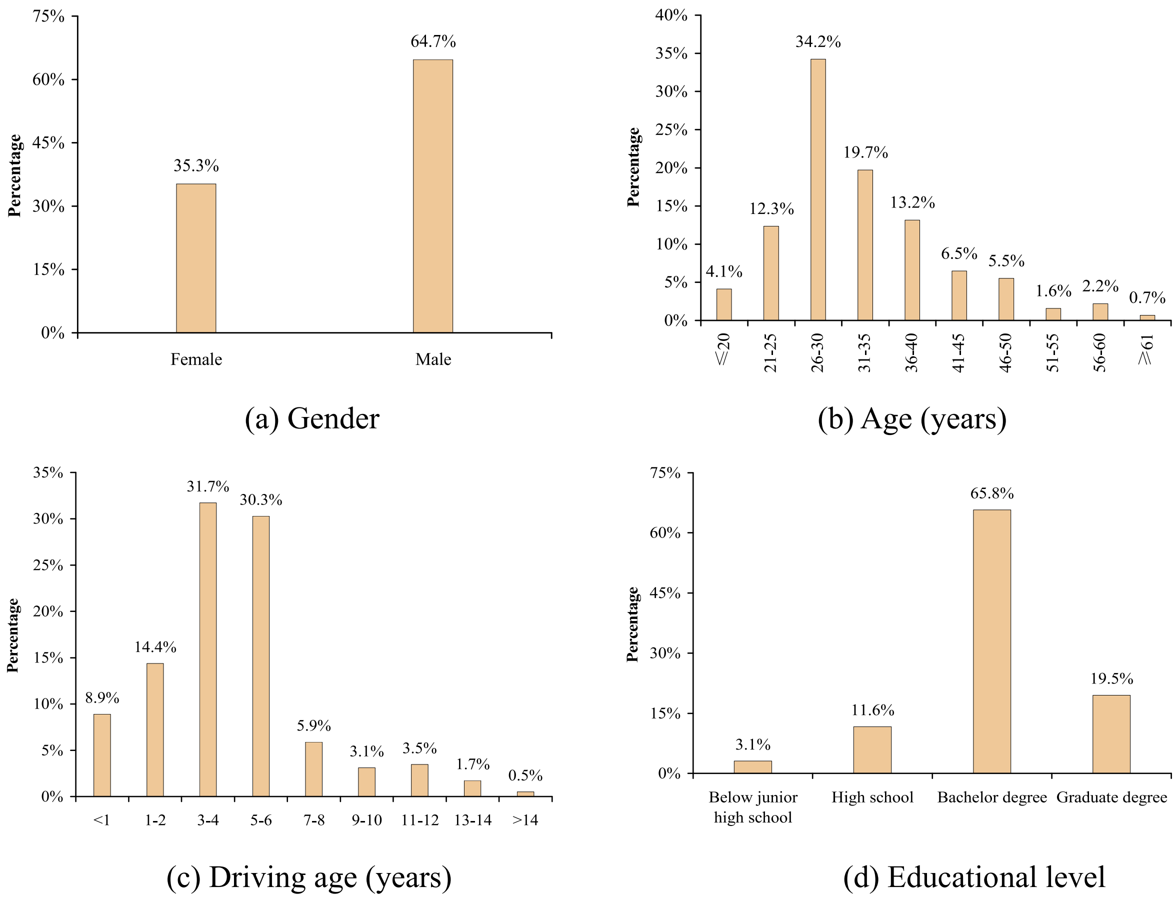


Fig. 3. Demographic characteristics of respondents.

only parking spaces in Wuhan such that they have no opportunity to experience. Fig. 4b shows that most of people (a proportion of 79%) have a positive attitude towards the women-only parking spaces, with a belief that the provision of the women-only parking spaces is a kind of care for the female drivers. Some feminists (about 7%) think that such a privilege for women is discrimination. Fig. 4c shows that over half of the respondents think that the provision of the women-only parking spaces is necessary to facilitate the parking of the female drivers. Moreover, it is expected to increase the investment of the women-only parking spaces in the future, as shown in Fig. 4d.

3. Determinants of influencing the time for completing parking process

In this section, we identify the determinants of influencing the time for completing the parking process at a parking space. Fig. 5 shows the distributions of such time for the women-only and regular parking spaces. From this figure, one can calculate the expected

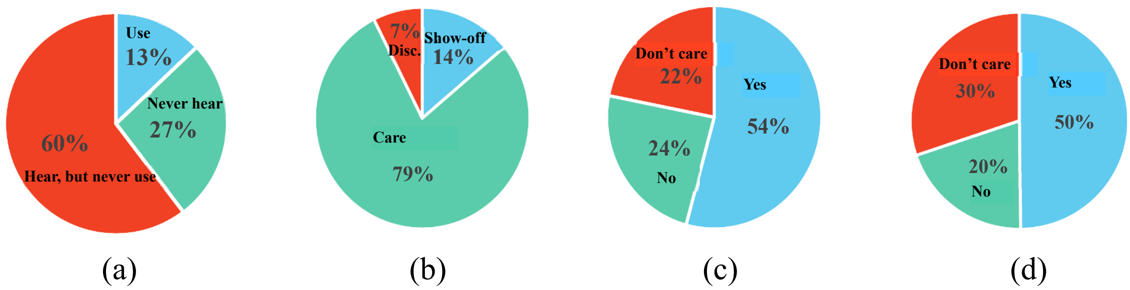


Fig. 4. Statistical distributions of respondents for four questions: (a) Q1, (b) Q2, (c) Q3, and (d) Q4.

values of the time of parking process for these two types of parking spaces, which are 33.59 and 36.23 s, with standard deviations of 18.10 and 18.98 s, respectively. This means that the women-only parking spaces can reduce drivers' average time for completing the parking process and its variance, compared to the regular parking spaces. Naturally, one wants to know what factors can significantly affect the drivers' time spent on parking process at a parking space.

To this end, we first determine the possible domain of the influencing factors. Note that the stakeholders in the parking system concerned include parkers, vehicles, and parking spaces. The factors influencing the time for completing the parking process should thus be related to these stakeholders, and their attributes should be included as many as possible. When designing the questionnaires for this study, we made a list of parker's attributes (including gender, age, driving age, parking accident), vehicle's attributes (including length, width), and parking space's attributes (including parking type, parking environment). A pilot survey is implemented, such that the characteristics of all stakeholders can be adequately reflected in the variables selected. Table 3 indicates the descriptive statistics of sample data. We then use a multiple regression model to identify the major ones among the candidate factors.

The multiple regression model adopted is as follows.

$$\begin{aligned} lnz = \alpha_0 + \alpha_1 lnAge + \alpha_2 lnDriving\_Age + \alpha_3 lnVeh\_Length + \alpha_4 lnVeh\_Width \\ +\alpha_5 Park\_Type + \alpha_6 Park\_Accident + \alpha_7 Single\_Obst + \alpha_8 Both\_Obst \\ +\alpha_9 Gender + \varepsilon, \end{aligned} \tag{1}$$

where

z= time for completing the parking process, counting from the time that vehicle starts toenter the parking trajectory (or turn the steering wheel) to the time that the vehicle stops steadily;
Age = age of driver;
Driving_Age = driving years of driver;
Veh_Length = vehicle's length (i.e., longest distance from the front of a vehicle to its rear);
Veh_Width = vehicle's width;
Park_Type = a dummy variable, equal to 1 if a driver chooses a women-only parking space, and 0 otherwise;

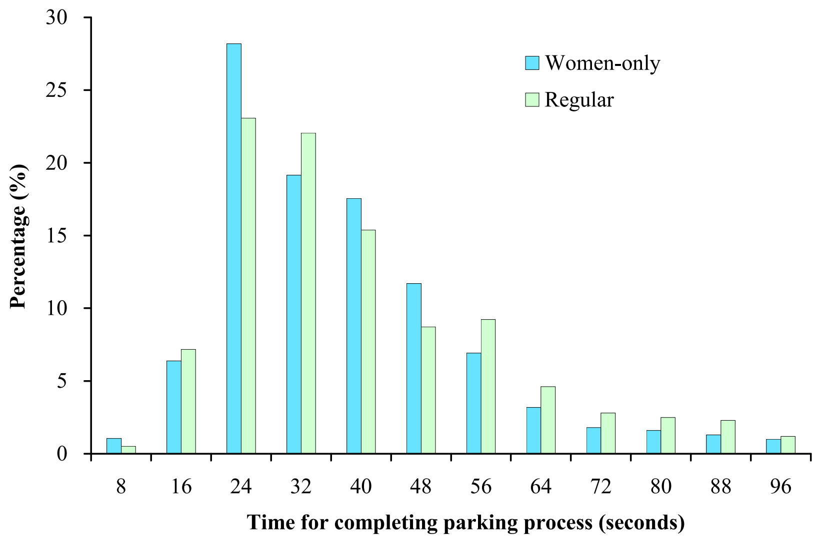


Fig. 5. Distributions of time for completing parking process for women-only and regular parking spaces.

Table 3
Descriptive statistics of sample data.

Variable	Mean	Std Dev	Min	Max
z	34.487	0.885	5.13	92.94
Park_Type	0.491	0.026	0	1
Age	33.904	0.436	18	65
Gender	0.353	0.025	0	1
Driving_Age	5.998	0.310	0.5	40
Park_Accident	0.060	0.012	0	1
Single_Obst	0.319	0.024	0	1
Both_Obst	0.553	0.025	0	1
Veh_Length	4.570	0.014	3.7	5.3
Veh_Width	2.067	0.005	1.75	2.8

Park_Accident = a dummy variable, equal to 1 if a driver suffers parking accident(s) in the past year, including scratches and collisions between vehicles;
Single_Obst = a dummy variable, equal to 1 if there is an obstacle (e.g., a car or a wall) on one side of the parking space only, and 0 if there is no obstacle on each side;
Both_Obst = a dummy variable, equal to 1 if there is an obstacle on each of two sides of the parking space, and 0 if there is no obstacle on each side;
Gender = a dummy variable, equal to 1 if the driver is female, and 0 otherwise;
α_i = parameters to be estimated, $i = 0, \ldots, 9$;
ε = an error term.

In order to weaken the multi-collinearity, a backward elimination method is adopted in this paper. This method sequentially removes the least significant variables, beginning with all the variables in the equation, until no non-significant variables remain. The retained explanatory variables and the estimation results with ordinary least squares (OLS) approach are shown in Table 4. The results show that compared to the regular parking spaces, the time for completing the parking process for the women-only parking spaces is reduced by 8.2%, which indicates that introducing the women-only parking spaces can indeed reduce driver's time spent on parking process and thus the total journey time. In addition, we find that the vehicle length, driver's gender, and parking environment (obstacles on one or both sides of a parking space) have positive effects on the time spent on the parking process, while driving age has negative effects, which is consistent with our anticipation.

