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Shared parking for ride-sourcing platforms to reduce cruising traffic

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ABSTRACT

Ride-sourcing platforms create cruising traffic that exacerbates road congestion in large cities. This paper considers shared parking services for ride-sourcing vehicles to reduce the cruising traffic. We propose a novel business model that integrates shared parking services into the ride-sourcing platform. In the proposed business model, the platform (a) provides app-based interfaces to connect passengers, drivers and garage operators (private or commercial garages); (b) matches passengers to ride-sourcing vehicles for ride services; (c) matches idle vehicles to vacant parking spaces to reduce cruising traffic; and (d) determines the ride fare, driver payment, and parking rates to maximize its profit. We model the integrated platform as a three-sided market (i.e., passenger, driver and garage) that involves two matching processes: passenger-driver matching and driver-garage matching. An economic equilibrium model is proposed to characterize the incentives of passengers, drivers, garage operators, and the ride-sourcing platform. Optimal pricing strategies and the corresponding market outcomes are derived by solving the platform's profit maximization problem. Based on the model, we show that integrated parking and ride-sourcing service can offer a Pareto improvement: in certain regime, it leads to higher passenger surplus, higher driver surplus, higher garage surplus, higher platform profit, and reduced cruising traffic. Further, we investigate the impacts of matching efficiency between drivers and garages on the proposed business model. We show that a higher matching efficiency always leads to increased platform profit and driver surplus, but it may not necessarily benefit passengers or garage operators in all cases. The above insights are validated through realistic numerical studies for San Francisco.

1. Introduction

The advances in mobile sensing technology and the popularization of smartphones have largely promoted the widespread success of ride-sourcing platforms. These platforms, such as Uber, Lyft and Didi, are also referred to as transportation network companies (TNCs). Unlike traditional taxis, they match willing passengers and idle drivers in real-time to provide on-demand mobility services through the mobile app. Due to the cost-effectiveness and convenience of their services, TNCs experienced an explosive growth in large cities, which fundamentally transformed the landscape of urban transportation. Uber announced that its ride-sourcing business has expanded to over 900 cities across the globe, in 93 countries. It has 5 million registered drivers on the platform globally, serving 18.7 million trips per day on average in the first quarter of 2020 (Iqbal, 2021). The expansion and proliferation of ride-sourcing businesses in metropolitan areas are particularly evident. The data collected by the New York City Taxi and Limousine Commission (NYCTLC) illustrated that there are about 85,000 TNC vehicles in New York affiliated with four major ride-sourcing platforms Uber, Lyft, Juno and Via. They totally made almost 720,000 ride-hailing trips per day in December 2019 (Schneider, 2021b). Their total number of average daily trips increased by 91% from 2017 to 2019. In Chicago, over 312,000 trips were undertaken by TNCs per

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day in February 2020, while about 38,000 taxi trips occurred each day on average (Schneider, 2021a). In San Francisco, 5700 ride-sourcing vehicles operate during peak hours. They daily complete over 170,000 TNC rides, approximately 12 times more than the number of taxi trips, 15% of all intra-San Francisco trips, and 9% of all San Francisco individual trips (Castiglione et al., 2016).

The rapid growth of ride-sourcing businesses has raised public concerns about its negative externalities on urban traffic conditions. The popularity of ride-sourcing services thanks notably to a very short passenger waiting time, which requires a substantial number of available but idle ride-sourcing vehicles (hereafter referred to as cruising vehicles/traffic). These idle vehicles cruise on the streets to seek the next passenger, incurring additional fuel cost and empty vehicle kilometers traveled (VKT). This is particularly harmful to the city as the cruising traffic occupies road spaces and contributes to traffic congestion without serving any travel demand. According to a detailed report (Schaller, 2017), from 2013 to 2017, the number of ride-sourcing vehicles in NYC increased by 59% and ride-sourcing trips increased by 15%. However, at the same time, the number of idle vehicles increased by 81%, ride-sourcing drivers spent more than 40% of their time empty and cruising for passengers, which contributes to an increase of 36% in VKT and a 15% reduction of traffic speed.

