



# Robust liner ship routing and scheduling schemes under uncertain weather and ocean conditions

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## ABSTRACT

Since sailing along the Northern Sea Route (NSR) through the Arctic Ocean comes into reality, commercial shipping developments including opening new liner services along this route have been put on the agenda. However, uncertainties within weather and ocean conditions in this region, especially during temperature-varying seasons, may cause failure to on-time arrivals, resulting unignorable or even unaffordable monetary losses. As a result, these uncertainties will inevitably affect cargo shippers' willingness of choosing this being-explored route, even if on average its shipping time is much shorter than that of the traditional Asia-Europe shipping lines. In this context, the present paper describes a liner ship routing and scheduling problem considering schedule-sensitive demand and late-arrival penalty for operating incoming NSR shipping lines in the future. By using two types of uncertainty sets, bounded and budget-bounded uncertainty sets, we construct and solve two robust counterparts of this problem. The deterministic model and bounded robust model can be seen as special cases of the budget-bounded robust model when the uncertainty budget is set to null and full, respectively. A multifaceted case study of planning liner shipping routes and schedules along the NSR is conducted to validate the efficacy and efficiency of the proposed models and identify the effects of uncertainty bounds and budgets on the solution performance. The following facts and conclusions are revealed from the optimization results of the case study: The budget-bounded robust model functions the best in the worst condition; low revenue rate and high demand sensitivity are the major factors for excluding a candidate port from the route; the order of a visited port in the route decides the weight of its uncertainty budget on the entire shipping line; setting the uncertainty budget in a decelerative way will enhance the scheduling robustness of solutions.

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## 1. Introduction

As a central component of global logistics systems, maritime transportation plays a key role in international seaborne trade. Liner shipping, mostly in the form of container ships, has evidently become the leading maritime transportation means, which carries about 60% of goods in terms of monetary value nowadays and the share of which in the seaborne transportation is still increasing. Liner shipping typically provides the service of transporting goods by means of high-capacity, ocean-going ships that transit along regular routes on fixed schedules. It arose as the fastest-growing maritime transportation sector in past decades, offering the most cost-effective and environment-friendly way to transfer massive goods.

Routing and scheduling are two important decisive aspects of liner shipping planning. Both planning decisions inherently influence the business development and daily operations of a liner shipping company and are closely related to other strategic, tactical and operational decisions. A routing plan determines, on the strategic or tactical level, which ports a ship or a fleet of ships should visit and in which sequence of ports to be visited. Scheduling is another tactical decision in liner shipping planning, where upper-level strategic decisions, such as fleet size and deployment and service network structure, are typically considered to be fixed. In a scheduling problem, the planned arrival and departure times at visited ports are main determinant variables. Earlier research often studied liner scheduling problems in a deterministic setting and often determines the schedule as a direct result of routing decision. However, uncertain factors increasingly arise from port operations, access to channels, weather and sea conditions, and so on and hence draw particular attention on shipping schedules from liner ship planners and managers. Additional considerations must be injected into scheduling decisions to reduce the negative impact caused by a variety of uncertainties. The negative impact on the shipping company includes various economic losses, such as the caused variation of the operating and depreciation costs and the revenue decline from lost customers.

We often use schedule reliability as one of punctuality performance indicators for liner shipping, which is referred to as the ability or probability for the liner ships of a shipping company or a service network to meet its published schedules. Public assessment reports on this indicator are regularly released by third-party companies, such as *Drewry Shipping Consultants*, *Lloyd's Register* and *SeaIntel Maritime Analysis*, to evaluate the performance of those top liner shipping companies in the world. These reports revealed that the schedule reliability of liner ships worldwide has decreased ever since 2013. On the other hand, schedule reliability varies with carriers. For instance, it ranges from 20% to 100% among the 15 routes operated by different carriers from Shanghai to New York. A report released by *Drewry* in 2006 monitored more than 5,410 vessels for 5 months and found out that over 40% of vessels arrived at least one day behind their planned schedules, indicating an unsatisfactory level of service on average in the liner shipping industry. The situation even turned worse in recent years. In another report released by *Drewry* in 2014, the average schedule reliability of the whole liner industry has dropped to 61% in recent years. The average schedule reliability published by *CargoSmart* of 12 routes used by 23 carriers in 2017 ranges from 55% to 70% only.

When it comes to the Northern Sea Route (NSR), a forthcoming transportation shortcut between East Asia and Europe, a major unmanageable uncertainty factor causing shipping schedule unreliability is its uncertain weather and ocean conditions. Many studies predicted that the NSR will be able to serve commercial shipping in the following decades. For example, [Smith and Stephenson \(2013\)](#) found that most normal liner ships will have had the capability of navigating along the NSR in late summer since early 2020s and the effective open-water width and navigable frequency of this shipping line are expected to continuously increase in the foreseeable future. However, the weather and ice conditions along the NSR often vary in a wide range and are hardly predictable in an accurate manner, especially during season-changing periods. The sailing time and cost are heavily subject to the variation of the uncertain weather and ocean conditions along the route. To minimize the potential economic loss, a liner shipping company that expects to operate shipping lines along the NSR must take into account these uncertainties in making its marketing plans and shipping itineraries. From the perspective of risk management, it is much safer and more cost-effective to accommodate these uncertainties in the planning stage in advance of taking supplementary actions to compensate potential losses, which typically should have been made 3–5 months ahead of the scheduled departure times.

The level and pattern of shipping demand across various ports poses as other key factors affecting the decisions on liner shipping routes and schedules, on which the NSR would not be an exclusive case. Liner shipping decisions must be made on the basis of forecasted shipping demand, for which the nature of forecasting adds another dimension of uncertainty to liner routing and scheduling ([Christiansen et al., 2004](#)). In previous research (e.g., [Chuang et al., 2010](#); [Wang et al., 2012](#)), shipping demand was typically treated as independent, deterministic or stochastic, quantities from routing and scheduling decisions. This assumption, however, underestimates the interaction between the market demand and the competition among shipping companies. A more competitive scheduling strategy undoubtedly wins more shipping orders, where it often implies a tighter schedule. A conservative schedule may reduce the risk of being unpunctual, yet it may be less attractive to consignors, causing a lower income to the carrier.

The focus of this paper is on modeling and solving a combined liner ship routing and scheduling problem within uncertain sailing environments, with an application for the forthcoming NSR route in the foreseeable future. Both decisive aspects are critical to the profitability and competitiveness of a liner shipping company that expects to operate commercial shipping lines along the route. Adding port calls along a shipping line can increase access to more cargo catchment areas and potentially generates additional revenues. Offering short transit times between ports is especially important for attracting those shippers who have time-sensitive cargos. In this sense, the tension and tradeoff between routing and scheduling is very important ([Notteboom, 2006](#)). Past research, however, observed that most of liner ship routing research did not take into account scheduling components ([Meng et al., 2014](#)), or shipping schedules are designated simultaneously with routing decisions, under the assumption that sailing time and port operations time are deterministically given. When the need of setting a certain amount of buffer time to deal with voyage delays arises and the amount of shipping demand is evidently sensitive to shipping durations, optimizing a critical schedule presents another important set of decisions

in addition to route planning. Very few studies in the literature have considered ship routing and scheduling decisions as two different decisive aspects, except [Agarwal and Ergun \(2008\)](#) and [Tierney et al. \(2019\)](#).

The differences of our paper from the above two exclusive studies are obvious. [Agarwal and Ergun \(2008\)](#) presented a fully deterministic routing and scheduling problem without considering any supply or demand uncertainty, while our paper explicitly considers a combined routing and scheduling problem with shipping time uncertainty and schedule-dependent demand. [Tierney et al. \(2019\)](#) optimized the schedule by using the chance-constrained optimization technique, ensuring ships to arrive at each designated port with a certain level of probability, while our paper optimizes the schedule taking into account the impact of late arrivals on elastic demand and delay penalty and reformulates and resolves the combined routing and scheduling problem in the robust optimization (RO) framework.

In this paper, we are concerned about how those uncertain sailing factors would affect routing and scheduling decisions and hence the profitability of a liner shipping company. In particular, we are interested in applying the RO technique for deriving the most robust routing and scheduling solution of a liner shipping line along the NSR. In such an uncertain circumstance, it is often neither the expected performance nor the best-case performance but the worst-case performance that determines the profitability and reputation of a liner shipping company. The shipping industry flows a huge amount of money for its daily operations, and once the worst case occurs, the loss to the company's financial conditions could be destructive. This is the major motivation of building RO models for optimizing the liner ship routing and scheduling decisions along the NSR.

The main body of this paper is on developing RO models for minimizing the negative impact from the worst case of varying weather and ocean conditions on the profitability of a liner ship sailing along the NSR. The remainder of the paper is structured as follows. First, [Section 2](#) reviews some applications of RO in the network design area with an emphasis on maritime transportation network design. Next, [Section 3](#) presents a deterministic liner ship routing and scheduling model and two robust counterparts of this model for uncertain weather and ocean conditions, all of which are in the mixed integer linear programming (MILP) form. In [Section 4](#), a case study of routing and scheduling a single liner ship along the NSR is carried out, in which the effectiveness and efficacy of these deterministic and robust models are widely evaluated and compared. Finally, [Section 5](#) summarizes a number of concluding remarks and offers some future extensions for this study.