4. Determinants of influencing women-only parking space choice

Driver's parking space choice depends on various factors, such as walking time, parking fee, occupation rate, and accessibility (Van Der Goot, 1982; Brooke, 2015; Shaaban and Pande, 2016; Christiansen et al., 2017; Soto et al., 2018). This section aims to identify the determinants of influencing driver's choice of the women-only parking spaces so as to improve the use efficiency of the women-only parking spaces. To do so, the drivers' characteristics, parking experience and attitudes towards the women-only parking spaces are considered. We divide the willingness to choose the women-only parking spaces into three categories: strong willingness which means that the respondents prefer to use the women-only parking spaces between two options: regular and women-only parking spaces; moderate willingness which means the respondents are indifferent in both options, i.e., 50% probability for each option; and low willingness which means that the respondents have no motivation to use the women-only parking spaces, including the case of never using the women-only parking spaces. Given that the willingness is characterized in an ascending order from low-level willingness to high-level willingness, the Ordered Logit (OL) modeling approach is adopted in this study. The OL model is a regression model for ordinal dependent variables. It can be regarded as an extension of the logistic regression model that is applied to dichotomous dependent variables, allowing for more than two (ordered) response categories. For a detailed description of the OL model, readers can refer to Crawford et al. (1998), Agresti (2007), Greene and Hensher (2010), Bellizzi et al. (2018), and Echaniz et al. (2019).

In this paper, the OL model is specified as follows:

Table 4
Estimation results of the variables.

Variable	Coefficient	Std Error	Prob.
Single_Obst	0.159**	0.081	0.049
Both_Obst	0.283***	0.075	0.000
ln*Driving_Age*	−0.060**	0.028	0.033
ln*Veh_Length*	1.185***	0.390	0.003
Gender	0.121**	0.052	0.020
Park_Type	−0.082*	0.048	0.093

Note: "*", "**" and "***" mean that the results are significant at 10%, 5% and 1% statistical levels, respectively.

$$Y_i^* = \boldsymbol{\beta}\boldsymbol{x}_i + \varepsilon_i, i = 1, 2, \ldots\ldots, n, \tag{2}$$

where Y_i^* is the ith ordinal dependent variable, n is the number of samples, and $\boldsymbol{\beta}$ is a m-dimensional vector to be estimated. $\boldsymbol{x}_i$ is a vector of m features of the ith sample, including age, driving years, educational degree etc. It is assumed to be independent of ε_i. The parameter ε_i is a continuous random variable, following a standard logistic distribution with a mean of 0 and a standard deviation of $\pi/\sqrt{3}$. This disturbance term reflects the fact that the variables may not be perfectly measured, and/or some relevant variables may not be completely introduced in the equation.

The ith latent dependent variable Y_i^* is observed in a discrete form through the following censoring mechanism:

$$\begin{cases} Y_i = 1, & \text{if } Y_i^* \leqslant c_1; \\ Y_i = 2, & \text{if } c_1 < Y_i^* \leqslant c_2; \\ & \ldots\ldots \\ Y_i = J, & \text{if } c_{J-1} < Y_i^*; \end{cases} \tag{3}$$

where J is the category or level of willingness with values 1 (low willingness), 2 (moderate willingness) and 3 (strong willingness). c_j ($j = 1,2,\ldots,J-1$) are called the threshold parameters in an increasing order, i.e., $c_1 < c_2 < \ldots < c_{J-1}$, which can be estimated based on n observed samples.

In this study, the dependent variable Y_i is defined as the driver's willingness to choose the women-only parking spaces, whereas the independent variable vector $\boldsymbol{x}_i$ represents the respondents' identity characteristics (e.g., gender, age, driving years, education), parking experience (e.g., parking difficulties due to narrow parking space, whether he/she suffers parking accidents in the past years), and their cognitions and attitudes towards the women-only parking spaces (e.g., necessity of setting up women-only parking spaces).

Table 5 shows some basic information of the demand and supply sides (including the characteristics of the respondents, parking experience and their attitudes towards the women-only parking spaces), and the variable representations. It should be pointed out that categorical variables can be denoted as dummy variables in the OL model, and J categories need J-1 dummy variables. For instance, age of drivers has 5 ranges, including ≤ 20, 20–30, 30–40, 40–50, and ≥ 50, and thus can be represented by four 0/1 variables. Specifically, the age interval [30, 40] is chosen as the reference age, "Age1 = 1" means an age less than 20, "Age2 = 1" means an age between 20 and 30, "Age3 = 1" means an age between 40 and 50, and "Age4 = 1" means an age exceeding 50.

We use an econometric software, called EViews, to estimate the parameters of the OL model.[4] Table 6 shows the estimation results of the OL model. It can be seen that the driver's gender, driving age, and whether he/she ever used the women-only parking spaces will have a significant impact on his/her choice of women-only parking spaces. In particular, female drivers are more likely to choose the women-only parking spaces, but male drivers tend not to choose. This is consistent with the original intention of constructing the women-only parking spaces. For a driver with over 6-year driving experience, he/she has not a strong willingness to choose a women-only parking space. A driver with an experience of parking at the women-only parking space tends to choose this type of parking space again, implying that the women-only parking space does bring some convenience for parking. In addition, the other variables, such as, the education level and whether the driver suffers from parking accidents in the past year, have no statistically significant effect on driver's choice of the women-only parking spaces.

5. Optimization of parking space quota and parking fees

In the previous sections, we have identified the factors of influencing the drivers' time for completing the parking process and their parking space choices. These factors significantly affect the parking demand in a parking lot, which further affects the private operator's revenue and thus investment decision on parking spaces, which are investigated in this section.

As previously stated, the women-only parking spaces are larger than the regular parking spaces in terms of length and width, leading to a larger land area requirement for the women-only parking spaces, as shown in Table 1. As a result, a women-only parking space requires a higher investment cost than a regular parking space, particularly for the parking lot at a CBD area in which the land procurement is more expensive than at the city's suburb area. However, the average time for completing the parking process for a women-only parking space is shorter than that for a regular parking space, according to Table 4 and Fig. 5. Hence, the demand for parking at the women-only parking spaces could be higher than that at the regular parking spaces. As a result, more customers are attracted to patronize the shopping mall and thus a higher profit, which is from the parking fees and the sale revenue of shopping mall, is incurred for a profit-driven operator of the building complex. There is, therefore, a trade-off between the increased investment cost in the women-only parking space and the associated increased profit. Consequently, for a given parking lot, the profit-driven operator of the building complex naturally expects to maximize its net profit by setting the proportion of the women-only parking spaces in a parking lot and/or the parking fees for the two kinds of heterogeneous parking facilities (i.e., differential products). In the following, we will determine the optimal numbers and/or optimal parking fees of the regular and women-only parking spaces for a given parking lot.

[4] EViews, as a statistical software, can be used for general statistical analysis and econometric analyses, such as cross-section and panel data analysis and time series estimation and forecasting. For more details, please see the website: http://www.eviews.com/home.html.

Table 5
Basic information of demand and supply sides and variable representations.

Feature descriptions	Value	Proportion	Variable representation
Characteristics of respondents			
Gender	Female	35.27%	Gender = 1
	Male	64.73%	Gender = 0
Age	≤20	4.11%	Age1 = 1
	20–30	46.58%	Age2 = 1
	30–40	32.88%	–
	40–50	11.99%	Age3 = 1
	≥50	4.45%	Age4 = 1
Driving age	≤2	23.29%	–
	2–6	61.99%	Driving_Age1 = 1
	≥6	14.73%	Driving_Age2 = 1
Education	Below junior high school	3.08%	Education1 = 1
	High school	11.64%	Education2 = 1
	Bachelor degree	65.75%	–
	Graduate degree	19.52%	Education3 = 1
Experience of parking			
Parking difficulties due to narrow parking space	Yes	54.79%	Trouble = 1
	No	45.21%	Trouble = 0
Parking accidents	Yes	21.23%	Park_Accident = 1
	No	78.77%	Park_Accident = 0
Attitudes towards women-only parking spaces			
Cognition	Never hear	26.37%	–
	Hear, but never used	60.62%	Cognition1 = 1
	Have already used	13.01%	Cognition2 = 1
Necessity	Low necessity	23.97%	–
	Don't care	22.26%	Necessity1 = 1
	Strong necessity	53.77%	Necessity2 = 1
Attitude	Discrimination for women	6.85%	–
	A care for women	79.11%	Attitude1 = 1
	A show-off	14.04%	Attitude2 = 1

Table 6
Estimation results of the OL model.

Variable	Coefficient	Std. Error	Prob.
Gender	0.654***	0.241	0.007
Driving_Age1	−0.377	0.277	0.173
Driving_Age2	−0.694*	0.374	0.064
Cognition1	0.124	0.262	0.634
Cognition2	0.994**	0.398	0.013
Necessity1	0.562*	0.322	0.081
Necessity2	0.409	0.283	0.148

Note: "*", "**" and "***" mean that the results are significant at 10%, 5% and 1% statistical levels, respectively.

Six schemes for optimizing the parking lot parameters are considered in terms of whether the parking fees / the numbers of the regular and women-only parking spaces are given or not.

- Scheme 1 (as the benchmark case) aims to jointly optimize the parking fees and numbers of the regular and women-only parking spaces so as to maximize the total net profit of the operator of the building complex.
- Scheme 2 is the same with Scheme 1, except that a uniform parking fee is applied, i.e., to determine the optimal numbers of the regular and women-only parking spaces and the optimal uniform parking fee.
- Scheme 3 determines the optimal numbers of the regular and women-only parking spaces and the optimal parking fee of the women-only parking spaces for a given parking fee of the regular parking spaces.
- Scheme 4 only optimizes the numbers of the regular and women-only parking spaces, with their parking fees given.
- Scheme 5 optimizes the (differential) parking fees of the regular and women-only parking spaces, with their respective number of parking spaces given.
- Scheme 6 optimizes a uniform parking fee of the regular and women-only parking spaces, with their respective number of parking spaces given.

Obviously, each of Schemes 2–6 is a special case of Scheme 1. The mathematical models for these schemes and their solutions are in turn formulated as follows.