Responding to the significant negative effects of ride-sourcing businesses, relevant authorities have proposed various congestion-limiting policies to regulate TNCs. Among these regulatory policies, the imposition of a congestion charge and a cap on the number of TNC vehicles are the most prevalent. In February 2019, New York City Taxi and Limousine Commission (NYCTLC) imposed a $2.75 charge on all ride-sourcing trips that pass through a designated congestion area of NYC in Manhattan (NYCTLC, 2019). In April 2019, the New York State Assembly authorized the Metropolitan Transportation Authority (MTA) to charge a cordon tax between $10 and $15 on vehicles entering south of 61st Street in Manhattan by 2021, by which MTA can collect $3.5B per year to upgrade its aging transit infrastructure. In November 2019 Chicago announced a congestion charge on ride-sourcing trips with single passenger on weekdays in the downtown area (Pierog, 2019). The raised congestion tax will be used to subsidize public transit and taxi drivers. Almost at the same time, San Francisco approved a Traffic Congestion Mitigation excise tax (effective January 1, 2020) of 1.5% to 3.25% on fares for rides originating in San Francisco that are facilitated by commercial ride-share companies or are provided by an autonomous vehicle or private transit services vehicle (Treasurer & Tax Collector, 2020). Compared to the congestion surcharge, imposing a cap on the number of TNC vehicles mitigates traffic congestion by directly reducing the ride-sourcing traffic. In 2018, the New York City Council passed a regulation freezing the number of TNC vehicles on the road for one year. Further in 2019, the NYCTLC required ride-sourcing platforms to cap their vehicle cruising time below 31% out of the overall operating time in Manhattan core at peak hours (Taxi and Limousine Commission, 2019). The city introduced the cruising cap as part of a range of regulations to promote TNCs to better utilize their drivers' resources and reduce the cruising traffic. The new policy was set to come into effect in February 2020. However, it was repealed by the Supreme Court of State of New York (CNBC, 2019), claiming that the city's cruising cap was "arbitrary and capricious".

This paper proposes an alternative approach to address the cruising traffic of TNCs. We submit that instead of cruising on the street, idle TNC vehicles should park in off-street parking spaces. This can save the fuel cost for drivers, reduce the cruising traffic on streets, and alleviate the traffic congestion in cities. Several previous works have investigated the parking provision for TNC vehicles. For instance, Xu et al. (2017) investigated the allocation of road spaces for on-street parking of vacant ride-sourcing vehicles to reduce cruising congestion. The optimal parking provision strategy was proposed to address the trade-off between reduced cruising traffic and reduced road spaces. Kondor et al. (2018) estimated the minimum parking spaces required for the autonomous mobility-on-demand systems and quantified the relationship between parking provision and traffic congestion. However, we emphasize that cruising traffic often concentrates in densely populated areas where the land space is severely limited. In these areas, providing extra spaces for TNC parking is a daunting task, which has not been well addressed before. To mitigate this challenge, *we seek strategies that exploit the underutilized parking infrastructures instead of creating a need for new parking resources.* In particular, we consider shared parking for TNC vehicles, which is enabled by a platform that connects idle parking spaces to cruising TNC vehicles for short-term parking services. By unlocking the potential of these underutilized parking spaces, our proposition can significantly boost the urban parking supply without requiring extra road spaces or building additional parking garages. This can reduce the cruising cost for TNC drivers, increase the revenue for TNC platforms, provide extra profit-earning opportunities for garage owners, and curb the cruising traffic without incurring negative externality on other transport modes. To fully realize these benefits, the following questions should be carefully investigated: (1) how to operate the integrated parking and ride-sourcing platform to extract the maximum benefit? (2) How does the proposed business model affect the TNC market?

This paper presents a comprehensive economic analysis to address the aforementioned questions. We propose a novel business model that integrates shared parking into the ride-sourcing platform. The integrated platform (a) designs a smartphone app and offers user interfaces to both passengers, drivers, and garage operators (private or commercial garages); (b) implements online dispatching algorithms to execute two matching processes for the market participants, namely, the matching between ride-sourcing vehicles and passengers requesting for ride services and the matching between cruising vehicles and shared parking garages; and (c) determines the ride fare, driver payment and parking rates to attract both passengers, drivers, and garage operators. Distinct from ordinary ride-sourcing platforms, our business model brings garage operators into the ride-sourcing market and capitalizes the underutilized parking capacity to reduce cruising traffic. This brings a Pareto improvement that benefits TNC passengers, TNC drivers, ride-sourcing platforms, garage operators, and all other road users. To quantify these benefit, we propose an economic equilibrium model that captures the interactions among passengers, TNC drivers, garage operators, and the ride-sourcing platform. The profit-maximizing pricing strategy of the platform and corresponding market outcomes are derived and analyzed under the proposed business model. The key contributions of this paper are summarized below:

- We develop an economic equilibrium model that captures the incentives of passengers, drivers and garage operators at the aggregate level and characterizes the involved two matching processes, i.e., the matching of passengers and vehicles and the matching of vehicles and garages. The optimal platform pricing and corresponding market outcomes on passenger demand, driver supply, parking provision, passenger waiting time, driver waiting time, cruising traffic, platform profit, etc., are derived by solving the platform's profit maximization problem.
- We quantify the effect of shared parking on the ride-sourcing market through numerical experiments based on reported San Francisco TNC data. The results show that the proposed business model has a dual effect on boosting the ride-sourcing economy and alleviating cruising traffic. Compared to the ordinary ride-sourcing business without parking service, by recruiting approximately 1% of the overall parking spaces in San Francisco for shared parking of TNC vehicles, the number of cruising vehicles decreases by 82.55%, passenger travel cost decreases by 1.28%, passenger demand increases by 5.27%, driver net wage increases by 3.33%, the total number of drivers increases by 7.34%, and the platform profit increases by 22.17%. Such effect could be more significant under more efficient matching of vehicles and garages.
- We investigate the impacts of TNC-parking integration on the ride-sourcing market by both numerical experiments and theoretical analysis. We find that shared-parking has distinct impacts on different stakeholders of the TNC market: (a) drivers always benefit from the TNC-parking integration; (b) passengers may either benefit or get hurt by the parking services depending on the matching efficiency between vehicles and parking garages, and (c) the platform always benefits from introducing shared parking services for drivers. In certain regime, a Pareto improvement can be achieved where the proposed business model yields higher passenger surplus, higher driver surplus, higher platform profit, and reduced cruising traffic.
- We examine the impacts of the vehicle-garage matching efficiency on the ride-sourcing market under TNC-parking integration. The results indicate that increasing matching efficiency always yields higher driver surplus and brings an increased profit for the platform. However, the impact on the passengers and garage operators exhibits a counter-intuitive pattern: as the vehicle-garage matching efficiency improves, the passenger surplus first decreases and then increases, while the optimal parking supply initially increases and then decreases. This indicates that passengers and garage operators may not always benefit from improved matching efficiency. The reasons for these counter-intuitive results are discussed in Section 5.3.

2. Related works

Ride-sourcing market has been extensively studied by various research communities. It is typically formulated as a two-sided market in which passengers (demand side) request mobility services via the ride-hailing platform and drivers (supply side) provide ride services to meet passengers' travel needs. The platform receives a ride fare from passengers and makes a payment to drivers for providing ride services, by which a commission is charged to make a profit. He and Shen (2015) proposed a network equilibrium model to understand the characteristics of the smartphone-based e-hailing taxi business and evaluated its impact on taxis. Zha et al. (2016) developed an aggregate model to provide a comprehensive economic analysis of ride-sourcing markets, in which regulatory interventions and platform competition are investigated under distinct market scenarios. Zha et al. (2017) proposed equilibrium models to investigate the impacts of surge pricing under different behavioral assumptions of labor supply in a ride-sourcing market. The performance of surge pricing on platform profit, driver revenue, and passenger surplus were quantitatively evaluated based on the established equilibrium models. Bai et al. (2019) presented an analytical model to study the market equilibrium outcomes under the profit maximization of the on-demand service platform. Fixed pricing scheme and time-varying pricing scheme were examined and compared by extensive numerical experiments based on real TNC data. Overall, most of the existing literature constructs equilibrium models to understand the structure and intrinsic properties of the ride-sourcing market. Essentially, the success of ride-sourcing business is largely attributed to the advanced matching algorithms and order dispatching techniques implemented at the ride-hailing platform. In the aggregate modeling and equilibrium analysis of ride-sourcing market, a prominent approach to characterize the matching process is through aggregate matching functions. The Cobb–Douglas matching function is usually introduced to capture the matching frictions between passengers and drivers (He and Shen, 2015; Zha et al., 2016; Nourinejad and Ramezani, 2020), by which the passenger waiting time can be explicitly formulated. Another strand of studies consider the matching of passengers and drivers as a queuing process to capture the stochastic dynamics of the ride-sourcing system (Bai et al., 2019; Li et al., 2019b; Banerjee et al., 2015). These studies lay a solid theoretical foundation for the construction of the market equilibrium model in this paper. Overall, extensive studies have focused on the field of ride-sourcing, covering spatial–temporal demand forecasting (Ke et al., 2017, 2018, 2021), driver behavior analysis (Xu et al., 2019; Gurvich et al., 2019; Sun et al., 2019), demand–supply matching and order dispatching (Xu et al., 2018; Li et al., 2019a; Ke et al., 2019b), impact analysis of ride-sourcing business (Babar and Burtch, 2017; Yu et al., 2017; Beojone and Geroliminis, 2021) and ride-sourcing market regulating (Li et al., 2019b, 2021), etc. A detailed review of the ride-sourcing system is available in Wang and Yang (2019).