## 2. Robust optimization and network design

The concept of robustness arises originally from control theory. In 1970 s, researchers including [Soyster \(1973\)](#) introduced this term to the operations research field, but it did not receive much attention until late 1990s, when [Ben-Tal and Nemirovski \(1998, 2000, 2004\)](#), [El Ghaoui and Lebret \(1997\)](#), and [El Ghaoui et al. \(1998\)](#), respectively, achieved some breakthroughs. Their research outcomes are mainly on set-based RO methods, which focus on the attributions of uncertainty sets. The other type of RO methods is scenario-based, proposed by [Kouvelis and Yu \(1997\)](#), providing a choice that functions best through all scenarios evaluated by a certain criterion like least regret or minimax. It is applied to solve problems with discrete variables where uncertain data are attached to a specific scenario.

Ever since then, both operations research and transportation research scholars have been practicing this approach in various network design problems. Among all sources of uncertainty, demand uncertainty received the most attention. [Chung et al. \(2011\)](#) addressed a network design problem integrating transportation dynamics and box-uncertainty set in a RO model, with an objective of minimizing the sum of transportation cost and expansion cost. They also proved that the model functions better under severe conditions compared to the deterministic counterpart. [Baghalian et al. \(2013\)](#) proposed a model that maximizes profits with demand uncertainty in several scenarios. This model is robust in the way that its solution remains close to the optimal one in all scenarios. [Atamtürk and Zhang \(2007\)](#) described a two-stage RO approach for solving network flow and design problems with uncertain demand and introduced a budget for demand uncertainty into this problem to control the conservativeness of the solutions. They also proved that when the topology structure of the two-stage network meets a certain criterion, the separation problem is tractable. [Chekuri et al. \(2007\)](#) threw light on the complexity status of a robust network design problem in undirected networks.

RO has also been applied in airline network design. Relevant research topics include: (1) Robust fleet assignment ([Smith and Johnson, 2006](#)); (2) robust aircraft routing ([Smith and Johnson, 2006](#); [Yan and Kung, 2018](#)); (3) robust crew scheduling ([Smith and Johnson, 2006](#); [Barnhart et al., 2003](#)), which basically covers almost all important decisions in airline network optimization. These studies experienced a change from the application of conceptual robustness to the implementation of RO methods. In early relevant research ([Smith and Johnson, 2006](#); [Gao et al., 2009](#); [Barnhart et al., 2003](#); [Lan et al., 2006](#); [Jiang and Barnhart, 2013](#)), instead of applying RO techniques, the solution algorithms are mainly developed from the column generation method or decomposition-based method. These studies considered uncertainty, but they merely introduced the conceptual robustness. In some recent studies, successful implementations of RO techniques have been conducted. For example, [Yan and Kung \(2018\)](#) successfully solved a route planning problem, formulated a robust model with the min-max principle, which provided a precise model and gave an exact solution algorithm for the problem of this type for the first time; [Murça \(2017\)](#) proposed a robust airport departure metering model under uncertain taxi-out time predictions by employing RO theory. These studies marked the transition from the conceptual robustness to the implementation of RO methods.

Compared to ground and airline network design, the number of RO applications in maritime network design is relatively less, despite a large number of problems of this type that have been investigated in the area ([Christiansen et al., 2004, 2013](#)). From the relevant literature, it is found that most of these network design problems are tackled by deterministic optimization models. However, uncertain factors arise widely from many components in the maritime logistics and supply chain sector. A similar transition from conceptual robustness to the implementation of RO methods can also be found in the research on maritime network design problems.

Christiansen and Fagerholt (2002) pioneered the robustness concept into this field. They set a time window in their schedule planning model to avoid port closure on weekends and defined this as robustness. Apparently, the robustness here is merely a concept; yet it inspired a series of follow-up research in this area. Almost simultaneously, Alvarez et al. (2011), Wang and Meng (2012a, 2012b) and Wang et al. (2012) introduced RO into this field. Alvarez et al. (2011) focused on the shipping finance practices, considering the impact from the uncertainties in purchase prices, sale prices, asset values and charter rates. Wang and Meng (2012a, 2012b) conducted a series of research with RO techniques: They addressed a liner ship route and schedule design problem with considering uncertain operations time and wait time at ports and developed a solution algorithm that incorporates a sample average approximation method, linearization techniques, and a decomposition scheme; they demonstrated the effectiveness of their model by testing it on extensive numerical experiments based on an Asia–America–Europe shipping route. In the same context, Wang et al. (2012) also studied a liner ship fleet design and deployment problem under uncertain demands, proposing a model to obtain an optimal liner fleet plan that is less sensitive to changes in uncertain container shipment demands.

In summary, even though a few studies have considered weather-induced uncertain sailing times, the mainstream modeling approach is either simulation-based or scenario-based, and few of them adopted the RO approach. Compared to these previous studies, this paper advances the liner ship routing and scheduling research in the following two aspects: 1) On the model development side, this paper proposes and solves an optimal liner ship routing and scheduling problem with schedule-dependent demand and late-arrival penalty in the RO framework, which considers the effect of uncertain weather and ocean conditions on sailing time and cost; 2) on the result analysis side, this paper identifies key factors affecting the port selection result and on-time arrival performance of a liner shipping line and quantifies the impacts of uncertainty bound size and budget trend on routing and scheduling decisions, through a real-world case study in the context of NSR, in which the sailing speed and cost are heavily affected by uncertain ocean and weather conditions.

### 3. Model development

#### 3.1. Modeling assumptions

For the sake of developing tractable models as well as adopting a commercial solver for their solutions, the following modeling assumptions are used throughout the paper. These assumptions jointly provide a basis for maintaining the linear structure of all deterministic and robust models and deducting any unnecessary integrality complexity or data processing difficulty.

1. The complete itinerary of a ship in a planning horizon is a round trip, which starts from an end port (e.g., port 1), stopping at selected intermediate ports and finally arriving at another end port (e.g., port  $n$ ) along the outbound direction, and then starts from port  $n$ , stopping at intermediate ports and returning to port 1 along the inbound direction. Such a round-trip itinerary was employed as a standard shipping network by many previous liner ship routing and scheduling studies.
2. The relationship between sailing cost and sailing time is approximately linear, for which the underlying reason is that the variable part of the sailing cost of a liner ship typically comprises fueling cost, mooring cost, crew and procurement cost, insurance and finance cost, depreciation cost, repair and maintenance cost, and so on, the majority of which is proportional to sailing time, despite some of the individual cost components may not have a linear relationship with sailing time. A recent statistical study reported by Bialystocki and Konovessis (2016) justified that the total sailing cost for a voyage segment can be well approximated by a linear function of the sailing time; Ying et al. (2018) found that a linear relationship exists approximately between fuel consumption and mooring time.
3. Cargo handling delay and waiting delay at ports as well as their uncertainties are ignored in this paper. The reason is twofold. First, as we discussed earlier, the applicable context of the proposed research problem in this paper is NSR, along which most of potentially visited ports, especially those along the Russian coast, are relatively uncongested. Second, if needed, these minor time components could be easily incorporated as constants into the proposed model and their associated randomness could be dealt with in the same way as uncertain sailing time. This inclusion does not increase the complexity of the presented modeling and solution methods, but only adds extra workload in data collection and process.
4. Elastic, schedule-dependent shipping demand between a port pair is depicted as a linear function of the difference between the claimed departure time of the origin port and arrival time of the destination port. This linear structure of the demand function may be quite a simplified modeling setting, but appears as a necessary part for ensuring the linear form of the models.
5. The penalty for a late arrival at any port is assumed to be a constant. This penalty is reflection of a loss of reputation or a decrease of competitiveness of the liner shipping company in the liner shipping market instead of a compensation paid to cargo demanders. The latter is certainly related to the cargo type and amount, delay length and cause, and many other unpredictable factors. This constant setting also makes the modeling and solution methods much simpler.
6. The number of cargos is evaluated in real numbers. Although cargos are in general counted in the number of twenty-foot equivalent units (TEUs), we believe that it is more appropriate to use real numbers instead of integer numbers to quantify the cargo demand and flow, considering that different shippers may share containers and using real numbers here can greatly reduce the integrality complexity of the models.
7. Weather and ocean conditions can be relatively accurately predicted and the average sailing speed on each segment remains the same in a short term (e.g., in the duration of sailing on a segment), though the latter may significantly change under varying weather and ocean conditions in a long term (e.g., in a period between the schedule announcement date and the ship departure date).

8. It is assumed that the actual sailing time along any segment is equal to its nominal sailing time when weather and ocean conditions are in the ideal state and the former is typically longer than the latter when weather and ocean conditions get deteriorated. Due to this setting, those actual sailing times used in the following deterministic model are just their nominal values, while the actual sailing times in the robust counterparts are not their nominal ones, but heavily determined by the actual weather and ocean conditions.

### 3.2. Notation

To offer good reading convenience, we summarize below the definition of all the notation used throughout the paper (see Table 1).

For discussion convenience, we distinguish the outbound and inbound voyage directions of the vessel whenever appropriate in the above notation list and denote them by  $\alpha$  and  $\beta$ , respectively. We call the journey from the starting port to the ending port the *outbound trip* and the journey from the ending port to the starting port the *inbound trip*, as defined in Meng et al. (2014) and many other studies. Complying with the origins and destinations of the demand pattern exhibited in the given case study, we presume here that any cargo demand in this study can be satisfied by either the vessel's outbound or inbound trip alone. If carrying a certain amount of demand needs to go beyond only one of the outbound and inbound directions (i.e., using both the outbound and inbound segments), we just need to simply modify the constraint set in the same modeling framework. This simple modification occurs merely on the notational side and does not create any extra modeling and algorithmic difficulty.