5.1. Scheme 1: Joint optimization of number of parking spaces and parking fees

In order to represent the total net profit of the operator of the building complex, we first define the demand and the full price for parking in a parking lot. Parking demand is usually elastic and depends on the full price for parking. Let i denote the type of parking spaces, with $i = w$ or r, where "w" denotes the women-only parking spaces, and "r" denotes the regular parking spaces. Let q_i be the parking demand at type i's parking space, and p_i be the full price for parking at type i's parking space. The inverse parking demand function is given as

$$p_i = a_i - b_i q_i, \ \ i = w, r, \tag{4}$$

where the intercept a_i denotes the maximum willingness-to-pay of the parkers for parking at type i's parking space, and the slope b_i denotes the price sensitivity of drivers using type i's parking space, measuring how demand q_i reacts to a change in the full price received by drivers. In reality, the willingness-to-pay of parkers for the women-only parking spaces may be greater than that for the regular parking spaces, and thus their price sensitivity towards the women-only parking spaces may be higher than that towards the regular parking spaces. It is therefore plausible to assume that $a_w > a_r$ and $b_r > b_w$ hold.

The full price p_i consists of the time for searching for an available parking space d_i, the time for completing the parking process z_i at an available parking space, and the parking fee f_i, expressed as

$$p_i = \tau_d d_i + \tau_z z_i + f_i, \ \ i = w, r, \tag{5}$$

where τ_d is the value of parking search time, and τ_z is the value of the time for completing the parking process at an available parking space. z_i is given by Eq. (1) in Section 3, which is exogenously determined through a statistical analysis of sampling survey. The parking search time d_i in a parking lot can be calculated by the following Bureau of Public Roads (BPR) type function (see Lam et al., 1999, 2006; Li et al., 2007, 2008)

$$d_i = t_0 + \alpha\left(\frac{q_i}{N_i}\right)^{\beta}, i = w, r, \tag{6}$$

where α and β are positive parameters, which can be calibrated by survey data. N_i is the number of type i's parking space in a car park, and t_0 is the average parking search time for an available parking space at the free-flow condition in a parking lot. The second term on the right-hand side denotes the parking congestion effects. As the parking demand in a parking lot increases, the parking search time increases, and vice versa. In reality, it has been observed that drivers spend a significant percentage of their total trip time in searching for an available parking space (Axhausen and Polak, 1991).

As previously stated, the operator of the building complex simultaneously operates the car park and the shopping mall. The total net profit of the operator is thus the sum of the total parking profit and the total sale profit of the shopping mall. The former equals the total parking revenue minus the parking operating cost and the parking lot investment cost due to appropriation of land. The latter consists of the profit from shopping of the parkers who use the underground car park and of the customers who do not use the underground car park. In this paper, it is assumed that the number of the customers who do not use the underground car park is not affected by introduction of the women-only parking spaces and thus the sale profit from their shopping is a constant. Therefore, ignoring the sale profit from these customers in the total net profit of the operator does not affect the operator's decision solutions.

Let c_i be the marginal operating cost per type i's parking space, and s_i be the area per type i's parking space. Let $\widehat{c}$ denote the opportunity cost or value of land per unit area in a parking lot. Let g represent the average profit of the operator from parkers' shopping. Let $\Lambda(\cdot)$ represent the total net profit of the operator. The profit maximization problem for the profit-driven operator can thus be formulated as

$$\max \ \ \Lambda(N_w, N_r, f_w, f_r, q_w, q_r) = f_w q_w + f_r q_r + g(q_w + q_r) - (c_w N_w + c_r N_r) - \widehat{c}(s_w N_w + s_r N_r), \tag{7}$$

subject to

$$s_w N_w + s_r N_r = S, \tag{8}$$

$$a_i - b_i q_i = \tau_d\left(t_0 + \alpha\left(\frac{q_i}{N_i}\right)^{\beta}\right) + \tau_z z_i + f_i, \ \ i = w, r, \tag{9}$$

$$f_i^{min} \leqslant f_i \leqslant f_i^{max}, \ \ i = w, r, \tag{10}$$

$$N_i \geqslant 0, q_i \geqslant 0, \ \ i = w, r, \tag{11}$$

where S denotes the total available land area for the parking lot. f_i^{min} and f_i^{max} are the lower and upper bounds of the parking fees of type i's parking space, respectively. The first two terms on the right-hand side of Eq. (7) are the total parking revenue from the regular and women-only parking services, the third term is the total profit from the parkers' shopping in the shopping mall, and the last two terms are the total parking operating cost and the total parking facility investment cost, respectively. Eq. (8) is the constraint condition of total land area of the parking lot. Eq. (9) is the parking demand-supply equilibrium condition, which is obtained by Eqs. (4)-(6).

Inequality (10) is the boundary constraint of parking fees, and Inequality (11) includes the nonnegative constraints of the number of parking spaces and parking demand. It should be pointed out that the boundary or nonnegative constraints do not pose any difficulty in finding the optimal solution. In fact, once a decision variable exceeds its constraint bound (upper or lower bound), then it is set at the corresponding bound (i.e., the case of corner solution). Therefore, one needs only to analyze the case of interior solution using the Karush-Kuhn-Tucker condition.

From Eqs. (8) and (9), one can obtain

$$N_r = \frac{S - s_w N_w}{s_r}, \tag{12}$$

$$f_i = a_i - b_i q_i - \tau_d \left(t_0 + \alpha \left(\frac{q_i}{N_i} \right)^{\beta} \right) - \tau_z z_i,\ \ i = w, r. \tag{13}$$

Substituting Eqs. (12) and (13) into Eq. (7) yields an unconstrained optimization problem:

$$\begin{aligned} & max\ \Lambda(N_w, q_w, q_r) = q_w \left(a_w - b_w q_w - \tau_d t_0 - \tau_z z_w - \alpha \tau_d q_w^{\beta} N_w^{-\beta} + g \right) \\ & + q_r \left(a_r - b_r q_r - \tau_d t_0 - \tau_z z_r - \alpha \tau_d q_r^{\beta} \left(\frac{S - s_w N_w}{s_r} \right)^{-\beta} + g \right) - \left(c_w N_w + c_r \frac{S - s_w N_w}{s_r} \right) - \widehat{c} S. \end{aligned} \tag{14}$$

For presentation purpose, we denote $\varsigma_w = a_w - \tau_d t_0 - \tau_z z_w$ and $\varsigma_r = a_r - \tau_d t_0 - \tau_z z_r$. From the first-order optimality condition of optimization problem (14), we have

$$\begin{cases} \left(\varsigma_w - b_w q_w - \tau_d \alpha \dfrac{q_w^{\beta}}{N_w^{\beta}} + g \right) - \left(b_w q_w + \tau_d \alpha \beta \dfrac{q_w^{\beta}}{N_w^{\beta}} \right) = 0, \\ \left(\varsigma_r - b_r q_r - \tau_d \alpha q_r^{\beta} \left(\dfrac{S - s_w N_w}{s_r} \right)^{-\beta} + g \right) - \left(b_r q_r + \tau_d \alpha \beta q_r^{\beta} \left(\dfrac{S - s_w N_w}{s_r} \right)^{-\beta} \right) = 0, \\ \tau_d \alpha \beta q_w^{\beta+1} N_w^{-(\beta+1)} \dfrac{1}{s_w} - \tau_d \alpha \beta q_r^{\beta+1} \left(\dfrac{S - s_w N_w}{s_r} \right)^{-(\beta+1)} \dfrac{1}{s_r} - \dfrac{c_w}{s_w} + \dfrac{c_r}{s_r} = 0. \end{cases} \tag{15}$$

From Eq. (15), one can determine the values of q_w, q_r and N_w. The economic implications of the optimality conditions shown in Eq. (15) can be explained as follows. The terms in the first brackets of the first equation represent the marginal revenue of the operator of the building complex generated from the parkers of the women-only parking spaces and their patronages in the shopping mall. The terms in the second brackets represent the marginal utility of the parkers of the women-only parking spaces. Hence, the first equation shows that at the optimal parking demand level of the women-only parking spaces, the marginal revenue of the operator equals the marginal utility of the parkers of the women-only parking spaces. The terms in the first brackets of the second equation represent the marginal increase in the total revenue of the operator due to a marginal increase in the parking demand of the regular parking spaces. The terms in the second brackets represent the marginal effects on the utility of the parkers due to a marginal increase in the parking demand of the regular parking spaces. The second equation states that at the optimal parking demand level of the regular parking spaces, the marginal revenue of the operator is equal to the marginal utility of the parkers of the regular parking spaces. The first two terms of the final equation represent the marginal effects on the total revenue of the operator due to a marginal increase in the numbers of the women-only and regular parking spaces, respectively. The last two terms represent the marginal operating costs of the women-only and regular parking spaces, respectively. It states that at the optimal numbers of the women-only and regular parking spaces, the difference of the marginal revenue and the marginal operating cost of the women-only parking spaces equals that of the regular parking spaces.

Taking the second-order partial derivatives of objective function $\Lambda(\cdot)$ in Eq. (14) with regard to N_w, q_w, and q_r, one can obtain its Hessian matrix, represented by $H(\Lambda(\cdot))$, as follows.

$$H(\Lambda(\cdot)) = \begin{pmatrix} -\tau_d \alpha \beta (\beta+1) \left(\dfrac{q_w^{\beta+1}}{N_w^{\beta+2}} + \dfrac{q_r^{\beta+1} s_r^{\beta} s_w^2}{(S - s_w N_w)^{\beta+2}} \right) & \tau_d \alpha \beta (\beta+1) \dfrac{q_w^{\beta}}{N_w^{\beta+1}} & -\tau_d \alpha \beta (\beta+1) \dfrac{q_r^{\beta} s_r^{\beta} s_w}{(S - s_w N_w)^{\beta+1}} \\ \tau_d \alpha \beta (\beta+1) \dfrac{q_w^{\beta}}{N_w^{\beta+1}} & -\left(2b_w + \tau_d \alpha \beta (\beta+1) \dfrac{q_w^{\beta-1}}{N_w^{\beta}} \right) & 0 \\ -\tau_d \alpha \beta (\beta+1) \dfrac{q_r^{\beta} s_r^{\beta} s_w}{(S - s_w N_w)^{\beta+1}} & 0 & -\left(2b_r + \tau_d \alpha \beta (\beta+1) \dfrac{q_r^{\beta-1} s_r^{\beta}}{(S - s_w N_w)^{\beta}} \right) \end{pmatrix}. \tag{16}$$

It is easy to show that the Hessian matrix $H(\Lambda(\cdot))$ is negative definite, meaning that the function $\Lambda(\cdot)$ is concave, and thus there is a unique solution to the optimization problem (14). The values of f_w, f_r and N_r can then be uniquely determined by Eqs. (12) and (13).

5.2. Scheme 2: Optimizing the numbers of women-only and regular parking spaces and a uniform parking fee for both parking types

The only difference of Schemes 2 and 1 is the decision on the parking fee. Scheme 2 aims to find a uniform parking fee, i.e., $f_r = f_w = f$. Substituting it into the model of Scheme 1, one obtains the model of Scheme 2 as

$$\max \ \Lambda(N_w, N_r, f, q_w, q_r) = (f+g)(q_w+q_r) - (c_w N_w + c_r N_r) - \widehat{c}(s_w N_w + s_r N_r), \tag{17}$$

subject to Eqs. (8)-(11).