There is a growing body of literature that illustrates the negative externalities of ride-sourcing businesses on traffic condition and investigates the regulatory interventions to mitigate the congestion caused by ride-sourcing platforms. Qian et al. (2020) quantified the impacts of the growth of ride-sourcing businesses on traffic congestion and emission based on the large-scale trajectory data from major TNCs in NYC. The results show that from 2017 to 2019, the number of TNC vehicles in NYC increased by 48%, the average speed in Manhattan decreased from 11.76 km/h to 9.56 km/h, and the average citywide speed on weekdays reduced by 22.5%. Erhardt et al. (2019) analyzed data from two major TNCs and found that ride-sourcing businesses largely contribute to the increasing traffic congestion in San Francisco. From 2010 to 2016, vehicle hours of delay on weekdays increased by 62% compared to 22% in a virtual simulation scenario without TNCs. Based on the survey results on Uber use by residents of Santiago, Chile, Rayle et al. (2016) developed a parametric model to determine the relationship between ride-sourcing business and vehicle kilometers traveled (VKT). The simulation results show that the ride-sourcing business has an impact on increasing VKT. Diao et al. (2021) systematically quantified the impacts of TNCs on urban mobility in the United States from different perspectives, i.e., road congestion, transit ridership and private vehicle ownership. The results showed that the ride-sourcing market accounted for increased road congestion (0.9% and 4.5% increases in density and duration, respectively) and declined transit ridership (8.9%). Ke et al. (2020) developed market equilibrium models combined with the macroscopic fundamental diagram (MFD) to investigate the congestion effects of the ride-sourcing business. Beojone and Geroliminis (2021) conducted a citywide agent-based simulation with a trip-based MFD model to experimentally analyze the impacts of ride-sourcing services on traffic congestion. Responding to the increased congestion caused by TNCs, relative transportation departments and authorities began to regulate the ride-sourcing market with congestion-limiting policies in recent years. There are several studies investigating the impacts of regulatory policies on the ride-sourcing market. Vignon et al. (2020) investigated the effectiveness of various regulations on regulating the ride-sourcing market considering congestion externality and product differentiation. Li et al. (2019b, 2021) established economic models to evaluate and compare the impacts of various regulations and congestion-limiting policies on the ride-sourcing market, including the driver minimum wage, the cap on the number of vehicles, the trip-based congestion charge and time-based congestion charge. Results indicate that time-based congestion charge is superior to the trip-based congestion charge since it penalizes idle vehicle time. However, a consensus is achieved in these studies that congestion-limiting policies imposed on TNCs improve traffic congestion problems at a cost of curbing the ride-sourcing business.

Several attempts have also been made to address the cruising traffic arising from ride-sourcing businesses via parking. The central idea is to park idle TNC vehicles when there is no passenger. Xu et al. (2017) investigated the allocation of road spaces to on-street parking for vacant ride-sourcing vehicles. The optimal parking provision was determined to balance the trade-off between road capacity loss and the reduction of cruising traffic. Kondor et al. (2018) estimated the minimum number of parking spaces for the autonomous mobility-on-demand services serving all trips made in private vehicles via a large-scale mobility simulation in Singapore. Ruch et al. (2019) studied the fleet operation of a mobility-on-demand system in which idle and staying vehicles were assigned to a limited set of parking spaces. The spatial dimension of parking spaces was considered and different parking operating policies were evaluated from both operational cost and the impact on system service level. Beojone and Geroliminis (2021) developed parking management strategies for the ride-sharing system to prevent idle vehicles from cruising. The large-scale agent-based simulation results showed that the parking strategy can mitigate the negative impacts of ride-sourcing on congestion and improve the service quality. However, the parking strategies discussed in these studies either occupied limited road spaces or required exclusive parking resources to fulfill the parking demand of ride-sourcing vehicles, which may lead to conflicts with the regular parking demand of private vehicles and cause additional traffic congestion.