It is noted here that in the above list of decision variables, except the segment usage indicator, all other variables are continuous. As a result, the models described below, including a deterministic model and two relevant robust models, are all of the mixed discrete–continuous type, or more specifically, in the mixed integer linear form.

### 3.3. Deterministic model

The deterministic model presented below is primarily inspired by the classic work in Rana and Vickson (1988), which provides a basic building block for many containership routing problems and also implies the essential structure of the routing part of the problem in this paper. Other similar single-ship liner routing work includes Shintani et al. (2007), Wang et al. (2013), Plum et al. (2014), for example; previous work on the single-ship liner scheduling includes Ting and Tzeng (2003), Wang et al. (2014), and Mulder and Dekker (2019), among others. A comprehensive review on liner ship routing and/or scheduling problems can be found in Ronen (1983), Ronen (1993), Christiansen et al. (2004), Christiansen et al. (2013), Meng et al. (2014). Based on the previous work, we

**Table 1**  
Notation.

Sets	
$N$	Set of candidate ports, where $N = \{1, \dots, i, \dots, n\}$
$A$	Set of candidate segments connecting two candidate ports, where $A = \{(i, j)\}$
$U$	Set of uncertainty
Parameters	
$n$	Total number of candidate ports in the network, where $n =  N $
$r_{pq}$	Revenue rate (in \$/TEU) per unit of goods from origin port $p$ to destination port $q$
$a_{ij}$	Cargo capacity (in TEU) of the ship on segment $(i, j)$
$t_{ij}$	Nominal sailing time (in days) on segment $(i, j)$
$t_i^{\alpha} (t_i^{\beta})$	Actual arrival time (in days) at port $i$ for outbound (or inbound) trips
$t_{ij}$	Actual sailing time (in days) on segment $(i, j)$
$\epsilon$	Uncertainty level bound
$\zeta_{ij}$	Actual uncertainty level for the sailing time on segment $(i, j)$
$\Gamma_j^{\alpha} (\Gamma_j^{\beta})$	Uncertainty budget at port $j$ for the outbound (or inbound) trip
$g_i$	Daily delay penalty rate (in \$/day) at port $i$
$D_{pq}$	Base cargo demand (in TEU) from origin port $p$ to destination port $q$
$s_{pq}^e$	Demand sensitivity parameter (in TEU/day) for the cargo demand from origin port $p$ to destination port $q$
$c_{ij}^1$	Fixed sailing cost (in \$) on segment $(i, j)$
$c_{ij}^2$	Variable sailing cost rate (in \$/day) on segment $(i, j)$
$M$	A very large number
Variables	
$x_{ij}$	Usage indicator for segment $(i, j)$ , where $x_{ij}$ is a 0–1 binary variable: If segment $(i, j)$ is included in the shipping route, $x_{ij} = 0$ ; otherwise, $x_{ij} = 1$
$e_i^{\alpha} (e_i^{\beta})$	Arrival delay at port $i$ for the outbound (or inbound) trip (in days)
$T_i^{\alpha} (T_i^{\beta})$	Claimed arrival time (in days) at port $i$ for the outbound (or inbound) trip
$y_{pq}$	Actually served cargo demand (in TEU) from port $p$ to port $q$
$d_{pq}$	Potentially induced cargo demand (in TEU) from port $p$ to port $q$
$u_i (v_i)$	Dual variable associated with the constraint on the level of protection or conservatism of the outbound (inbound) trip at port $i$
$w_{kj} (z_{kj})$	Dual variable associated with the constraint on the actual uncertainty level $\zeta_{kj}$ of the outbound (inbound) trip on segment $(k, j)$



develop a deterministic liner ship routing and scheduling problem below. Its objective function reads:

$$\max \sum_{p \in N} \sum_{q \in N} r_{pq} y_{pq} - \sum_{i \in N} \sum_{j \in N} (c_{ij}^1 + t_{ij} c_{ij}^2) x_{ij} - \sum_{i \in N} (e_i^\alpha + e_i^\beta) g_i \quad (1)$$

The above objective function in (1) aims at maximizing the profit of operating a liner vessel in a round trip. The first item represents the total revenue; the second item is the total sailing cost; the last item denotes the total penalty cost caused by late arrivals. The total sailing cost for any segment  $(i, j)$  includes two parts, fixed sailing cost (e.g., port service fee, segment-specific sailing cost, and possible ice-breaking service fee),  $c_{ij}^1$ , which is port- and segment-specific, and variable sailing cost (e.g., fuel consumption, crew and procurement cost, depreciation cost, insurance and finance cost, and repair and maintenance cost),  $t_{ij} c_{ij}^2$ , which is segment-specific and proportional to sailing time,  $t_{ij}$ . For simplicity, this objective function neglects some minor cost components like cargo handling cost, for the reason that the induced linear terms by these cost components do not result in any algorithmic or computational change but only cause some extra workload in data collection and processing. Different from Rana and Vickson's (1989) model, our objective function introduces a penalty term to represent the schedule reliability.

Then we continue to construct the constraint set of the deterministic model as follows. First, on any sailing segment, a vessel should never load cargos exceeding its capacity, which results in the following *capacity constraints*:

$$\sum_{p=1}^i \sum_{q=j}^n y_{pq} \leq a_{ij} + M(1 - x_{ij}) \quad \forall i = 1, \dots, n-1; j = i+1, \dots, n \quad (2a)$$

$$\sum_{p=i}^n \sum_{q=1}^j y_{pq} \leq a_{ij} + M(1 - x_{ij}) \quad \forall j = 2, \dots, n; i = 1, \dots, j-1 \quad (2b)$$

where  $M$  is a very big number that far exceeds any possible variable value in this model and the constraints in (2a) and (2b) are developed, respectively, for the outbound and inbound trips. It is worth mentioning that the capacity of a vessel varies with season, temperature, location and density of the sea water and hence has different values over different sailing segments. The left-hand side of either of the above constraints represents the total cargo flow carried on segment  $(i, j)$ , while the right-hand side denotes the capacity of the vessel on  $(i, j)$ . When ports  $i$  and  $j$  are directly connected in the route, i.e.,  $x_{ij} = 1$ , the right-hand side indicates the capacity; when the two ports are not directly connected, i.e.,  $x_{ij} = 0$ , the right-hand side turns into a very big number, implying that no capacity constraint needs to be imposed on the cargo flow carried on segment  $(i, j)$ .

Second, a set of so-called *arrival constraints* are required to ensure the arrival of any cargo demand to its destination port. Such constraints for the outbound and inbound trips are given in (3a) and (3b), respectively. For the outbound trip, for example, the constraint implies that if there is a certain amount of cargo demand from port  $p$  to port  $q$ , then there must be at least a segment connecting port  $p$  (or another port between port  $p$  and  $q$ ) and port  $q$ .

$$y_{pq} \leq M \sum_{i=p}^{q-1} x_{iq} \quad \forall p = 1, \dots, n-1; q = p+1, \dots, n \quad (3a)$$

$$y_{pq} \leq M \sum_{i=q+1}^p x_{iq} \quad \forall p = 2, \dots, n; q = 1, \dots, p-1 \quad (3b)$$

Similarly, to ensure the departure of any cargo demand from its origin port, a set of *departure constraints* are also required, where the one in (4a) is for the outbound trip and the one in (4b) is for the inbound trip, respectively.

$$y_{pq} \leq M \sum_{j=p+1}^q x_{pj} \quad \forall p = 1, \dots, n-1; q = p+1, \dots, n \quad (4a)$$

$$y_{pq} \leq M \sum_{j=q}^{p-1} x_{pj} \quad \forall p = 2, \dots, n; q = 1, \dots, p-1 \quad (4b)$$

where  $M$  in (3) and (4) is still a very big number.

Third, the following *demand constraint* indicates that between any origin–destination port pair the actually served cargo demand by the vessel is no more than the potentially induced cargo demand, implying that in some cases the potentially induced cargo demand between an origin–destination port pair may not be fully satisfied.

$$y_{pq} \leq d_{pq} \quad \forall p \in N, q \in N \quad (5)$$

where the potentially induced cargo demand is specified by the demand elasticity function in (9).

It should be noted that the constraints given above in (3)–(5) presents an alternative form to the corresponding constraints in Rana and Vickson's (1988) model. In our model, the cargo demand between any origin–destination port pair is a variable dependent on the published schedule for this port pair. We found that directly adopting constraints from Rana and Vickson's model will introduce a

quadratic item into the constraint set, destructing the linear structure of the model and adding extra computational efforts. Nevertheless, the set of alternative constraints in our model are equivalent to, but appear as a decomposed form of the relevant constraints in Rana and Vickson's model.