From the first-order optimality condition of the above optimization problem, one obtains

$$\begin{cases} q_w + q_r - \dfrac{f+g}{b_w + \tau_d \alpha\beta q_w^{\beta-1} N_w^{-\beta}} - \dfrac{f+g}{b_r + \tau_d\alpha\beta q_r^{\beta-1}\left(\dfrac{S - s_w N_w}{s_r}\right)^{-\beta}} = 0, \\ \dfrac{\tau_d \alpha\beta (f+g) q_w^\beta}{s_w N_w \left(b_w N_w^\beta + \tau_d\alpha\beta q_w^{\beta-1}\right)} - \dfrac{\tau_d\alpha\beta(f+g)s_r^\beta q_r^\beta}{(S - s_w N_w)\left(b_r(S-s_wN_w)^\beta + \tau_d\alpha\beta s_r^\beta q_r^{\beta-1}\right)} + \dfrac{c_r}{s_r} - \dfrac{c_w}{s_w} = 0, \\ \varsigma_w - b_w q_w - \tau_d \alpha q_w^\beta N_w^{-\beta} - f = 0, \\ \varsigma_r - b_r q_r - \tau_d\alpha q_r^\beta \left(\dfrac{S - s_w N_w}{s_r}\right)^{-\beta} - f = 0. \end{cases} \tag{18}$$

Solving the above system of equations, one can obtain the values of variables f, N_w, q_w and q_r. Once f, N_w, q_w and q_r are determined, then N_r can be calculated by $N_r = \frac{S - s_w N_w}{s_r}$.

The economic implications of Eq. (18) are as follows. The first two terms in the first equation represent the direct effects on the total revenue of the operator due to a marginal increase in the uniform parking fee of the women-only and regular parking spaces, respectively. The last two terms represent the indirect marginal effects on the total revenue through the changes in the parking demands of the women-only and regular parking spaces caused by the marginal increase in the uniform parking fee, respectively. The first equation means that at the optimal parking fee, the marginal revenue of the operator equals the marginal parking operating cost. The second equation shows that at the optimal parking numbers, the marginal net profit (marginal parking revenue minus marginal parking operating cost) of the women-only parking spaces is equal to that of the regular parking spaces. The final two equations states that at the optimal uniform parking fee, the full prices of the women-only and regular parking spaces are equal to the marginal utility of the associated parkers, respectively.

5.3. Scheme 3: Optimizing the numbers of women-only and regular parking spaces and the parking fee of women-only parking spaces, given the parking fee of regular parking spaces

We now consider Scheme 3, in which the parking fee of the regular parking spaces f_r is pre-given and fixed, and one needs to optimize the parking fee of the women-only parking spaces f_w, and the numbers of the women-only and regular parking spaces N_w and N_r. Similar to Scheme 1, the model for Scheme 3 can be given as

$$\max \ \Lambda(N_w, N_r, f_w, q_w, q_r) = f_w q_w + f_r q_r + g(q_w + q_r) - (c_w N_w + c_r N_r) - \widehat{c}(s_w N_w + s_r N_r), \tag{19}$$

subject to Eqs. (8), (9), (11) and $f_w^{min} \leqslant f_w \leqslant f_w^{max}$.

Compared to Eqs. (7)-(11) in Scheme 1, f_r is not any more a decision variable for Scheme 3. From the first-order optimality condition of model (19), one can determine the values of q_w, N_w, f_w and q_r by

$$\begin{cases} (f_w + g) - \left(b_w q_w + \tau_d\alpha\beta\left(\dfrac{q_w}{N_w}\right)^\beta\right) = 0, \\ \dfrac{\tau_d\alpha\beta}{s_w}\left(\dfrac{q_w}{N_w}\right)^{\beta+1} - \dfrac{(f_r+g)\tau_d\alpha\beta s_r^\beta q_r^\beta}{(S - s_w N_w)\left(b_r(S - s_w N_w)^\beta + \tau_d\alpha\beta s_r^\beta q_r^{\beta-1}\right)} - \dfrac{c_w}{s_w} + \dfrac{c_r}{s_r} = 0, \\ \varsigma_w - b_w q_w - \tau_d\alpha\left(\dfrac{q_w}{N_w}\right)^\beta - f_w = 0, \\ \varsigma_r - b_r q_r - \tau_d\alpha\left(\dfrac{q_r s_r}{S - s_w N_w}\right)^\beta - f_r = 0. \end{cases} \tag{20}$$

Once the values of q_w, N_w, f_w and q_r are calculated by Eq. (20), one can then determine the value of N_r in terms of $N_r = \frac{S - s_w N_w}{s_r}$.

The economic implications of Eq. (20) can be given below. The terms in the first brackets of the first equation represent the marginal revenue brought by the parkers of the women-only parking spaces and their patronages in the shopping mall. The terms in the second brackets represent the effects on the utility of the parkers due to a marginal increase in the parking demand of the women-only parking spaces. The first equation states that at the optimal demand level of the women-only parking spaces, the marginal revenue of the operator is equal to the marginal utility of the parkers. The second equation shows that at the optimal numbers of the women-only and regular parking spaces, the marginal net profit of the women-only parking spaces equals that of the regular parking spaces. The last two equations state that the full prices for the women-only and regular parking spaces equal the marginal utility of the associated parkers, respectively.

5.4. Scheme 4: Optimizing the numbers of women-only and regular parking spaces, given their parking fees

In this scheme, the parking fees f_w and f_r for the women-only and regular parking spaces are pre-given and fixed, and one only needs to optimize the numbers of the women-only and regular parking spaces N_w and N_r. The corresponding model for Scheme 4 is

$$\max\ \Lambda(N_w, N_r, q_w, q_r) = f_w q_w + f_r q_r + g(q_w + q_r) - (c_w N_w + c_r N_r) - \widehat{c}(s_w N_w + s_r N_r), \tag{21}$$

subject to Eqs. (8), (9), and (11).

From the first-order optimality condition of model (21), one can derive the solutions of q_w, q_r and N_w, satisfying the following system of equations:

$$\begin{cases} \varsigma_w - b_w q_w - \tau_d \alpha \dfrac{q_w^\beta}{N_w^\beta} - f_w = 0, \\ \varsigma_r - b_r q_r - \tau_d \alpha \dfrac{q_r^\beta}{N_r^\beta} - f_r = 0, \\ \dfrac{\tau_d \alpha \beta q_w^\beta (f_w + g)}{s_w N_w \left(b_w N_w^\beta + \tau_d \alpha \beta q_w^{\beta-1}\right)} - \dfrac{\tau_d \alpha \beta q_r^\beta (f_r + g)}{s_r N_r \left(b_r N_r^\beta + \tau_d \alpha \beta q_r^{\beta-1}\right)} - \dfrac{c_w}{s_w} + \dfrac{c_r}{s_r} = 0. \end{cases} \tag{22}$$

The first two equations state that the full prices for the women-only and regular parking spaces are equal to the marginal utility of the associated parkers, respectively. The last equation states that at the optimal numbers of the women-only and regular parking spaces, the marginal net profit of the women-only parking spaces is equal to that of the regular parking spaces.

5.5. Scheme 5: Optimizing the parking fees of the women-only and regular parking spaces, with the numbers of their parking spaces given

In this scheme, the numbers, N_w and N_r, of the women-only and regular parking spaces are pre-given and fixed. One only thus needs to optimize their parking fees f_w and f_r and the associated parking demands q_w and q_r. The model for Scheme 5 is

$$\max\ \Lambda(f_w, f_r, q_w, q_r) = f_w q_w + f_r q_r + g(q_w + q_r) - (c_w N_w + c_r N_r) - \widehat{c}(s_w N_w + s_r N_r), \tag{23}$$

subject to Eqs. (8)-(10), and $0 \leqslant q_i \leqslant N_i,\ \ i = w, r$.

From the first-order optimality condition of the above model, one can derive q_w and q_r, satisfying

$$\begin{cases} \left(\varsigma_w - b_w q_w - \tau_d \alpha q_w^\beta N_w^{-\beta} + g\right) - \left(b_w q_w + \tau_d \alpha \beta q_w^\beta N_w^{-\beta}\right) = 0, \\ \left(\varsigma_r - b_r q_r - \tau_d \alpha q_r^\beta N_r^{-\beta} + g\right) - \left(b_r q_r + \tau_d \alpha \beta q_r^\beta N_r^{-\beta}\right) = 0. \end{cases} \tag{24}$$

One can then determine the values of f_w and f_r by $f_w = \varsigma_w - b_w q_w - \tau_d \alpha q_w^\beta N_w^{-\beta}$ and $f_r = \varsigma_r - b_r q_r - \tau_d \alpha q_r^\beta N_r^{-\beta}$.

The terms in the first brackets of the first equation represent the marginal effects on the total revenue of the operator due to the marginal increase in the parking demand of the women-only parking spaces. The terms in the second brackets represent the marginal effects on the utility of the parkers due to the marginal increase in the parking demand of the women-only parking spaces. The first equation states that for the women-only parking spaces, the marginal revenue of the operator equals the marginal utility of the parkers. Economic implication of the second equation is similar to that of the first equation. It shows that for the regular parking spaces, the marginal revenue of the operator equals the marginal utility of the parkers.

For such a special scheme 5, one can check the effects of the average profit parameter g (see Eq. (7)) through identifying the signs of $\frac{\partial q_w}{\partial g}$, $\frac{\partial q_r}{\partial g}$, $\frac{\partial f_w}{\partial g}$ and $\frac{\partial f_r}{\partial g}$, expressed as

$$\frac{\partial q_w}{\partial g} = \frac{1}{2b_w + \tau_d \alpha \beta (\beta + 1) q_w^{\beta-1} N_w^{-\beta}} > 0, \tag{25}$$

$$\frac{\partial q_r}{\partial g} = \frac{1}{2b_r + \tau_d \alpha \beta (\beta + 1) q_r^{\beta-1} N_r^{-\beta}} > 0, \tag{26}$$

$$\frac{\partial f_w}{\partial g} = \frac{b_w + \tau_d \alpha \beta q_w^{\beta-1} N_w^{-\beta}}{2b_w + \tau_d \alpha \beta (\beta + 1) q_w^{\beta-1} N_w^{-\beta}} < 0, \tag{27}$$

$$\frac{\partial f_r}{\partial g} = \frac{b_r + \tau_d \alpha \beta q_r^{\beta-1} N_r^{-\beta}}{2b_r + \tau_d \alpha \beta (\beta + 1) q_r^{\beta-1} N_r^{-\beta}} < 0. \tag{28}$$

Eqs. (25)-(28) indicate that the parking demands, q_w and q_r, of the women-only and regular parking spaces are monotonically increasing with regard to g. However, their parking fees f_w and f_r are monotonically decreasing with regard to g.