On the other hand, shared parking, as another business model in sharing economy, can help address the shortage of parking spaces in the city by making unused parking spaces publicly available for rent. Shared parking boosts urban parking supply by a more efficient usage of limited parking facilities without requiring building new parking lots, which offers an ideal solution to the provision of parking spaces for ride-sourcing platforms to reduce cruising traffic. Generally, the parking sharing business can be also modeled as a two-sided market involving three groups of decision-makers, i.e., the parking garage owners/operators, the drivers in need of parking spaces, and the parking sharing platform. In recent years, more and more studies have centered on the area of shared parking. Xu et al. (2016) first attempted to address the private parking lot sharing problem during regular working hours through mechanism design. The results demonstrated that the proposed mechanisms would result in remarkable social welfare gain and budget balance for the parking-sharing platform in cities with a large population. Shao et al. (2016) proposed a framework to coordinate the regular parking demand of residents and shared parking of public users. Two parking lots allocation schemes, namely, an optimization-based allocation and a first-book-first-serve allocation, are examined and the results showed that shared parking maximizes the use of private parking facilities and benefits the overall community. Zhang et al. (2020) developed a parking choice equilibrium model to characterize the shared parking in urban cities considering the spatial distribution of both curbside spaces and private parking spaces. The platform's pricing strategies were examined under the platform-profit-maximization and social-cost-minimization, respectively. In Shao (2018), an market equilibrium model was developed to investigate the operating mechanism of shared parking, in which an aggregate matching function was introduced to represent the demand–supply meeting friction. One can refer to Shao (2018) for a detailed investigation of the shared parking business.

Despite the increasing studies focusing on the modeling and analysis of shared parking services, there are few works that consider shared parking for TNCs. Jian et al. (2020) considered a carsharing operator that integrates parking-sharing services to provide bundled services to travelers in the context of an integrated sharing platform. The traveler choice equilibrium model was developed and the platform's optimal pricing and supply strategies were explored under profit maximization and social welfare maximization, respectively. However, the proposed equilibrium model is dedicated to the integration of carsharing and parking sharing service, which does not involve the intimate interactions among passengers, drivers, and the platform in the TNC market. Our previous work (Li et al., 2020) first proposed the idea of integrating the shared parking service into the ride-sourcing platform and developed a market equilibrium model to characterize the interactions among the involved stakeholders (i.e., passengers, drivers, garage operators and the platform), which provides a preliminary investigation of the novel business model of integrating ride-sourcing with shared parking. The numerical results indicate that shared parking always yields a Pareto improvement: at the profit-maximizing parking supply, both passengers, drivers and the platform benefit from the proposed business model. However, the developed equilibrium model did not capture the matching frictions between cruising vehicles and shared parking spaces, which is not realistic enough to reveal the underlying operating mechanism of the business model. Through a Cobb–Douglas matching function that characterizes the vehicle-garage matching process, this paper complements our preliminary work (Li et al., 2020) and offers a more comprehensive analysis of TNC-parking integration under distinct matching efficiencies. Taking into account the matching friction between TNC vehicles and parking garages, we show that share parking always benefits drivers and the platform while it may hurt passengers when the parking supply and matching efficiency is relatively low.

3. The proposed business model

The proposed business model integrates shared parking into the ride-sourcing platform. This leads to a three-sided market structure that covers both passengers, drivers and the garage operators. The integrated platform offers designated apps for each group to enable on-demand ride-hailing services or parking services in real time. Passengers can request mobility services through the user app from anywhere at any time. Drivers can flexibly adjust their work schedules by logging on/off their apps. Garage operators can offer or withdraw any number of parking spots at any time depending on the availability of vacant parking spaces. Under the proposed business model, each ride starts with a passenger requesting and submitting his/her ride service. The platform then dispatches a driver to pick up the passenger and deliver the passenger to his/her destination. After the passenger alights, the platform recommends a shared parking garage to the driver for[1] short-term parking (e.g., 5–8 min Taxi and Limousine Commission, 2019) until the next passenger arrives. Several transactions occur during this process: (a) each passenger pays a ride fare to the platform based on the trip distance and trip time; (b) each driver receives a payment from the platform based on the ride fare and the commission rate; and (c) each garage operator receives a parking fee from the platform based on the accumulated parking time of ride-sourcing vehicles at a per-minute parking rate. These fares and payments are determined by the integrated platform to maximize its profit. The system diagram is shown in Fig. 1. Several remarks are highlighted below:

- The parking service should be integrated into the ride-sourcing platform instead of any other third-party platforms. This is because to provide shared parking service to ride-sourcing vehicles, the platform needs to retrieve the real-time location of the vehicles and accumulate a large number of users on both demand side (ride-sourcing vehicles) and supply side (shared parking spaces). The ride-sourcing platform is in the unique position to perform such integration.
- In the proposed business model, TNC vehicles would not compete with the exogenous regular parking demand for parking spaces. This is because on the demand side, TNC parking is short. The waiting time of drivers for the next passenger request usually lasts for 5–8 min, thus each TNC vehicle only needs to occupy the shared parking spaces for several minutes. On the supply side, garage operators have the maximum level of flexibility. Through the app-based user interface, they can offer more parking spaces to the platform when the regular parking demand is low or withdraw the shared parking spots from the platform if the regular parking demand is high. These two factors together indicate that the garage operators can dynamically fill in the gaps between regular long-term parking demands (which usually last for several hours) using the short-term shared parking of TNC vehicles, which only takes the spaces that would have been unused if they are not reserved for TNC parking.
- There should be adequate unused parking spaces to support the proposed business model. This is possible when the spatial–temporal patterns of the ride-souring demand and the regular parking demand are complementary. For instance, the data from San Francisco County Transportation Authority shows that the peak of the TNC market is between 7 am–9 am and 6 pm−8 pm, with over half of ride-sourcing trips occurring in the northeast corner of San Francisco (Castiglione et al., 2016). However, during the peak hours of TNC market, the occupancy rates of parking garages in the northeast area of San Francisco range between 20%–60% with an average of 40% (SFpark, 2013). This indicates that the regular parking demands and the TNC market are temporally and spatially complementary.
- As TNC vehicles only utilize idle parking spaces, the marginal cost of the parking spaces offered to the ride-sourcing platform is minimum. This enables ride-sourcing vehicles to enjoy a heavy parking discount compared to other parking demands. The rates of the shared parking services are determined by the platform, while the rates for other regular parking demands are determined by the garage operator. These two parking services are operated in parallel. The garage operator determines the allocation of the parking spaces in each service to maximize his/her profit.[2]
- The proposed business model requires minimum hardware and software modification on the existing parking infrastructures. The TNC platform will develop an app-based interface for the garage operator. To reduce the inbound/outbound time in garages and improve the efficiency of shared parking, the platform will communicate with the garage operator to guarantee smooth entry and exit of TNC vehicles. Garage operators need to reserve easy-access parking spots for TNC vehicles to accommodate their short stay. Parking data should be submitted periodically through the interface for financial settlements.

4. Mathematical formulation

The platform determines the ride fare, driver payment, and the parking rate to maximize its profit. In response to the platform's pricing strategy, the decision making of passengers, drivers and garage operators interact with each other and constitute the market equilibrium. In this section, we present a mathematical model to investigate the optimal pricing strategy of the platform under these market equilibrium constraints. The incentives of passengers, drivers, and garage operators will be characterized. The matching processes among the three groups will be formulated.

[1] In practice, drivers may choose to relocate to a distinct location instead of following the platform's recommendation to park at a designated garage. This may lower the vehicle-garage matching efficiency. Such effects will be quantified via numerical studies in Section 5.3.

[2] We do not investigate the optimal allocation of parking spaces between TNC parking and regular parking. The model for the decisions of the garage operator is left for future work.

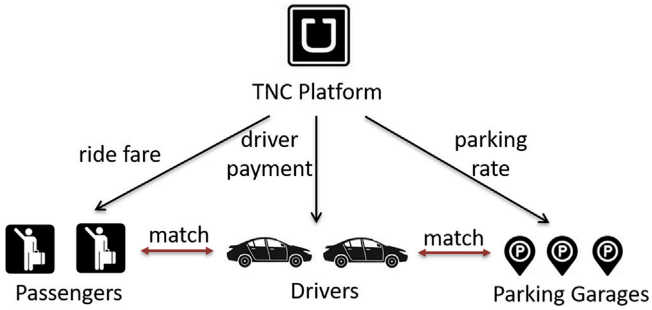


Fig. 1. The system diagram for the business model.

4.1. Passenger incentives

Passengers decide whether to take e-hailing services based on the total travel cost of the ride-sourcing trip. We model the total travel cost of a TNC trip as the weighted sum of *average* ride fare p_f and *average* waiting time w^c:

$$c = p_f + \alpha w^c, \tag{1}$$

where p_f is the average ride fare determined by the platform, w^c is the average passenger waiting time, and α specifies the trade-off between time and money. The total cost c is also referred to as the generalized cost. Due to the heterogeneous passenger travel time, travel distance, and perceptions of value of time, different riders may experience distinct costs. We assume that the average passenger arrival rate λ depends on the average generalized cost,[3] which leads to the following passenger demand function:

$$\lambda = \lambda_0 F_p(p_f + \alpha w^c), \tag{2}$$

where λ_0 is the arrival rate of potential passengers (total travel demand in the city); $F_p(\cdot) \in [0,1]$ determines the proportion of potential passengers choosing ride-sourcing services. We assume that $F_p(\cdot)$ is a strictly decreasing and continuously differentiable function, so that a higher travel cost leads to a lower arrival rate of passengers.