Fourth, the shipping route must consist of a series of consecutive sailing segments, which are specified by the following *route constraints*. The constraints in (6), (7) and (8) are applied to the first port, intermediate ports, and last port, respectively; moreover, the constraints in (6a), (7a) and (8a) are for the outbound trip and the ones in (6b), (7b) and (8b) for the inbound trip. It should be noted that under our current setting the first and last ports to be visited (i.e., port 1 and port  $n$ ) have been determined in advance to the routing and scheduling decisions. If this is not the case, we can extend the shipping network by adding two virtual ports, port 0 and port  $n + 1$ , and set the two virtual ports as the first and last ports.

$$\sum_{j=2}^n x_{1j} = 1 \quad (6a)$$

$$\sum_{j=1}^{n-1} x_{nj} = 1 \quad (6b)$$

$$\sum_{i=1}^{p-1} x_{ip} - \sum_{j=p+1}^n x_{pj} = 0 \quad \forall p = 2, \dots, n-1 \quad (7a)$$

$$\sum_{i=p+1}^n x_{ip} - \sum_{j=1}^{p-1} x_{pj} = 0 \quad \forall p = 2, \dots, n-1 \quad (7b)$$

$$\sum_{i=1}^{n-1} x_{in} = 1 \quad (8a)$$

$$\sum_{i=2}^n x_{i1} = 1 \quad (8b)$$

Fifth, the elastic demand functions specify the relationship between the induced cargo demand and the announced sailing time, which are in the following linear forms for the outbound and inbound trips, respectively, as shown in (9a) and (9b), where  $D_{pq}$  is the base cargo demand from origin port  $p$  to destination port  $q$  and  $s_{pq}^e$  is a parameter to indicate the demand elasticity or sensitivity degree to the announced sailing time along the segment from origin port  $p$  to destination port  $q$ :

$$d_{pq} = D_{pq} - s_{pq}^e (T_q^\alpha - T_p^\alpha) \quad \forall i \in N \quad (9a)$$

$$d_{pq} = D_{pq} - s_{pq}^e (T_q^\beta - T_p^\beta) \quad \forall i \in N \quad (9b)$$

where using a constant demand elasticity like  $s_{pq}^e$  to specify the relationship between transportation cost/time and induced demand has been widely employed in the freight transportation practice (Wilson, 1978; Beuthe et al., 2001).

Sixth, we also write down a set of *delay constraints* to properly evaluate the delay of the vessel: If the vessel arrives at a port later than its claimed arrival time, the delay is the actual arrival time minus the claimed arrival time; otherwise, the delay is zero. The delay at each port will be introduced into the objective function for calculating the delay penalty.

$$e_i^\alpha \geq t_i^\alpha - T_i^\alpha \quad \forall i \in N \quad (10a)$$

$$e_i^\beta \geq t_i^\beta - T_i^\beta \quad \forall i \in N \quad (10b)$$

$$e_i^\alpha \geq 0 \quad \forall i \in N \quad (11a)$$

$$e_i^\beta \geq 0 \quad \forall i \in N \quad (11b)$$

In the above delay constraints, the arrival time at any port is defined as the accumulated sailing time over all used segments before the arrived port, if we set the departure time at the first port to be 0. Such relationships between arrival times and sailing times for the outbound and inbound trips are presented below, respectively:

$$t_i^\alpha = \sum_{k=1}^{i-1} \sum_{j=k+1}^i t_{kj} x_{kj} \quad \forall i = 2, \dots, n$$

$$t_i^\beta = \sum_{k=1}^{n-1} \sum_{j=k+1}^n t_{kj} x_{kj} + \sum_{k=n}^{i+1} \sum_{j=k-1}^i t_{kj} x_{kj} \quad \forall i = n-1, \dots, 1$$

where it must be noted that in this deterministic model, the actual sailing time along any segment is just its *nominal sailing time*, which is defined as the sailing time under the ideal weather and ocean conditions or the sailing time unaffected by inclement weather and ocean conditions.

Apparently, in the case of sailing time uncertainty arising, which will be elaborated in the next two subsections, if a published schedule is too tight, the risk of violating on-time arrivals and causing delay penalties would be higher (see the constraints in (10) and (11)), which in turn reduces the total profit; in contrast, if a published schedule is too loose, it may attract less cargo demand (see the constraints in (9)), which decreases the total profit as well. Therefore, the scheduling plan must be carefully determined to achieve an optimal trade-off for profit maximization.

Seventh, we specify the announced sailing time along any segment (i.e., the difference between the arrival time at the downstream port and the departure time from the upstream port) is not less than its nominal sailing time. This specification may not be required in terms of the solution feasibility and mathematical property of the model, but it guarantees delivering an acceptable and reasonable announced schedule to the public. Such announced sailing time constraints for the outbound and inbound trips are given as follows, as shown in (12a) and (12b), respectively:

$$T_j^\alpha - T_i^\alpha \geq t_{ij} x_{ij} \quad \forall i \in N, j \in N, i < j \quad (12a)$$

$$T_j^\beta - T_i^\beta \geq t_{ij} x_{ij} \quad \forall i \in N, j \in N, i > j \quad (12b)$$

Finally, it is noted that the arrival time at the last port, i.e., port  $n$ , appears in both the outbound and inbound trips, implying the following equivalency relationship:

$$T_n^\alpha = T_n^\beta \quad (13)$$

and for modeling convenience, the initial departure time from the first port, i.e., port 1, is set to 0:

$$T_1^\alpha = 0 \quad (14)$$

### 3.4. Robust counterpart with bounded uncertainty

We first consider a RO case that each uncertain parameter has its own uncertainty interval, as independent of that of any other uncertain parameter. This implies the situation that the uncertain weather and ocean conditions on a sailing segment are little related to those on another sailing segment. From the above deterministic model, it is readily observed that the objective function in (1) and the constraint in (10) contain the data related to weather and ocean conditions, actual sailing time  $t_{ij}$ , and hence will be affected by the sailing time uncertainty. As for the constraint in (12), it merely contains nominal sailing time  $t_{ij}$ , which is used for creating an acceptable announced schedule for the shipping community instead of evaluating arrival delay or sailing benefit and cost. Therefore, the following robust counterpart model with bounded uncertainty can be derived from the above deterministic model with updating the objective function in (1) and the constraints in (10).

Suppose that the actual sailing time on segment  $(k, j)$  ranges in the following interval:

$$|t_{kj} - t_{kj}| < \epsilon |\mathbb{I}_{\mathcal{A}\mathbb{I}}|$$

where  $\epsilon > 0$  is a given (relative) uncertainty level bound. According to RO theory, all constraints hold true with the fluctuations of uncertain parameters and the optimal objective function value can be obtained regardless of the change. For the constraint in (10a), for example, its robust counterpart can be rewritten as:

$$e_i^\alpha \geq \max_{t_{kj}} \left\{ \sum_{k=1}^{i-1} \sum_{j=k+1}^i t_{kj} x_{kj} : |t_{kj} - t_{kj}| < \epsilon |\mathbb{I}_{\mathcal{A}\mathbb{I}}| \right\} - t_\gamma^\alpha$$

which is equivalent to:

$$e_i^\alpha \geq \sum_{k=1}^{i-1} \sum_{j=k+1}^i (t_{kj} + \epsilon |\mathbb{I}_{\mathcal{A}\mathbb{I}}|) \mathcal{I}_{\mathcal{A}\mathbb{I}} - t_\gamma^\alpha \quad (15a)$$

Following a similar fashion, the constraint in (10b) can be converted as:

$$e_i^\alpha \geq \sum_{k=1}^{n-1} \sum_{j=k+1}^n (t_{kj} + \epsilon |\mathbb{I}_{\mathcal{A}\mathbb{I}}|) \mathcal{I}_{\mathcal{A}\mathbb{I}} + \sum_{\mathcal{A}=3}^{\mathcal{Q}+1} \sum_{\mathbb{I}=\mathcal{A}-\mathcal{Q}}^{\mathcal{Q}} (\mathbb{I}_{\mathcal{A}\mathbb{I}} + \epsilon |\mathbb{I}_{\mathcal{A}\mathbb{I}}|) \mathcal{I}_{\mathcal{A}\mathbb{I}} - t_\gamma^\alpha \quad (15b)$$

On the other hand, the objective function in (1) can be rewritten as follows:



$$\max \sum_{p \in N} \sum_{q \in N} r_{pq} y_{pq} - \max_{t_{ij}} \left\{ \sum_{i \in N} \sum_{j \in N} \left( c_{ij}^1 + t_{ij} c_{ij}^2 \right) x_{ij} : \left| t_{ij} - t_{ij} \right| < \epsilon \mathbf{I}_{\mathcal{I}_1} \right\} - \sum_{\gamma \in n} \left( \cdot_{\gamma}^{\alpha} + \cdot_{\gamma}^{\beta} \right) \mathcal{P}_{\gamma}$$

Here we apply the maximin criterion to evaluate the net system profit in the worst case. When all sailing times reach their upper bounds, the demand shrinks to the lowest, while the cost increases to a climax, leading to the least profit. Therefore, as a result, the objective function in (1) can be further simplified as:

$$\max \sum_{p \in N} \sum_{q \in N} r_{pq} y_{pq} - \sum_{i \in N} \sum_{j \in N} \left( c_{ij}^1 + (t_{ij} + \epsilon \mathbf{I}_{\mathcal{I}_1}) \mathbf{J}_{\mathcal{I}_1}^R \right) \mathcal{I}_{\mathcal{I}_1} - \sum_{\gamma \in n} \left( \cdot_{\gamma}^{\alpha} + \cdot_{\gamma}^{\beta} \right) \mathcal{P}_{\gamma} \quad (16)$$

To sum up, the robust counterpart with bounded uncertainty is comprised of the objective function in (16) and the constraints in (2)-(9) and (11)-(15), which presents an MILP problem.

### 3.5. Robust counterpart with budget-bounded uncertainty

We then consider a more complex, less conservative RO case, in which a set of relevant uncertain parameters are collectively subject to a so-called budget bound. This implies another situation that the uncertain weather and ocean conditions on different sailing segments are affected by each other: The sum of uncertainties over a series of consecutive sailing segments is bounded by a prespecified budget.