5.6. Scheme 6: Optimizing a uniform parking fee for both the women-only and regular parking spaces, with the numbers of their parking spaces given

We now further look at a special case of Scheme 5, i.e., Scheme 6, in which a uniform parking fee f for different types of parking spaces is determined, given the number of each type of parking spaces. The model for Scheme 6 is

$$\max\ \Lambda(f, q_w, q_r) = (f+g)(q_w+q_r) - (c_w N_w + c_r N_r) - \widehat{c}S, \tag{29}$$

subject to Eqs. (8)-(10), and $0 \leqslant q_i \leqslant N_i,\ \ i = w, r$.

From the first-order optimality condition of model (29), one can derive the solutions of f, q_w and q_r, satisfying

$$\begin{cases} \varsigma_w - \tau_d \alpha \dfrac{q_w^\beta}{N_w^\beta} - b_w q_w - f = 0, \\ \varsigma_r - \tau_d \alpha \dfrac{q_r^\beta}{N_r^\beta} - b_r q_r - f = 0, \\ q_w + q_r - \dfrac{f+g}{b_w + \tau_d \alpha \beta q_w^{\beta-1} N_w^\beta} - \dfrac{f+g}{b_r + \tau_d \alpha \beta q_r^{\beta-1} N_r^\beta} = 0. \end{cases} \tag{30}$$

The first two equations state that the full price equals the marginal utility for each of the women-only and regular parking spaces. The third equation states that at the optimal uniform parking fee, the marginal revenue of the operator equals the marginal parking operating cost.

In light of the above discussions, one can easily identify the interrelations between the solutions of different schemes, as shown in Fig. 6. In fact, Scheme 1 is a relaxation of all other schemes, meaning that the solution space of Scheme 1 covers those of all other schemes. Thereby, the optimal solution of Scheme 1 (shown in red dot in Fig. 6), which is a global optimum due to its concavity, provides an upper bound for Schemes 2–6. It is especially important for evaluating the solution qualities of Schemes 2–6. Note that when the parking fees of the women-only and regular parking spaces in Schemes 3 and 4 just take the values of the optimal parking fee solutions of Scheme 1, the resultant optimal solutions of Schemes 3 and 4 are identical with the optimal solution of Scheme 1. When the numbers of the women-only and regular parking spaces in Scheme 5 just take the optimal numbers of parking spaces of Scheme 1, the optimal solution of Scheme 5 is the same as that of Scheme 1. Consequently, the optimal solution of Scheme 1 (i.e., the red dot) must fall into the common area of the solution spaces of Schemes 1, 3, 4 and 5, as shown in Fig. 6. Similarly, when the parking fees of the women-only and regular parking spaces in Schemes 3 and 4 just take the optimal uniform parking fee solution of Scheme 2, the resultant optimal solutions of Schemes 3 and 4 are the same as the optimal solution of Scheme 2 (shown in black dot in Fig. 6). When the numbers of the women-only and regular parking spaces in Scheme 6 just take the optimal numbers of Scheme 2, the optimal solution of Scheme 6 is identical with that of Scheme2. The optimal solution of Scheme 2 must thus lie in the common area of the solution spaces of Schemes 1–6. Given an identical parking fee of the regular parking spaces for Schemes 3 and 4, Scheme 4 is a special case of Scheme 3. As a result, the solution space of Scheme 4 must fall in the solution space of Scheme 3. Note that Scheme 6 is a special

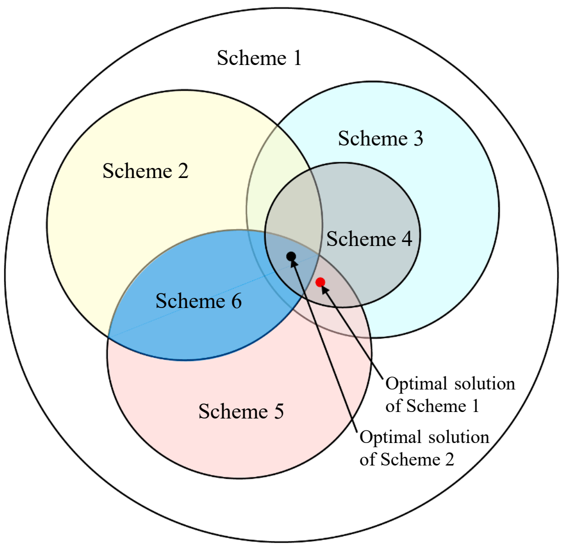


Fig. 6. The interrelations between the solutions of different schemes.

case of Schemes 2 and 5, and thus the solution space of Scheme 6 is the common area of the solution spaces of Schemes 2 and 5.

6. A special case: Linear parking search time function

It is difficult to compare the properties of solutions of general models presented in the previous section due to difficulty in mathematical tractability. In this section, we discuss a special case with a linear parking search time function, i.e., $\beta = 1$ in Eq. (6). Substituting $\beta = 1$ into the models in Section 5 yields the solutions of different schemes, summarized in Table 7. Based on the solutions for the special case, we can answer the following interesting and important issues: (i) the motivation of jointly operating the parking lot and shopping mall for the operator; (ii) the critical condition of introducing the women-only parking spaces for the operator; and (iii) the difference of the uniform and differential parking pricing schemes.

We first look at the opertor's motivation of jointly operating the parking lot and the shopping mall. Table 7 shows that for Schemes 1, 2, 3, 5 and 6 (except Scheme 4 with given parking fees), the optimal parking fees decrease by a proportion of the average profit parameter g, and thus the parking fees of the women-only and regular parking spaces are negatively correlated to g. This means that the operator of the building complex would like to use the revenue of shopping mall to subsidize the parkers using the underground car park of that shopping mall so as to attract more customers and thus increase its total net profit.

We now examine the critical condition of introducing the women-only parking spaces for the operator. Note that the women-only parking spaces generally have a higher marginal operating cost and a higher investment cost than the regular parking spaces. As a result, investment in the women-only parking spaces may lead to a loss in the net profit of parking operator. Therefore, the critical condition of introducing a women-only parking space should be that its marginal net profit exceeds the marginal net profit of a regular parking space, i.e., $\frac{\partial \Lambda}{\partial N_w} \geqslant \frac{\partial \Lambda}{\partial N_r}$. Taking Scheme 1 (benchmark model) as an example, one can easily derive the critical condition of investing in the women-only parking spaces as $\frac{(\varsigma_w+g)^2}{4\tau_d \alpha s_w} - \frac{c_w}{s_w} > \frac{(\varsigma_r+g)^2 \tau_d \alpha s_r}{4(b_r S+\tau_d \alpha s_r)^2} - \frac{c_r}{s_r}$ in terms of the solution of Scheme 1 in Table 7. This means that the marginal net profit (i.e., the difference of the marginal revenue and the marginal parking operating cost) of the women-only parking spaces is greater than that of the regular parking spaces. Thereby, the investment of the women-only parking spaces can cause a higher net profit compared that of the regular parking spaces. Otherwise, the profit-driven operator has no motivation to introduce the women-only parking spaces.

In the following, we compare the solutions of the uniform and differential parking pricing schemes. Note that the willingness-to-pay of parkers for the women-only parking spaces is generally higher than that for the regular parking spaces, and the women-only parking spaces can also reduce driver's parking process time compared to the regular parking spaces. Therefore, $f_r < f_w$ holds for Schemes 1 and 5, meaning that the optimal parking fee for the women-only parking spaces is higher than that for the regular parking spaces, as shown in Table 7. Table 7 also shows that the uniform parking fees f under Schemes 2 and 6 are higher than the optimal parking fees of the regular parking spaces f_r, but lower than the optimal parking fees of the women-only parking spaces f_w under the differential pricing schemes 1 and 5, i.e., $\frac{\varsigma_r - g}{2} = f_r < f < f_w = \frac{\varsigma_w - g}{2}$. Therefore, compared to the differential parking fees, the uniform parking fees lead to a higher parking demand q_w for the women-only parking spaces, but a lower parking demand q_r for the regular parking spaces. This

Table 7
The solutions of different schemes with $\beta = 1$.