Remark 1. In practice, passenger trips demonstrate heterogeneous spatial–temporal patterns: trips may occur from distinct locations at different times. This paper neglects these aspects and presents an aggregate model that predicts the average market outcomes over space and time. We emphasize that this aggregate model is sufficient for our purpose since we focus on quantifying the overall benefits of TNC-parking integration and the long-term impacts of regulatory policies at the city level. The development of a temporal–spatial model is left for future work.

The passenger waiting time w_c is an endogenous variable representing the quality of ride-sourcing services. It arises from the matching/meeting process between ride-sourcing vehicles and passengers and consequently depends on both passenger demand and driver supply. Typically, the passenger waiting time consists of the ride confirmation time (from the ride request being submitted to a vehicle being assigned) and the pickup time (from a vehicle being assigned to passenger pickup). Since the ride confirmation time (usually several seconds) is negligible compared to the pickup time (between three to six minutes), we ignore the ride confirmation time and approximate the passenger waiting time as the passenger pickup time. Assuming that the platform matches the passenger to the nearest idle vehicle, the pickup time depends on the distance of the nearest idle vehicle to the passenger. In this case, the average passenger waiting time w^c can be characterized as a monotone function of the average number of idle vehicles N^I. We denote the passenger waiting time function as $w^c = F_c(N^I)$ and impose the following assumption:

Assumption 1. $F_c(N^I)$ is positive, strictly decreasing with respect to N^I, and $\lim_{N^I \to 0} F_c(N^I) = \infty$.

We further assume that the passenger waiting time follows the "square root law", which was well-established and widely applied in street-hailing taxi market (Douglas, 1972), radio dispatching taxi market (Arnott, 1996), and online ride-hailing market (Li et al., 2019b), among others. This leads to the following passenger waiting time function:

$$w^c = F_c(N^I) = \frac{A}{\sqrt{N^I}}, \tag{3}$$

where A is the scaling parameter capturing possible factors in the matching of idle vehicles and passengers, such as the size of the city, the average traffic speed on the road network, and the demand/supply distribution, etc. Eq. (3) indicates that passengers' average waiting time is inversely proportional to the square root of the number of idle vehicles. The intuition is that if both waiting

[3] This assumption indicates that the distribution of generalized cost does not matter in the determination of average passenger arrival rate. It is consistent with the well-established logit model.

passengers and idle vehicles are uniformly and independently distributed across the city, the distance between a passenger and her closest idle vehicle is inversely proportional to the square root of the total number of idle vehicles, which further determines the passenger waiting time. A detailed justification of the square root law can be found in Li et al. (2019b).

Remark 2. We assume that idle vehicles are uniformly distributed when deriving (3). However, when idle vehicles are parked, their spatial distribution is determined by the locations of parking garages. We argue that in practice, parking garages are usually highly distributed across the city, which can be reasonably approximated by the uniform distribution. However, we conjecture that even if they are not uniformly distributed, the square-root law (3) should still hold with a different value of A. We leave it as future work to validate this conjecture.

Remark 3. While traffic speed affects the scaling parameter A and trip completion time μ, this paper does not model traffic speed as an endogenous variable that depends on the number of ride-sourcing vehicles. We argue that this simplification is acceptable as it only leads to a conservative estimate of the benefits of TNC-parking integration in curbing the cruising traffic. This is because if congestion is considered, the benefit of TNC-parking integration would be even larger as reducing cruising traffic plays a positive role on mitigating traffic congestion, making ride-sourcing vehicles operate more efficiently on roads.

4.2. Driver incentives

Drivers decide whether to join the ride-sourcing platform based on the net hourly wage offered by the platform. The net hourly wage is the gross income minus the vehicle expenses. The gross income is the hourly payment drivers receive from the platform, which is further determined as the per-trip driver payment offered by the platform times the average number of served trips per hour. The vehicle expense comprises the fixed costs (e.g., vehicle registration and insurance) and the time-based operating cost (e.g., fuel cost and working time). The fixed costs are diminished over the long term and are irrelevant to the short-term decision-making of drivers in the ride-sourcing market. Thus, we neglect the fixed costs and only consider an average hourly operating cost of ride-sourcing vehicles.