We know that the optimal solution of the above RO model is fully immune to the sailing time uncertainty. In many cases, however, it may not be unrealistic and irrational to take every possible deviation of sailing time into consideration. In spite of lack of explicit information on the distribution of sailing time, there are other factors that may guide us to seek a more conservative solution for dealing with uncertainty. For example, a frequently used strategy in risk management is to utilize insurance to cover risks, but insurance is provided at a price. A full-price insurance is usually too pricey and unnecessary. Very often, a wiser action is to evaluate the extent of risks that one can take and leave the rest to the insurance. In a similar way, RO provides an approach to insure system optimization with uncertainties, which can relieve the extreme effect of deviations of uncertain parameters. It is important to evaluate the bearable extent of risks, and then leave the rest to be protected by RO, or in other words, to decide the number of parameters to be protected by RO. The amount of protection added to the  $i$  th constraint is controlled through a given budget  $\Gamma_i$ , which indicates the number of parameter deviations from which we wish to protect. To be more specific, the optimal solution is protected against all cases in which up to  $\lfloor \Gamma_i \rfloor$  number of these parameters in the  $i$  th constraint are allowed to change completely, and one parameter changes by at most  $\Gamma_i - \lfloor \Gamma_i \rfloor$ . The level of protection employed in the model reflects the level of conservativeness of the planner. It is the planner's responsibility to decide what extent of the impact of uncertainty on the feasibility of the solution to be accepted and the level of profits to be sacrificed. How to set a trade-off between the reduced revenue and the increased robustness can be found in, for example, Bertsimas and Sim (2003).

In the budget-bounded case of our liner routing and scheduling problem, each actual sailing time  $t_{kj}$  is modeled as an independent and bounded random parameter that falls within an interval  $((1 - \epsilon) \mathbf{I}_{\mathcal{I}_1}, (Q + \epsilon) \mathbf{I}_{\mathcal{I}_1})$ , provided that the bound of the sailing time is known a priori. If we set  $\zeta_{kj}$  equal to  $(t_{kj} - t_{kj}) / \epsilon \mathbf{I}_{\mathcal{I}_1}$ , then  $\zeta_{kj}$  can be regarded as the extent of deviation off the nominal value, where  $\zeta_{kj} \in (-1, 1)$ . In the RO context, it can also be used to indicate the level of protection or conservatism described before.

Following the budget-bounded uncertainty definition, a subproblem, which will be used for constructing the robust counterpart with budget-bounded uncertainty, is given for each constraint in (10) in the following form:

$$\begin{aligned} & \max \sum_k \sum_j \epsilon \mathbf{I}_{\mathcal{I}_1} \mathcal{I}_{\mathcal{I}_1} \zeta_{kj} \\ & \text{subject to } \sum_k \sum_j \zeta_{kj} \leq \Gamma_i \\ & 0 \leq \zeta_{kj} \leq 1 \quad \forall k, j \end{aligned}$$

where two important formulation issues should be clearly stated: 1)  $\zeta_{kj}$ , not  $x_{kj}$ , is the decision variable of this subproblem; 2) since  $\zeta_{kj}$  would have a negative impact on optimal solutions only when its value is greater than 0 (i.e., the value of actual sailing time  $t_{kj}$  is greater than its nominal value), it is sufficient to use the range of  $[0, 1]$  instead of  $[-1, 1]$  for  $\zeta_{kj}$  in this subproblem. The objective function of the subproblem is to maximize a linear combination of all  $\zeta_{kj}$ 's. Given the simple linear structure of the constraints, the optimal solution of this subproblem can be obtained by ranking all  $\zeta_{kj}$ 's in terms of the decreasing order of their coefficients,  $\epsilon \mathbf{I}_{\mathcal{I}_1} \mathcal{I}_{\mathcal{I}_1}$ , where the first  $\lfloor \Gamma_i \rfloor$  of  $\zeta_{kj}$ 's are set to 1, the next  $\zeta_{kj}$  is set to  $\Gamma_i - \lfloor \Gamma_i \rfloor$ , and all the remaining  $\zeta_{kj}$ 's are set to 0. This is exactly what the level of protection or conservatism means.

By incorporating the above subproblem, the robust counterpart with budget-bounded uncertainty can be derived as follows. First, the constraints in (10) are converted to:

$$e_i^\alpha \geq \max_{\zeta_{kj}} \left\{ \sum_{k=1}^{i-1} \sum_{j=k+1}^i (t_{kj} + \epsilon \mathbb{I}_{\mathcal{A}\mathbb{I}} \zeta_{\mathcal{A}\mathbb{I}}) \mathcal{I}_{\mathcal{A}\mathbb{I}} \leq \sum_{\mathcal{A}=\mathcal{Q}}^{\mathcal{I}-\mathcal{Q}} \sum_{\mathbb{I}=\mathcal{A}+\mathcal{Q}}^{\mathcal{I}} \zeta_{\mathcal{A}\mathbb{I}} \leq \Gamma_{\mathcal{I}}^\alpha, \mathbf{P} \leq \zeta_{\mathcal{A}\mathbb{I}} \leq \mathbf{Q}, \forall \mathcal{A}, \mathbb{I} \right\} - t_i^\alpha \quad (17a)$$

$$e_i^\beta \geq \max_{\zeta_{kj}} \left\{ \sum_{k=1}^{n-1} \sum_{j=k+1}^n (t_{kj} + \epsilon \mathbb{I}_{\mathcal{A}\mathbb{I}} \zeta_{\mathcal{A}\mathbb{I}}) \mathcal{I}_{\mathcal{A}\mathbb{I}} + \sum_{\mathcal{A}=\exists}^{\mathcal{I}+\mathcal{Q}} \sum_{\mathbb{I}=\mathcal{A}-\mathcal{Q}}^{\mathcal{I}} (\mathbb{I}_{\mathcal{A}\mathbb{I}} + \epsilon \mathbb{I}_{\mathcal{A}\mathbb{I}} \zeta_{\mathcal{A}\mathbb{I}}) \mathcal{I}_{\mathcal{A}\mathbb{I}} \leq \sum_{\mathcal{A}=\mathcal{Q}}^{\exists-\mathcal{Q}} \sum_{\mathbb{I}=\mathcal{A}+\mathcal{Q}}^{\exists} \zeta_{\mathcal{A}\mathbb{I}} + \sum_{\mathcal{A}=\exists}^{\mathcal{I}+\mathcal{Q}} \sum_{\mathbb{I}=\mathcal{A}-\mathcal{Q}}^{\mathcal{I}} \zeta_{\mathcal{A}\mathbb{I}} \leq \Gamma_{\mathcal{I}}^\beta, \mathbf{P} \leq \zeta_{\mathcal{A}\mathbb{I}} \leq 1, \forall k, j \right\} - T_i^\beta \quad (17b)$$

It is difficult to directly solve the inner problem in (17) with  $\zeta_{ij}$  as its decision variable. Duality theory is employed here so that the inner problem can be rewritten. Obviously, the dual of the inner problem is a minimization problem. The maximum objective function value of the main problem can be obtained when the minimum objective function value of this dual is derived. Therefore, for each  $i \in N$ , the inner problems in (17) can be accommodated by the main problem through simply representing the corresponding dual problems by using a combination of the constraints of these duals, which are synthetically presented as follows:

$$e_i^\alpha \geq \sum_{k=1}^{i-1} \sum_{j=k+1}^i (t_{kj} x_{kj} + w_{kj}) + \Gamma_i^\alpha u_i - T_i^\alpha \quad (18a)$$

$$e_i^\beta \geq \sum_{k=1}^{n-1} \sum_{j=k+1}^n (t_{kj} x_{kj} + w_{kj}) + \sum_{k=n}^{i-1} \sum_{j=k-1}^i (t_{kj} x_{kj} + z_{kj}) + \Gamma_i^\beta v_i - T_i^\beta \quad (18b)$$

$$\sum_{i=j}^n u_i + \sum_{i=n}^1 v_i + w_{kj} \geq \epsilon \mathbb{I}_{\mathcal{A}\mathbb{I}} \mathcal{I}_{\mathcal{A}\mathbb{I}} \quad \mathcal{Q} \leq \mathcal{A} < \mathbb{I} \leq \exists \quad (18c)$$

$$\sum_{i=j}^1 v_i + z_{kj} \geq \epsilon \mathbb{I}_{\mathcal{A}\mathbb{I}} \mathcal{I}_{\mathcal{A}\mathbb{I}} \quad \exists \geq \mathcal{A} > \mathbb{I} \geq \mathcal{I} \quad (18d)$$

$$w_{kj} \geq 0 \quad 1 \leq k < j \leq n \quad (18e)$$

$$z_{kj} \geq 0 \quad n \geq k > j \geq i \quad (18f)$$

$$u_i, v_i \geq 0 \quad (18g)$$

where  $u_i$  is the dual variable corresponding to constraint  $\sum_{k=1}^{i-1} \sum_{j=k+1}^i \zeta_{kj} \leq \Gamma_i^\alpha$  in (17a),  $v_i$  is the dual variable to constraint  $\sum_{k=1}^{n-1} \sum_{j=k+1}^n \zeta_{kj} + \sum_{k=n}^{i-1} \sum_{j=k-1}^i \zeta_{kj} \leq \Gamma_i^\beta$  in (17b),  $w_{kj}$  is the dual variable to constraint  $\zeta_{kj} \leq 1$  in (17a) and (17b), and  $z_{kj}$  is the dual variable to constraint  $\zeta_{kj} \leq 1$  in (17b).