Scheme 1 $\begin{cases} N_w: \ \frac{c_r}{s_r} - \frac{c_w}{s_w} + \frac{(\varsigma_w+g)^2 \tau_d \alpha}{4 s_w (b_w N_w + \tau_d \alpha)^2} - \frac{(\varsigma_r+g)^2 \tau_d \alpha}{4 s_r (b_r N_r + \tau_d \alpha)^2} = 0 \\ N_r = (S - s_w N_w)/s_r \\ q_w = \frac{(\varsigma_w + g) N_w}{2(b_w N_w + \tau_d \alpha)} \\ q_r = \frac{(\varsigma_r + g)(S - s_w N_w)}{2(b_r (S - s_w N_w) + \tau_d \alpha s_r)} \\ f_w = (\varsigma_w - g)/2 \\ f_r = (\varsigma_r - g)/2 \end{cases}$	**Scheme 2** $\begin{cases} N_w: \ \frac{c_r}{s_r} - \frac{c_w}{s_w} + \frac{\tau_d \alpha (f+g)(\varsigma_w - f)}{s_w (b_w N_w + \tau_d \alpha)^2} - \frac{\tau_d \alpha (f+g)(\varsigma_r - f)}{s_r (b_r N_r + \tau_d \alpha)^2} = 0 \\ N_r = (S - s_w N_w)/s_r \\ q_w = \frac{\varsigma_w - f}{b_w + \tau_d \alpha N_w^{-1}} \\ q_r = \frac{\varsigma_r - f}{b_r + \tau_d \alpha N_r^{-1}} \\ f = \frac{(\varsigma_w - g)(b_r + \tau_d \alpha N_r^{-1}) + (\varsigma_r - g)(b_w + \tau_d \alpha N_w^{-1})}{b_r + \tau_d \alpha N_r^{-1} + b_w + \tau_d \alpha N_w^{-1}} \end{cases}$
Scheme 3 $\begin{cases} N_w: \frac{c_r}{s_r} - \frac{c_w}{s_w} + \frac{\tau_d \alpha q_w^2}{s_w N_w^2} - \frac{(f_r + g) \tau_d \alpha q_r^2}{(\varsigma_r - f_r) s_r N_r^2} = 0 \\ N_r = (S - s_w N_w)/s_r \\ q_w = \frac{\varsigma_w + g}{2(b_w + \tau_d \alpha N_w^{-1})} \\ q_r = \frac{\varsigma_r - f_r}{b_r + \tau_d \alpha s_r (S - s_w N_w)^{-1}} \\ f_w = (\varsigma_w - g)/2 \end{cases}$	**Scheme 4** $\begin{cases} N_w: \ \frac{c_r}{s_r} - \frac{c_w}{s_w} + \frac{\tau_d \alpha (f_w + g) q_w^2}{s_w (\varsigma_w - f_w) N_w^2} - \frac{\tau_d \alpha (f_r + g) q_r^2}{s_r (\varsigma_r - f_r) N_r^2} = 0 \\ N_r = (S - s_w N_w)/s_r \\ q_w = \frac{\varsigma_w - f_w}{b_w + \tau_d \alpha N_w^{-1}} \\ q_r = \frac{\varsigma_r - f_r}{b_r + \tau_d \alpha N_r^{-1}} \end{cases}$
Scheme 5 $\begin{cases} q_w = \frac{N_w(\varsigma_w + g)}{2(b_w N_w + \tau_d \alpha)} \\ q_r = \frac{N_r(\varsigma_r + g)}{2(b_r N_r + \tau_d \alpha)} \\ f_w = (\varsigma_w - g)/2 \\ f_r = (\varsigma_r - g)/2 \end{cases}$	**Scheme 6** $\begin{cases} q_w = \frac{(b_r + \tau_d \alpha N_r^{-1})(\varsigma_w + g) + (b_w + \tau_d \alpha N_w^{-1})(2\varsigma_w - \varsigma_r + g)}{2(b_w + \tau_d \alpha N_w^{-1})(b_w + \tau_d \alpha N_w^{-1} + b_r + \tau_d \alpha N_r^{-1})} \\ q_r = \frac{(b_r + \tau_d \alpha N_r^{-1})(2\varsigma_r - \varsigma_w + g) + (b_w + \tau_d \alpha N_w^{-1})(\varsigma_r + g)}{2(b_r + \tau_d \alpha N_r^{-1})(b_w + \tau_d \alpha N_w^{-1} + b_r + \tau_d \alpha N_r^{-1})} \\ f = \frac{(\varsigma_w - g)(b_r + \tau_d \alpha N_r^{-1}) + (\varsigma_r - g)(b_w + \tau_d \alpha N_w^{-1})}{2(b_w + \tau_d \alpha N_w^{-1} + b_r + \tau_d \alpha N_r^{-1})} \end{cases}$

means that the drivers prefer the women-only parking spaces to the regular parking spaces under the uniform parking pricing. Taking Schemes 5 and 6 as an example (for the two schemes, the numbers of the women-only and regular parking spaces are pre-given), one can easily obtain the total profit of the operator in terms of the last row of Table 7 below

$$\Lambda^d = \frac{\left(b_r + \tau_d \alpha N_r^{-1}\right)(\varsigma_w + g)^2 + \left(b_w + \tau_d \alpha N_w^{-1}\right)(\varsigma_r + g)^2}{4\left(b_w + \tau_d \alpha N_w^{-1}\right)\left(b_r + \tau_d \alpha N_r^{-1}\right)} - (c_w N_w + c_r N_r) - \hat{c} S, \tag{31}$$

$$\Lambda^u = \frac{\left[\left(b_r + \tau_d \alpha N_r^{-1}\right)(\varsigma_w + g) + \left(b_w + \tau_d \alpha N_w^{-1}\right)(\varsigma_r + g)\right]^2}{4\left(b_w + \tau_d \alpha N_w^{-1}\right)\left(b_r + \tau_d \alpha N_r^{-1}\right)\left(b_w + \tau_d \alpha N_w^{-1} + b_r + \tau_d \alpha N_r^{-1}\right)} - (c_w N_w + c_r N_r) - \hat{c} S, \tag{32}$$

where superscripts "d" and "u" represent the differential and uniform parking pricing schemes, respectively. Λ^d and Λ^u represent the total net profit of the operator under Schemes 5 and 6, respectively. The profit difference of thses two schemes is thus

$$\Lambda^d - \Lambda^u = \frac{(\varsigma_w - \varsigma_r)^2}{4\left(b_w + \tau_d \alpha N_w^{-1} + b_r + \tau_d \alpha N_r^{-1}\right)} > 0. \tag{33}$$

A positive profit difference shown in Eq. (33) implies that the differential parking pricing schemes for the women-only and regular parking spaces can lead to higher profit than the uniform parking pricing scheme.

7. A case study of Wuhan China

7.1. Parameter calibration

In this section, we apply the proposed models in the previous sections to the underground parking lot of Han Street, Wuhan China. According to data from the operator, the total area of the parking lot is $S = 18160$ square meters. The areas of a women-only parking space and a regular parking space are $s_w = 14.5638$ and $s_r = 12.8712$ square meters, respectively. The marginal operating costs of the women-only parking spaces and the regular parking spaces per hour are $c_w = 4.0$ and $c_r = 1.5$ (RMB/h), respectively. "RMB" is the Chinese currency "Renminbi", and US\$1 approximates RMB6.53 as of January 1, 2021. Without loss of generality, the values of the parking search time and the time for completing the parking process, τ_d and τ_z, are assumed to be the same, with a value of RMB35 per hour. The average profit parameter g is set to be RMB1.0 per hour. The average time for searching an available parking space under the free-flow condition in the parking lot is 3 min, i.e., $t_0 = 0.05$ hour.

The parking demand function plays an important role in the model applications. Hence, the parameters a_i and b_i in the parking demand function (4) should be calibrated. By the questionnaire survey, we collect the sample data of the willingness-to-pay prices of the respondents to use the women-only parking spaces and regular parking spaces. After implementing a regression analysis, we obtain the inverse demand functions for the women-only parking spaces and the regular parking spaces, as shown in Fig. 7. In this figure, the slopes of the inverse demand functions for the women-only parking spaces and the regular parking spaces are -0.0037 and -0.0041 (RMB/veh), and their intercepts are RMB13.464 and RMB13.4, respectively. This means that the price sensitivity of drivers towards the women-only parking spaces is higher than that towards the regular parking spaces, i.e., the decreased number of parkers due to

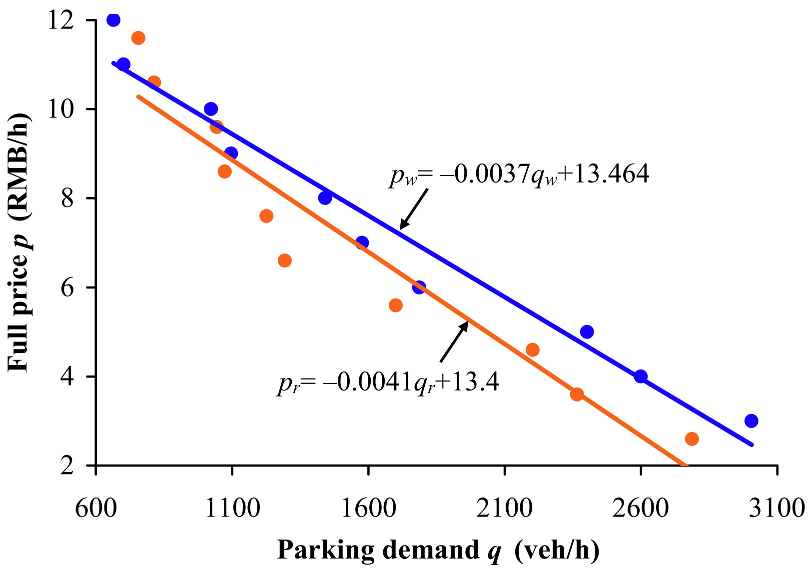


Fig. 7. Calibrated inverse demand functions for women-only and regular parking spaces.

marginal increase of parking price for the women-only parking spaces is larger than that for the regular parking spaces.

7.2. Comparison of efficiencies of different schemes

In order to look at the effects of introducing the women-only parking spaces on the total net profit of the operator and the layout of the parking spaces, different schemes in terms of parking configuration and parking pricing are compared, including:

- Scheme 1 (the benchmark model) represents the case with optimal parking configuration and optimal parking fees for both women-only and regular parking spaces.
- Scheme 2 represents the case with optimal parking configuration and optimal uniform parking fee for both women-only and regular parking spaces.
- Scheme 3 represents the case with optimal parking configuration for both women-only and regular parking spaces and optimal parking fee for the women-only parking spaces subject to the existing parking fee of RMB 5.0 per hour for the regular parking spaces.
- Scheme 4 represents the case with optimal numbers of the women-only and regular parking spaces subject to the existing uniform parking fees of RMB 5.0 per hour for all parking spaces regardless of the parking type.
- Scheme 5 represents the case with optimal parking fees for both women-only and regular parking spaces subject to 23 female parking spaces (status quo).
- Scheme 6 represents the case with optimal uniform parking fee for both women-only and regular parking spaces subject to 23 women-only parking spaces (status quo).
- Scheme 7 represents the status quo with 23 female parking spaces and the uniform parking fee of RMB 5.0 per hour for all parking spaces, regardless of parking type.

Table 8 shows the model solutions under different schemes. It can be seen in Table 8 that for Scheme 1, the optimal parking fee for the women-only parking spaces (RMB8.5 per hour) is RMB1.3 per hour higher than that for the regular parking spaces (RMB7.2 per hour). The optimal number of the women-only parking spaces is 258 spaces and the associated parking demand is 174 vehicles per hour, leading to an occupancy rate of 67.44%. The optimal number of the regular parking spaces is 1119 spaces and the corresponding parking demand is 668 vehicles per hour, yielding an occupancy rate of 59.70%. As a result, the total number of parking spaces is 1377 spaces and the optimal proportion of the number of the women-only parking spaces to the total number of parking spaces in the parking lot is 18.74%. It can also be seen that Scheme 1 yields the highest net profit of RMB4425 per hour among all schemes because it is a relaxation of all other schemes. Thereby, Scheme 1 is the best choice for a profit-driven operator.

In practice, differential parking prices in Scheme 1 may be difficult to be implemented. Scheme 2 assumes a uniform parking fee for all parking spaces. It can be seen in Table 8 that the optimal uniform parking fee is RMB7.4 per hour, which is higher than the optimal parking fee of the regular parking spaces of RMB7.2 per hour in Scheme 1, but lower than the optimal parking fee of the women-only parking spaces of RMB8.5 per hour in Scheme 1. The associated parking demand of the women-only parking spaces is 195 vehicles per hour, meaning an increase by 21 vehicles per hour (from 174 to 195) compared to Scheme 1. The parking demand of the regular parking spaces is 649 vehicles per hour, leading to a decrease by 19 vehicles per hour (from 668 to 649) in comparison with Scheme 1. As a result, the proportion of the number of the women-only parking spaces to the total number of parking spaces is 19.10%, which is the highest one compared to the other schemes. The resultant total net profit reaches RMB4367 per hour, which is 98.7% of the total

Table 8
Comparison of model solutions under different schemes.