Finally, by making use of duality theory, the objective function of the robust counterpart with budget-bounded uncertainty can be developed as follows:

$$\max \sum_{p \in N} \sum_{q \in N} r_{pq} y_{pq} - \sum_{i \in N} \sum_{j \in N} (c_{ij}^1 + t_{ij} c_{ij}^2) x_{ij} - \left( \sum_{i \in N} \Gamma_i^\alpha u_i + \sum_{i \in N} \Gamma_i^\beta v_i + \sum_{k=1}^{n-1} \sum_{j=k+1}^n w_{kj} + \sum_{k=n}^2 \sum_{j=k-1}^1 z_{kj} \right) - \sum_{i \in N} (e_i^\alpha + e_i^\beta) g_i \quad (19)$$

In summary, the formulation of the robust counterpart with budget-bounded uncertainty can be represented collectively by (19), (2)-(9), (11)-(14), and (18). Obviously, this is an MILP problem as well.

### 3.6. Solver and computing environment

As shown above, by using some formulation transformation processes in the RO framework, the two resulting robust models both have a deterministic MILP formulation, for which previously developed state-of-the-art solution algorithms can be directly applied for their solutions. A number of existing commercial software packages including LINGO, CPLEX, and GAMS can be used to solve this kind of problems in an efficient manner. In this study, LINGO is chosen for solving all the three MILP problems (including the deterministic model and two robust models) with its built-in branch-and-bound procedure. All the resulting computational experiments were run on a desktop computer equipped with 3.6 GHz Quad Core CPU and a 16 GB RAM.

## 4. Case study

The main purpose of this case study is to illustrate the applicability of the developed models, explore insights behind the obtained solutions, and understand how the impacts occur and vary from circumstance uncertainties and how to reduce those impacts.

#### 4.1. Context and data

As known as a “Golden Channel” through the Arctic Ocean, we used the NSR as the context of the case study for examining the three models presented in this paper. According to the definition given by the Russian legislation, the NSR is a shipping line connecting the Atlantic Ocean and the Pacific Ocean along the Russian coast stretching from the Barents Sea, via Siberia, to the Far East. The line has been a tempting treasure for the maritime shipping industry, not only because it may cut down the sailing time by up to 40 percent compared to the existing shipping lines, but also because of its geographical and geopolitical values in connecting East Asia to Russia, North Europe and other Arctic areas. Traditionally, there are three major routes to sail from Europe to the Pacific Ocean, namely via the Suez Canal, the Panama Canal, and the Cape of Good Hope. From Shanghai to Amsterdam, the corresponding sailing distances along these three routes are 10,762 kn, 14,139 kn and 12,071 kn, respectively; in contrast, the nominal distance of the NSR is merely 6,726 kn. Apparently, it is a more economical choice, especially for the liner shipping industry, where the daily operation cost of a single ship can reach hundreds of thousands of U.S. dollars. Even though the present high bunker price and constructional cost of ice-breaking vessels made it controversial that adopting the NSR is economically advantageous, there is no question that the route can significantly save the sailing time and cost in most cases.

With the global warming process speeding, the ice in the Arctic area is melting massively, increasing the possibility of taking advantage of the NSR. In August 2009, two German heavy lift vessels, *Beluga Fraternity* and *Beluga Foresight*, commenced an east-to-west passage of the NSR, which marked a new era when non-Russian vessels are capable of sailing through the NSR. Since then, the commercial development of the NSR has been discussed by the media and seriously considered by the shipping sector. In 2013, [Smith and Stephenson \(2013\)](#) reported that the feasibility for normal open-water ships to navigate along the NSR in late summer both increases in frequency and expands geographically and the emergence of unprecedented new shorter navigation routes for ice-strengthened ships is also expected in the following decades.

Previous studies on the NSR shipping mainly focused on economic viability (e.g., [Verny and Grigentin, 2009](#); [Ding et al., 2020](#); [Wang et al., 2020](#)) and sailing feasibility (e.g., [Ho, 2010](#); [Shibata et al., 2013](#); [Aksenov et al., 2016](#)). The research on the planning and management of NSR shipping lines is very limited ([Lin and Chang, 2018](#); [Ding and Xie, 2021](#)), for which part of the reason is insufficient related data and experience to the port infrastructure, shipping vessels, cost structures, demand type and amount, and sea conditions along the NSR. In view of this literature, this paper, thus, plays a role of attempting to fill, at least partially, the research gap from a shipping line planning perspective.

Most existing economic analyses on the NSR focus on shipping costs, evaluating and comparing the costs of using NSR and traditional routes like the Suez Canal. However, since the route has not been officially open yet, the insufficiency of real data or using hypothetical data poses technical challenges to these studies. Our case study is not an exclusive case. In addition, unlike those studies evaluating the overall performance of a liner fleet or the whole market, this research is concerned only with the performance of a single liner ship, posing a very low level of data requirements. An economic analysis on the revenue and demand is a complex and challenging process, involving a wide range of information, including the category of goods, shipping distance, shipping company, size of containers and so on, which is far beyond the scope of this study. In this study, the relevant revenue, base demand and demand sensitivity values are roughly estimated in terms of the previous results from [Liu and Kronbak \(2010\)](#), [Schøyen and Bråthen \(2011\)](#), [Omre \(2012\)](#), [Furuichi and Otsuka \(2013\)](#), as shown in [Table 3 to 5](#), respectively.

Moreover, key parameters in the case study are set as follows. The uncertainty bound  $\epsilon$  is first arbitrarily set to 0.4. The fixed cost  $f_{ij}^1$  for each sailing segment is set as US\$20,000 on average. A liner ship consumes 45 ton of fuel per day at the speed of 14 kn; with a fuel price of 600 US\$/ton, the daily variable cost  $f_{ij}^2$  can be roughly set to US\$30,000/day. The delay penalty rate is set as US\$80,000/day. The ship capacity is set as 4,000 TEUs, as suggested by [Wang and Shou \(2013\)](#) and [Shou and Feng \(2015\)](#), in which these previous studies found that due to the current water depth in the Arctic area, the largest capacity of a container ship allowed to use the NSR may not be significantly larger than 4,000 TEUs.

#### 4.2. Comparative evaluation of solution performance

Following our model development work presented earlier, the resulting three models were implemented in the case study, namely, the deterministic model, where the actual sailing time on any segment is expected to be exactly its nominal sailing time, the robust

**Table 2**  
Nominal sailing time between ports.<sup>1</sup>

		1 Shanghai	2 Provideniya	3 Pevek	4 Tiksi	5 Dikson Island	6 Archangel	7 Murmansk	8 Rotterdam
1	Shanghai	–	10	12	15	17	20	20	24
2	Provideniya	10	–	2	5	7	10	10	14
3	Pevek	12	2	–	3	5	8	8	12
4	Tiksi	15	5	3	–	3	6	6	10
5	Dikson Island	17	7	5	3	–	3	3	7
6	Archangel	20	10	8	6	3	–	1	5
7	Murmansk	20	10	8	6	3	1	–	5
8	Rotterdam	24	14	12	10	7	5	5	–

<sup>1</sup> The unit of the nominal sailing time is days.

**Table 3**Revenue for shipping a unit of goods between ports.<sup>1</sup>

		1 Shanghai	2 Provideniya	3 Pevek	4 Tiksi	5 Dikson Island	6 Archangel	7 Murmansk	8 Rotterdam
1	Shanghai	–	900	1,200	1,350	1,700	1,800	2,000	2,160
2	Provideniya	900	–	180	400	630	840	900	1,120
3	Pevek	1,200	180	–	270	500	720	800	1,080
4	Tiksi	1,350	400	270	–	270	480	540	800
5	Dikson Island	1,700	630	500	270	–	270	300	630
6	Archangel	1,800	840	720	480	270	–	90	400
7	Murmansk	2,000	900	800	540	300	90	–	450
8	Rotterdam	2,160	1,120	1,080	800	630	400	450	–

<sup>1</sup> The unit of the revenue for shipping a unit of goods is U.S. dollars per TEU.**Table 4**Base shipping demand between ports.<sup>1</sup>

		1 Shanghai	2 Provideniya	3 Pevek	4 Tiksi	5 Dikson Island	6 Archangel	7 Murmansk	8 Rotterdam
1	Shanghai	–	2,000	2,000	2,500	2,500	2,500	2,000	2,000
2	Provideniya	2,000	–	1,500	2,000	2,000	2,000	1,500	1,500
3	Pevek	2,000	1,500	–	2,000	2,000	2,000	1,500	1,500
4	Tiksi	2,500	2,000	2,000	–	2,500	2,500	2,000	2,000
5	Dikson Island	2,500	2,000	2,000	2,500	–	2,500	2,000	2,000
6	Archangel	2,500	2,000	2,000	2,500	2,500	–	2,000	2,000
7	Murmansk	2,000	1,500	1,500	2,000	2,000	2,000	–	1,500
8	Rotterdam	2,000	1,500	1,500	2,000	2,000	2,000	1,500	–

<sup>1</sup> The unit of shipping demand is TEUs.**Table 5**Shipping demand sensitivity between ports.<sup>1</sup>

		1 Shanghai	2 Provideniya	3 Pevek	4 Tiksi	5 Dikson Island	6 Archangel	7 Murmansk	8 Rotterdam
1	Shanghai	–	30	20	30	30	20	30	20
2	Provideniya	30	–	30	50	50	30	50	30
3	Pevek	20	30	–	30	30	20	30	20
4	Tiksi	30	50	30	–	50	30	50	30
5	Dikson Island	30	50	30	50	–	30	50	30
6	Archangel	20	30	20	30	30	–	30	20
7	Murmansk	30	50	30	50	50	30	–	20
8	Rotterdam	20	30	20	30	30	20	20	–

<sup>1</sup> The unit of shipping demand sensitivity is TEUs per day.

counterpart model with bounded uncertainty and the robust counterpart model with budget-bounded uncertainty. For the budget-bounded model, its budget is set to 5, based on our estimation on the overall variation of sailing times and our acceptance level on the worst conditions. The solutions obtained from the three models look very different for the same input data, which makes it interesting to compare the solution performance, profitability and robustness of these models.