Solution	Sch 1	Sch 2	Sch 3	Sch 4	Sch 5	Sch 6	Sch 7(Status quo)
Optimal number of women-only parking spaces	258	263	229	0	23	23	23
Optimal number of regular parking spaces	1119	1114	1152	1411	1385	1385	1385
Total number of parking spaces	1377	1377	1381	1411	1408	1408	1408
Optimal ratio of women-only parking spaces (%)	18.74	19.10	16.58	0	1.63	1.63	1.63
Optimal parking fee for women-only parking spaces (RMB/h)	8.5	7.4	8.6	5.0	8.9	7.0	5.0
Optimal parking fee for regular parking spaces (RMB/h)	7.2	7.4	5.0	5.0	6.9	7.0	5.0
Parking demand of women-only parking spaces (veh/h)	174	195	154	0	16	18	20
Parking demand of regular parking spaces (veh/h)	668	649	830	965	794	788	952
Total parking demand (veh/h)	842	844	984	965	810	806	972
Occupancy rate of women-only parking spaces (%)	67.44	74.14	67.25	0	69.57	78.26	86.96
Occupancy rate of regular parking spaces (%)	59.70	58.26	72.05	68.39	57.33	56.90	68.74
Resultant total net profit (RMB/h)	4425 (100%)	4367 (98.7%)	3818 (86.3%)	3673 (83.0%)	4251 (96.1%)	4239 (95.8%)	3661(82.7%)

Note: The numbers in the brackets of the last row represent the proportion of net profit of a scheme to that of Scheme 1.

net profit of Scheme 1. Note that a uniform parking pricing scheme is easier to be implemented for a parking lot than a differential parking pricing scheme in practice. Therefore, in reality Scheme 2 (a uniform parking pricing scheme) can be used as a substitution for Scheme 1 (a differential parking pricing scheme) due to its convenient implementation.

In contrast to Scheme 1 (benchmark case), Scheme 3 can attract more customers to park at the regular parking spaces due to a low pre-given parking fee of RMB5.0 per hour (which is RMB2.2 per hour lower than the optimal parking fee of RMB7.2 per hour under Scheme 1). As a result, the optimal number of the regular parking spaces increases by 33 spaces (from 1119 to 1152 spaces), and the corresponding parking demand increases by 162 (from 668 to 830) vehicles per hour compared to Scheme 1. The resultant occupancy rate of the regular parking spaces reaches 72.05%, which is the highest one among all schemes. On the other hand, the optimal number of the women-only parking spaces decreases by 29 spaces (from 258 to 229 spaces), leading its parking fee to increase by RMB0.1 per hour (from RMB8.5 to RMB8.6 per hour) due to a decreased provision. The resultant total net profit of Scheme 3 is RMB3818 per hour, accounting for 86.3% of the total net profit of Scheme 1. This means that Scheme 3 can realize 86.3% of the efficiency of Scheme 1 in terms of total net profit.

Compared to Schemes 1, 2 and 3, the operator under Scheme 4 has no motivation to invest in the women-only parking spaces due to a lower uniform parking fee of RMB5.0 per hour (such a parking fee is currently being used in the Han Street parking lot). The whole parking lot is used to invest in the regular parking spaces (1411 spaces), yielding a parking demand of 965 vehicles per hour and thus an occupancy rate of 68.39%. The resultant net profit is RMB3673 per hour, realizing 83.0% of the efficiency of Scheme 1 in terms of total net profit.

Scheme 5 has 23 women-only parking spaces and 1385 regular parking spaces. According to Table 8, in contrast to other schemes, Scheme 5 leads to the highest parking fee of RMB8.9 per hour for the women-only parking spaces, yielding a parking demand of 16 vehicles per hour and an occupancy rate of 69.57% for such parking spaces. The parking fee of the regular parking spaces is RMB6.9 per hour, and its parking demand is 794 vehicles per hour. The associated total net profit is RMB4251 per hour, achieving 96.1% of the efficiency of Scheme 1 from the perspective of total net profit.

Scheme 6, as a special case of Scheme 5, aims to find the optimal uniform parking fee for both women-only and regular parking spaces. It can be seen in Table 8 that for Scheme 6, the optimal uniform parking fee is RMB7.0 per hour, which decreases by RMB 0.4 compared to Scheme 2 (also a uniform scheme). The resultant total net profit is RMB4239 per hour, achieving 95.8% of the total net profit of Scheme 1. In addition, compared to the optimal differential parking pricing scheme 5, the optimal uniform parking fee scheme 6 leads to a decrease in the total net profit by RMB12 per hour.

Scheme 7 is a scheme currently being adopted in the Han Street parking lot, with 23 women-only parking spaces and 1385 regular parking spaces as well as a fixed and uniform parking fee of RMB5.0 per hour for all parking spaces. According to Table 8, Scheme 7 leads to the lowest total net profit of RMB3661 per hour among all schemes. In particular, compared to Scheme 4 with the same parking fee of RMB5.0 for all parking spaces, Scheme 7 causes a decrease in the total net profit of the operator by RMB12 per hour. This means that providing the women-only parking spaces may lead to a decrease in the total net profit for the operator (i.e., profit loss). In addition, compared to Scheme 6 (with an optimal uniform parking fee), Scheme 7 leads to a decrease in the total net profit by RMB578 per hour. This means that the present scheme in practice should be improved so as to enhance the profit of the operator.

Fig. 8 further examines the effects of the parking fees and number of women-only parking spaces on the total net profit of the operator, with a given parking fee of RMB5.0 per hour for the regular parking spaces. It can be observed in Fig. 8 that given a medium parking fee (e.g., between RMB6.5 and RMB10.5 per hour), as the number of the women-only parking spaces increases, the resultant total net profit first increases and then decrease. As a result, an optimal scheme with a parking fee of RMB8.6 per hour and 229 women-

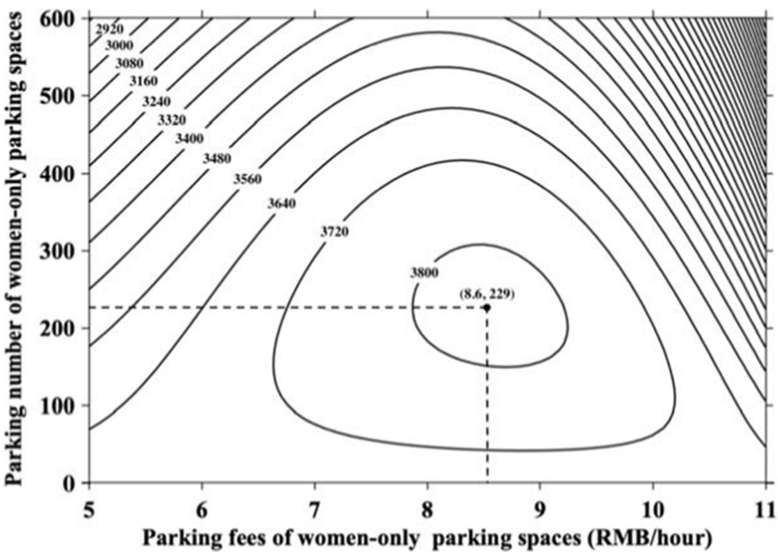


Fig. 8. Contour of operator's total net profit with regard to parking fees and number of women-only parking spaces.

only parking spaces can be found, yielding the maximum total net profit of RMB3818 per hour. However, for a low (e.g., <RMB6.5 per hour) or high (e.g., >RMB10.5 per hour) parking fee of the women-only parking spaces, as the number of the women-only parking spaces increases, the resultant total net profit would decrease. As a result, the total net profit after introducign the women-only parking spaces may be lower than that before introducing them. Again, it shows that providing the women-only parking spaces may lead to a loss of net profit for the operator. Therefore, in practice, the investment decision on the women-only parking spaces should be carefully made for the operator.

8. Conclusion and further studies

This paper examined the effects on drivers' parking behavior and park operator's investment behavior of introducing the women-only parking spaces. Based on a field survey on a parking lot in Wuhan, the major factors of influencing the time for completing the parking process at a parking space and the choice of the women-only parking spaces were identified. Optimization models were presented to determine the optimal numbers of the women-only and regular parking spaces in a parking lot and/or their optimal parking fees. The properties of the models are analytically explored, particularly for a linear parking search time function case. A case study from Wuhan China was conducted.

Some new and important insights have been obtained. First, introducing the women-only parking spaces can reduce driver's time spent on the parking process at a parking space. Driver's gender, vehicle length, and parking environment (obstacles on one or both sides of a parking space) have positive effects on the time for the parking process at a parking space, while driving age has negative effects. Second, driver's gender, driving age and whether he/she ever used the women-only parking spaces have a significant impact on his/her choice of women-only parking spaces, whereas the driver's education level and whether he/she suffers from parking accidents in the past year have no significant effect on his/her choice of women-only parking spaces. Third, there is an optimal uniform parking pricing scheme that can at most achieve over 98% of the efficiency of the optimal differential parking pricing scheme in terms of total net profit of operator, and thus can be used as a substitution for the differential parking pricing scheme due to its easy implementation. Fourth, providing the women-only parking spaces may lead to a loss of net profit for the operator, and thus the investment decision on the women-only parking spaces should be carefully made in practice.

The models proposed in this paper not only provide a useful research method for determining the optimal solutions of the number and parking fees of different parking spaces, but also provides the operator with a useful tool for rational planning and management of parking spaces. This is especially important for an initial development stage of the women-only parking spaces, in which the operator usually determines the number and/or parking fees of the women-only parking spaces in a parking lot only according to their intuition and perception. It is anticipated that the proposed research method in this paper is the first step towards scientific decision-making of the women-only parking spaces.

Although the models proposed in this paper have provided many new and important insights into the women-only parking spaces, there are still some possible directions for further extensions, as follows. First, there are various parking lots in a city, e.g., off-street vs. on-street, and downtown vs. suburb. It is important to investigate the competition and collaboration between parking lots, particularly when the women-only parking spaces are introduced. Second, more and more urban public facilities in some large Chinese cities are introducing the women-only parking spaces for attracting more customers, e.g., Wuhan *K*11 shopping malls and Beijing Daxing international airports. Different facilities have different functions and attributes, including service purposes, geographical location, customers' characteristics, demand intensity, and environment etc. It is therefore meaningful to set up criterions for efficient planning and layout of the women-only parking spaces for the heterogeneous facilities. Third, in reality, a parking lot in a city may be operated by a private sector or a public sector, with different operating objectives. The private sector aims to maximize its own net profit, whereas the public sector generally expects to maximize the social surplus of the system. There is thus a need to examine the effects of parking lot ownership on the development of women-only parking spaces in a future study.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgments

The authors would like to thank Prof. Hai Yang, Prof. Hai-Jun Huang, and two anonymous referees for their helpful comments and suggestions on earlier versions of the paper. The work described in this paper was jointly supported by grants from the National Natural Science Foundation of China (72131008, 71890974/71890970), the NSFC-JPI Urban Europe (71961137001), and the Fundamental Research Funds for the Central Universities (2021GCRC014).