Here we distinguish solution profitability and robustness in the following manner. Profitability is simply measured by the expected objective function value gained from the optimal solutions. When the robustness needs to be evaluated, solutions are tested under such a situation that the actual sailing times might be longer than the original nominal values so that the robustness of the solutions against uncertainties can be observed. The difference between profitability and robustness lies in the circumstance where the objective function value is evaluated. For example, all the sailing times are exactly consistent with those values listed in Table 2 when calculating the profitability for the deterministic model, while the sailing times would be different from those in Table 2 when uncertainties are considered and the robustness needs to be evaluated.

The optimal routing (i.e., which ports are chosen) and scheduling (i.e., announced arrival times at the chosen ports) results of the three models applied for the case study are shown below in Fig. 1 and Table 6. Fig. 1 shows a route with chosen ports along the NSR line, as shared by the optimal solutions of the three models, which shows the routing decision is very stable despite various uncertainties. Among the three models, the bounded model provides the most conservative schedule and the least profit, while the solution to the deterministic model presents a set of most radical routing and scheduling decisions without considering any uncertainty.

When it comes to the scheduling decision, it is interesting to look at the symmetry of the solutions between an inbound trip and its corresponding outbound one. The deterministic model provides a symmetrical solution with the midpoint at port  $n$ , while the bounded

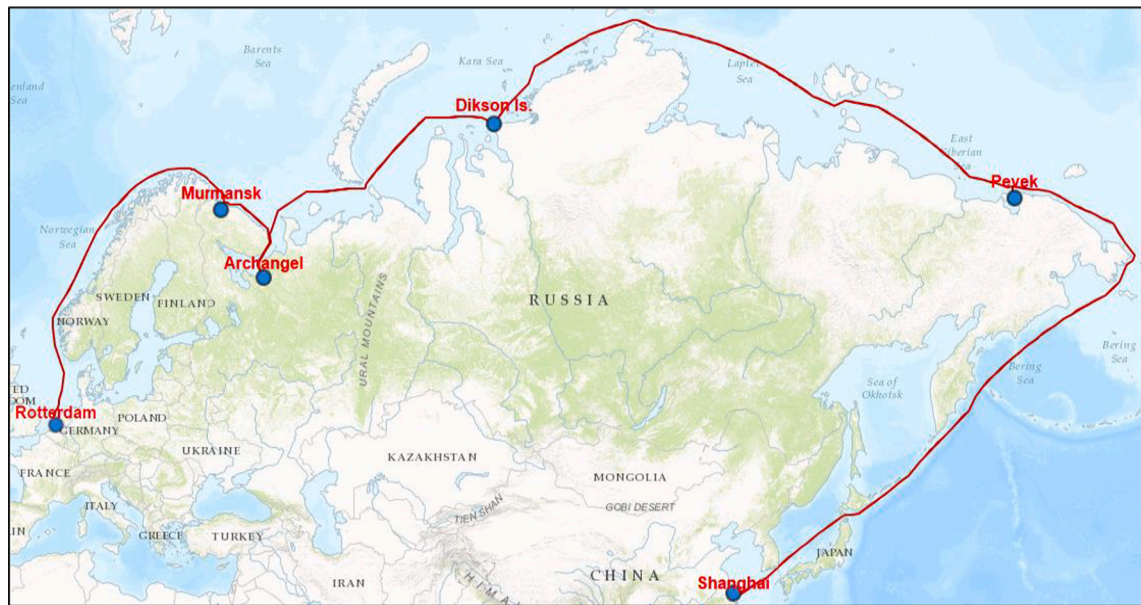


Fig. 1. The common route map of the solutions of the three models.

Table 6

Routing and scheduling solutions from the three models.

Model	Deterministic	Bounded	Budget-bounded
Objective function value (\$)	14,448,600	14,119,500	14,283,600
Chosen ports	Claimed arrival date (day)		
1	0	0	0
3	12	17	13
5	18	25	20
6	21	30	24
7	23	40	32
8	29	46	37
7	35	52	43
6	37	54	45
5	40	57	48
3	46	63	53
1	58	83	65

model provides an asymmetrical one. For instance, the planned sailing time in the deterministic model for segment (3, 5) is 6 (=18–12) days and that of its corresponding inbound trip equals to 6 (=46–40) days as well. In contrast, in the solution of the bounded model, the planned sailing time on segment (3, 5) is 8 (=25–17) days while that of its inbound counterpart is 6 (=63–57) days.

This above result is intuitively understandable for the deterministic case since all the supply and demand data are set symmetrical. The solution difference can be attributed to the minimum time constraints in (12). The set of constraints enforce that the minimum difference of the claimed arrival times between any two adjacent visited ports in the route should be no less than the corresponding nominal sailing time. In the deterministic model, where the actual sailing time is exactly the nominal sailing time, the solutions are hardly influenced by the constraints in (12). However, for the bounded model, there may be a gap between the nominal sailing time and the actual sailing time. Given the fact that a tighter schedule will attract more orders, it might be more lucrative to tighten the schedule and for some ports, as a result, the accumulated uncertainty causes the schedule asymmetry.

It is also worth mentioning that in the above scheduling results none of the solutions leads to any delay (i.e., all delays are less than or equal to 0). The major cause for this result is that the penalty is relatively high compared to the revenue. With reference to the routing result, it should be pointed out that ports 2 and 4 are never selected in any case. What they have in common are the low revenue rate and high demand sensitivity, so the usage of these ports will cause a severe decline in the total revenue.

Based on a discussion on the profitability of the solution from the three models, it seemingly suggests that the most lucrative model goes to the deterministic one. However, an emphasis should be made that the optimal objective function value is not the actual profit, but the expected profit instead. The deterministic model yields the most optimistic profit under an unrealistic assumption that the actual sailing times will be exactly the nominal sailing times. The actual sailing times will be affected by various sources of uncertainties, deviating more or less from the nominal sailing times. As this study mainly concentrates on improving the robustness to

uncertainties in sailing times under the worst conditions, it is therefore more interesting to derive the solutions with uncertain sailing times, compare the actual arrival times with the published schedule, and recalculate the corresponding punctuality indicators and actual profits. The actual profits here are employed to measure the robustness of the solutions.

Table 7 lists the performance of the solutions obtained from the three models when all the sailing times reach their upper bounds in the worst case. Once again, it is noted that when all uncertainty budgets are set to 0, the budget-bounded model is equal to the deterministic model, so the former is not focused here. It can be observed that the most lucrative model is the bounded model, because the solution it provides can assure the punctuality in any case. For the other two models, as no buffer time is reserved in the schedule, hardly can they resist any uncertainty in sailing time caused by uncertain weather and ocean conditions. Moreover, as a result of the domino effect, a delay at any port will be accumulatively added into the delays at the subsequent ports, so that the actual profits are typically much lower than the expected. As is shown in Table 6, the amount of the sacrificed profit by adopting the bounded model rather than the deterministic model is US\$329,100. In contrast, the profit protected by the adoption of the bounded model shown in Table 7 is US\$3,731,000, roughly 11 times the amount of the sacrificed. In this case, compared to the sacrificed profit, the profit saved by the bounded robust model is much higher. Hence it is highly recommended to adopt the bounded model rather than the deterministic model when the importance of solution robustness arises.

#### 4.3. Effect of uncertainty bounds

Another interesting result in the case study is to explore the effect of varying uncertainty bounds of the bounded model on its objective function values. For this purpose,  $\epsilon$  is reset in an increasing order from 0.5 to 1 to represent varying uncertainty bounds. The optimization results, including routing and scheduling decisions and profit values, are summarized in Table 8.

It is observed from this table that the optimal objective function value becomes more conservative as the bound increases. The routing decisions are consistent over the three different uncertainty bounds, where only ports 2 and 4 are omitted in all the solutions. As for the scheduling decisions, all schedules keep in a punctual manner; the schedules extend in accordance with the bounds to avoid late arrivals. The solutions remain asymmetrical with more conservative schedules on the outbound trips, as can be seen from that the announced arrival times at port 8 are always later than half of the claimed arrival times at port 1.

#### 4.4. Impact of increasing trends of uncertainty budgets

A practical issue arising from implementing the budget-bounded model is how to set an uncertainty budget. A budget is undoubtedly a monotonic increasing function along the order of visited ports as it accumulates over time, but its increasing trend might be set in different ways.

A preliminary conjecture is that the increasing trend of an uncertainty budget can be set in at least three forms: Accelerative, stable and decelerative. The uncertainty budget at a port is calculated as the sum of all the possible deviation extents of the uncertainty parameters from the nominal values in the constraint in (17) for that port, and the scale of budget is dependent on both the total number of uncertain parameters and the deviation extent. The increasing rate is then defined as the average deviation extent of the emerging uncertain parameters at a given port and the trend can be obtained from comparing the deviation extent between two neighboring candidate ports. Let us take the uncertainty budgets of the first three ports as an example. There is only one uncertainty parameter in the constraints of the outbound trip at port 2, which is  $\zeta_{12}$ . Suppose that the deviation extent is 50%, so uncertainty budget  $\Gamma_2^\alpha$  is 0.5. As for port 3, the uncertain parameters are  $\zeta_{13}$  and  $\zeta_{23}$ , following  $\zeta_{12}$ . Notice that  $\zeta_{13}$  and  $\zeta_{23}$  first appear in the constraint for port 3 and then they remain in all the punctuality constraints for the subsequent candidate ports. They are assumed to share a common deviation extent.  $\Gamma_3^\alpha$  can be obtained from adding  $\Gamma_2^\alpha$  to the total deviation extent of  $\zeta_{13}$  and  $\zeta_{23}$ . A 60% extent of  $\zeta_{13}$  and  $\zeta_{23}$  makes an accelerative budget with  $\Gamma_3^\alpha$  equal to 1.7. A decelerative budget can be 1.3 with a 40% extent of  $\zeta_{13}$  and  $\zeta_{23}$ . If the extent remains 50% for  $\zeta_{13}$  and  $\zeta_{23}$ , it becomes a stable budget with  $\Gamma_3^\alpha$  equal to 1.5.

The uncertainty budgets in this case are calculated as follows. The total number of uncertain factors at a port equals to the total number of all candidate segments located before this port, which are 0, 1, 3, 6, 10, ..., 49, and 56. The average deviation extents take

**Table 7**  
Profits and delays of the three models in the worst conditions.

Model		Deterministic	Bounded	Budget-bounded
Actual profits (\$)		10,388,500	14,119,500	12,145,800
Chosen ports	Actual arrival date (day)	Delay (day)		
1	0	0	0	0
3	17	-5	0	-4
5	25	-8	0	-5
6	30	-10	0	-6
7	40	-19	0	-8
8	46	-20	0	-9
7	52	-21	0	-9
6	54	-22	0	-9
5	57	-22	0	-9
3	63	-23	0	-10
1	83	-31	0	-18



**Table 8**

Solutions from the bounded model with varying uncertainty bounds.

Uncertainty bound	0.5	0.75	1.0
Objective function value (\$)	14,438,550	14,055,280	13,638,330
Chosen ports	Claimed arrival date (day)		
1	0	0	0
3	18	21	24
5	27	30	36
6	31	37	45
7	47	44	50
8	52	53	58
7	59	60	68
6	63	78	88
5	67	82	91
3	73	87	100
1	85	100	114

values from 1/16, 2/16, ..., and 16/16. The budgets are then calculated and shown in Fig. 2.

The solutions of the budget-bounded model obtained from applying the three budget-increasing trends are shown in Table 9. For scheduling decisions, the solutions remain asymmetrical and more conservative in the outbound trip. The rationale behind this lies in the different weights of the uncertainty budgets at different ports. For instance,  $\zeta_{12}$  appears in all the punctuality constraints in the budget-bounded model so that it influences all the candidate ports, while  $\zeta_{78}$  apparently affects only the arrival punctualities at fewer ports in the inbound trip. As described earlier, the uncertainty-related parameters are restrained to the uncertainty budget where it first appears, so that the budget of the ports that are visited earlier weighs more over the entire shipping line.

It is also interesting to point out that, even with the highest uncertainty budget, the model with the accelerative budget trend yields the most optimistic results: The highest expected profit and the tightest schedule. Despite sharing a common expected profit value, the model with the decelerative budget trend yields a much more conservative solution than the stable budget. These results can be explained with the finding about the weights of uncertainty budgets. Notice that before the uncertainty budget at port 4 for the inbound sailing, the accelerative budget trend maintains the lowest among the three trends, so the solution that it provides turns out the most optimistic in both the schedule and the expected profit. Moreover, a higher budget at the earlier ports visited leads to the most conservative schedule as well as the expected profit for the decelerative budget.

However, it is important to distinguish the expected profit from the solutions and the possible actual profit due to the influence of uncertain factors. The analysis above does not suggest using the accelerative budget trend. As the aim of this study lies in improving the robustness of the solutions, the most robust schedule recommended is the decelerative schedule, for which the most buffer time is preserved.

It is also important to point out that this developed robust function is merely one but not the only reference for making liner shipping routing and scheduling decisions. As described in developing the robust counterpart with budget-bounded uncertainty, one should always make a comprehensive and prudent evaluation on the scale of risk that he or she can take, and leave the remaining part to be protected.

## 5. Concluding remarks

In this paper, a new ship routing and scheduling problem for liner shipping services under uncertain weather and ocean conditions is defined and studied, taking into account late arrival penalty and shipping demand elasticity. A deterministic model is constructed, followed by its two robust counterparts with independently bounded uncertainty sets and budget-bounded uncertainty sets, respectively. The deterministic model and independently bounded model can be seen as special cases of the budget-bounded model when the uncertainty budget is set to null and full, respectively. The objectives of these models are set as maximizing the profit of operating a single liner ship for a round trip, which includes the service income, sailing and operations cost, and late arrival penalty. All these models merely contain a linear objective function and a set of linear constraints, and hence are of the appealing MILP form, which makes them readily solvable by many commercial software packages.

A case study is carried out in evaluating the solution performance of the three models under different uncertainty bounds and budgets in the context of NSR shipping lines. The solution conservativeness, symmetry and punctuality are examined and compared from different perspectives. The case study reveals some common features and different behaviors of the solutions: 1) The reason for which some ports are less favored in the routing solutions is due to their low revenue rate and high demand sensitivity; 2) the solutions of the deterministic model are symmetrical while the solutions of other models are mostly asymmetrical due to the gap between nominal sailing time and actual sailing time; 3) in terms of profit acquirement, the actual performance of the bounded model far exceeds that of the deterministic one; 4) since the uncertainty budget of an earlier visited port takes a higher weight over the entire shipping line, setting the budget in a decelerative manner will improve the robustness of solutions.

This study presents our initial effort to apply RO methods for modeling, solving and evaluating liner routing and scheduling problems with uncertain circumstance factors. Though we use a hypothetical liner shipping case along NSR as the problem context, the applicability of our modeling and solution methods may be extended to other decision-making cases under uncertain factors within liner shipping networks. Further research may be extended along a few different directions, including, but not limited to: 1) More

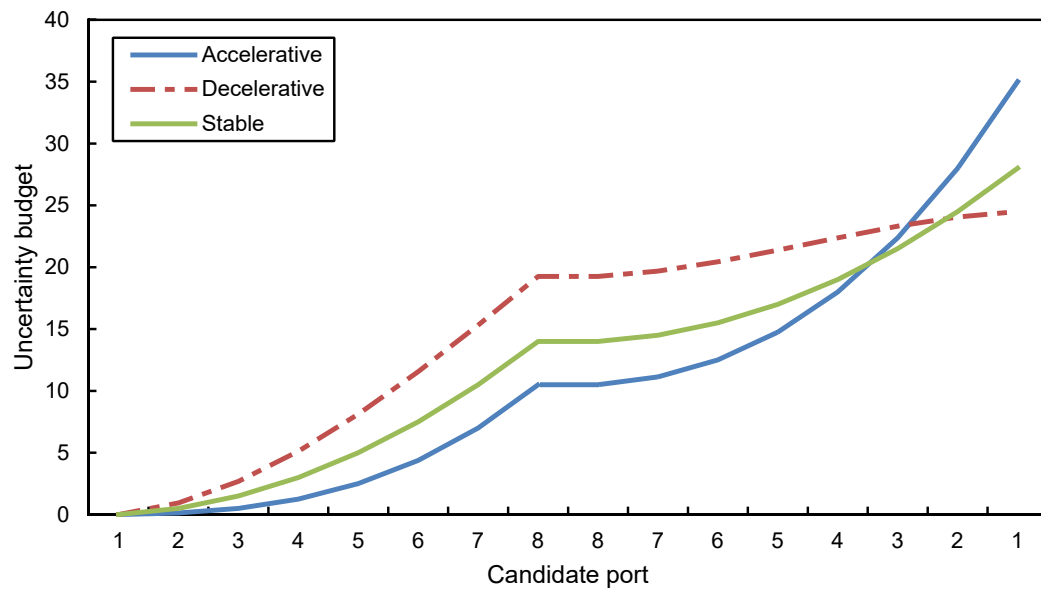


Fig. 2. Uncertainty budgets with different increasing trends.

Table 9

Solutions of the budget-bounded model from uncertainty budgets with different increasing trends.

Increasing trend of uncertainty budgets	Decelerative	Stable	Accelerative
Objective function value (\$)	13,975,300	14,119,800	14,358,400
Chosen ports	Claimed arrival date (day)		
1	0	0	0
3	17	17	15
5	25	25	23
6	29	29	27
7	48	47	45
8	54	53	51
7	60	59	57
6	77	60	58
5	81	64	62
3	87	70	68
1	99	82	80

complex liner ship routing and scheduling problems accommodating other types of supply and demand uncertainties; 2) more specific cost components and their dependent factors should be incorporated into the model instead of presenting the two rough cost components, namely, fixed sailing cost and variable sailing cost only; 3) more realistic liner ship routing and scheduling problems involving fleet management, frequency determination, alternative network structures, heterogeneous shipping demand, and multi-carrier competition or alliance; 4) applications of other stochastic optimization techniques such as stochastic programming and chance-constrained programming as well as the corresponding algorithm development and implementations, if more information about uncertainties is available.

### Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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