References

Agresti, A., 2007. An Introduction to Categorical Data Analysis. Wiley, Hoboken, New Jersey.
Arjmand, R., 2017. Public urban space, gender and segregation: Women-only urban parks in Iran. Routledge, London.
Alemi, F., Rodier, C., Drake, C., 2018. Cruising and on-street parking pricing: A difference-in-difference analysis of measured parking search time and distance in San Francisco. Transp. Res. Part A 111, 187–198.
Al-Sahili, K., Hamadneh, J., 2016. Establishing parking generation rates/models of selected land uses for Palestinian cities. Transp. Res. Part A 91, 213–222.

Anderson, S.P., de Palma, A., 2004. The economics of pricing parking. J. Urban Econ. 55 (1), 1–20.
Axhausen, K.W., Polak, J., 1991. Choice of parking: Stated preference approach. Transportation 18, 59–81.
Balac, M., Ciari, F., Axhausen, K.W., 2017. Modeling the impact of parking price policy on free-floatingcarsharing: Case study for Zurich, Switzerland. Transp. Res. Part C 77, 207–225.
Bellizzi, M.G., Eboli, L., Forciniti, C., Mazzulla, G., 2018. Air transport passengers' satisfaction: An ordered logit model. Transp. Res. Proc. 33, 147–154.
Bifulco, G.N., 1993. A stochastic user equilibrium assignment model for the evaluation of parking policies. Eur. J. Oper. Res. 71, 269–287.
Brooke, S., 2015. Factors influencing urban on-street parking search time using a multilevel modelling approach. PhD Dissertation, Loughborough University, Loughborough, UK. Available at: https://hdl.handle.net/2134/20180.
Christiansen, P., Engebretsen, Ø., Fearnley, N., Hanssen, J.U., 2017. Parking facilities and the built environment: Impacts on travel behaviour. Transp. Res. Part A 95, 198–206.
Crawford, D.L., Pollak, R.A., Vella, F., 1998. Simple inference in multinomial and ordered logit. Econometric Reviews 17 (3), 289–299.
Dave, S.M., Joshi, G.J., Ravinder, K., Gore, N., 2019. Data monitoring for the assessment of on-street parking demand in CBD areas of developing countries. Transp. Res. Part A 126, 152–171.
Du, Y., Yu, S., Meng, Q., Jiang, S., 2019. Allocation of street parking facilities in a capacitated network with equilibrium constraints on drivers' traveling and cruising for parking. Transp. Res. Part C 101, 181–207.
Dunckel-Graglia, A., 2013. Women-only transportation: How "pink" public transportation changes public perception of women's mobility. J. Public Transp. 16 (2), 85–105.
Echaniz, E., Ho, C.Q., Rodriguez, A., dell'Olio, L., 2019. Comparing best-worst and ordered logit approaches for user satisfaction in transit services. Transp. Res. Part A 130, 752–769.
Guan, Y., Wang, Y., Yan, X., Guo, H., Zhou, Y., 2020. A big data-driven framework for parking demand estimation in urban central districts. J. Adv. Transp. Article ID 8898848.
Gur, Y.J., Beimborn, E.A., 1984. Analysis of parking in urban centers: Equilibrium assignment approach. Transp. Res. Rec. 957, 55–62.
Greene, W.H., Hensher, D.A., 2010. Modeling Ordered Choices: A Primer. Cambridge University Press, Cambridge UK.
Hensher, D.A., King, J., 2001. Parking demand and responsiveness to supply, pricing and location in the Sydney central business district. Transp. Res. Part A 35, 177–196.
Hess, S., Polak, J., 2004. Mixed logit estimation of parking type choice. Presented at the 83rd annual meeting of the Transportation Research Board, Washington D.C.
Hunt, J.D., Teply, S., 1993. A nested logit model of parking location choice. Transp. Res. Part B 27, 253–265.
Iqbal, A., 2018. How safe are women-only parks perceived to be? Security J. 31, 859–881.
Khordagui, N., 2019. Parking prices and the decision to drive to work: Evidence from California. Transp. Res. Part A 130, 479–495.
Lam, W.H.K., Li, Z.C., Huang, H.J., Wong, S.C., 2006. Modeling time-dependent travel choice problems in road networks with multiple user classes and multiple parking facilities. Transp. Res. Part B 40, 368–395.
Lam, W.H.K., Tam, M.L., Yang, H., Wong, S.C., 1999. Balance of demand and supply of parking spaces. In: Ceder, A. (Ed.), Transportation and Traffic Theory. Elsevier, Oxford, pp. 707–731.
Lam, W.C.H., Fung, R.Y.C., Wong, S.C., Tong, C.O., 1998. The Hong Kong parking demand study. Proc. Inst. Civil Eng., Transport 129, 218–227.
Lau, W.W.T., Poon, P.S.T., Tong, C.O., Wong, S.C., 2005. The Hong Kong second parking demand study. Proceedings of the Institution of Civil Engineers, Transport 158, 53-59.
Lei, C., Ouyang, Y., 2017. Dynamic pricing and reservation for intelligent urban parking management. Transp. Res. Part C 77, 226–244.
Leurent, F., Boujnah, H., 2014. A user equilibrium, traffic assignment model of network route and parking lot choice, with search circuits and cruising flows. Transp. Res. Part C 47, 28–46.
Li, Z.C., Huang, H.J., Lam, W.H.K., Wong, S.C., 2007. A model for evaluation of transport policies in multimodal networks with road and parking capacity constraints. J. Math. Model. Algorith. 6, 239–257.
Li, Z.C., Lam, W.H.K., Wong, S.C., Huang, H.J., Zhu, D.L., 2008. Reliability evaluation for stochastic and time-dependent networks with multiple parking facilities. Netw. Spatial Econ. 8, 355–381.
Lim, H., Williams, G.T., Abdelqader, D., Amagliani, J., Ling, Z., Priester, D.W., Cherry, C.R., 2017. Alternative approach for forecasting parking volumes. Transp. Res. Procedia 25, 4171–4184.
Liu, W., 2018. An equilibrium analysis of commuter parking in the era of autonomous vehicles. Transp. Res. Part C 92, 191–207.
Liu, W., Yang, H., Yin, Y., 2014. Expirable parking reservations for managing morning commute with parking space constraints. Transp. Res. Part C 44, 185–201.
Mejia-Dorantes, L., Soto Villagran, P., 2020. A review on the influence of barriers on gender equality to access the city: A synthesis approach of Mexico City and its metropolitan area. Cities 96, 102439.
Mo, B., Kong, H., Wang, H., Wang, X., Li, R., 2021. Impact of pricing policy change on on-street parking demand and user satisfaction: A case study in Nanning, China. Transp. Res. Part A 148, 445–469.
Nourinejad, M., Roorda, M.J., 2017. Impact of hourly parking pricing on travel demand. Transp. Res. Part A 98, 28–45.
Parmar, J., Das, P., Dave, S.M., 2020. Study on demand and characteristics of parking system in urban areas: A review. J. Traffic Transp. Eng. (English Ed.) 7, 111–124.
Qian, Z., Rajagopal, R., 2014. Optimal dynamic parking pricing for morning commute considering expected cruising time. Transp. Res. Part C 48, 468–490.
Qin, H., Yang, X., Wu, Y.J., Guan, H., Wang, P., Shahinpoor, N., 2020. Analysis of parking cruising behaviour and parking location choice. Transportation Planning and Technology 43 (7), 717–734.
Rivadeneyra, A.T., Dodero, A.L., Mehndiratta, S.R., Alves, B.B., Deakin, E., 2015. Reducing gender-based violence in public transportation strategy design for Mexico City, Mexico. Transp. Res. Rec. 2531, 187–194.
Shaaban, K., Pande, A., 2016. Classification tree analysis of factors affecting parking choices in Qatar. Case Stud. Transport Policy 4, 88–95.
Shahrokni, N., 2014. The mothers' paradise: Women-only parks and the dynamics of state power in the Islamic Republic of Iran. J. Middle East Womens Stud. 10 (3), 87–108.
Shao, C., Yang, H., Zhang, Y., Ke, J., 2016. A simple reservation and allocation model of shared parking lots. Transp. Res. Part C 71, 303–312.
Shoup, D., 1999. The trouble with minimum parking requirements. Transp. Res. Part A 33, 549–574.
Soto, J.J., Márquez, L., Macea, L.F., 2018. Accounting for attitudes on parking choice: An integrated choice and latent variable approach. Transp. Res. Part A 111, 65–77.
Steiner, R.L., 1998. Trip generation and parking requirements in traditional shopping districts. Transp. Res. Rec. 1617, 28–37.
Thompson, R.G., Richardson, A.J., 1998. A parking search model. Transp. Res. Part A 32, 159–170.
Tong, C.O., Wong, S.C., Lau, W.W.T., 2004a. A demand-supply equilibrium model for parking services in Hong Kong. Hong Kong Inst. Eng. Trans. 11, 48–53.
Tong, C.O., Wong, S.C., Leung, B.S.Y., 2004b. Estimation of parking accumulation profiles from survey data. Transportation 31 (2), 183–202.
Van Der Goot, D., 1982. A model to describe the choice of parking places. Transp. Res. Part A 16 (2), 109–115.
Wang, X., Wang, X., 2019. Flexible parking reservation system and pricing: A continuum approximation approach. Transp. Res. Part B 128, 408–434.
Wong, S.C., Li, Z.C., Lam, W.H.K., 2021. Parking demand models. Int. Encyclopedia Transp. 519–523.
Wong, S.C., Tong, C.O., Lam, W.C.H., Fung, R.Y.C., 2000. Development of parking demand models in Hong Kong. J. Urban Plann. Dev. 126 (2), 55–74.
Young, W., Thompson, R.G., Taylor, M.A., 1991. A review of urban car parking models. Transport Rev. 11 (1), 63–84.
Zainon, M., 2013. Perception of women drivers towards women-only parking space: A case of KLCC. Universiti Teknologi Malaysia. Master Thesis.
Zulhasni, Z., 2015. Gender identification system in women-only parking lots by using gait analysis. Universiti Teknologi Petronas, Malaysia. Bachelor Thesis.