In the ordinary ride-sourcing market, ride-sourcing vehicles receives p_d per trip from the platform and they constantly travel on the road at an average hourly cost of l. Therefore the average driver net hourly wage can be derived as $\frac{\lambda p_d - Nl}{N}$, where λp_d is the total driver payment offered by the platform per hour, Nl is the total vehicle operating cost per hour, and the difference between them divided by the total number of drivers N gives the average net hourly wage. With shared parking, idle vehicles can either cruise on the street at the hourly cost of l or park in the shared parking garage subject to parking space availability. Suppose there are N^p parked vehicles at the market stationary state, then the average net hourly wage of drivers can be written as:

$$w_n = \frac{\lambda p_d - (N - N^p)l}{N}. \tag{4}$$

The driver net hourly wage w_n in (4) is still derived as the difference between the total driver payment and the total vehicle operating cost divided by the total number of drivers. The only difference is that due to shared parking services, the hourly vehicle operating cost reduces to $(N - N^p)l$ as there are only $N - N^p$ vehicles operating on the road at the stationary state. Assume that the total number of drivers N only depends on the *average* driver net wage w_n,[4] then we have the following driver supply function:

$$N = N_0 F_d\left(\frac{\lambda p_d - (N - N^p)l}{N}\right), \tag{5}$$

where N_0 is the number of potential drivers (all drivers seeking a job); $F_d(\cdot) \in [0,1]$ decides the proportion of potential drivers joining the platform. We assume that $F_d(\cdot)$ is a strictly increasing and continuously differentiable function with respect to w_n, so that a higher net hourly wage attracts more drivers to join the ride-sourcing platform. In (5), the number of parked vehicles N^p affects driver net wage level and consequently determines the driver supply of the ride-sourcing market integrated with shared parking. In the meanwhile, N^p is an endogenous variable determined by both the ride-sourcing parking demand and shared parking supply. To capture this, we will introduce the Cobb–Douglas matching function that characterizes the matching process between vehicles and garages in Section 4.4.

Remark 4. We do not consider the exogenous inbound/outbound time in garages and assume it is negligible compared to the actual parking time. In practice, the impact of the inbound and outbound time can be mitigated through various approaches. For instance, the garage operators can cooperate with the platform to enable online payment, so that drivers can enter or exit the garage without stopping, which largely reduces the inbound/outbound time of TNC vehicles. In addition, the garage operator can also reserve some convenient spots (i.e., close to garage gate) for TNC vehicles, if available, to further reduce the inbound/outbound time. This will introduce little disruption to the regular parking demand because most of the garage users park for a long period of time and spending a few more minutes to find a parking space within the garage only represents a negligible cost for them.

[4] The wage for distinct drivers can be different and this captures logit model as the special case.

4.3. Garage operator incentives

Garage operators decide whether to offer their unoccupied parking spots to the ride-sourcing platform based on the real-time inventory of their parking spaces. In real-time, the garage operator can strategically adjust their offering strategies to serve ride-sourcing vehicles without negatively affecting the regular parking demand of other private vehicles: they can offer more parking spaces to the platform when regular parking demand is off-peak, and withdraw the shared parking spaces from the platform when the regular parking demand is high. Since each ride-sourcing vehicle only parks for several minutes, this allows the garage operators to dynamically adjust the number of shared parking spaces offered to the integrated platform in accordance with the fluctuating regular parking demand. In this case, we model the decision-making of garage operators as an aggregate parking supply function that relates the total number of shared parking spaces K to the average hourly parking revenue e_g. Garage operators receive an hourly[5] parking rate of p_g per spot from the platform if it is occupied by ride-sourcing vehicles, therefore the average hourly parking revenue e_g can be written as:

$$e_g = \frac{N^p p_g}{K}. \tag{6}$$

The average hourly parking revenue e_g in (6) is derived as follows. The total parking revenue that garage operators receive from the platform sums to $N^p p_g$ \$/hour, thus the average hourly parking revenue per parking space is $N^p p_g$ divided by the total number of shared parking spaces K. The total number of shared parking spaces K in turn depends on the average hourly parking revenue e_g, which leads to the following parking supply function:

$$K = K_0 F_g\left(\frac{N^p p_g}{K}\right), \tag{7}$$

where K_0 is the average potential vacant parking spaces (unoccupied parking spaces in the city after satisfying the regular parking demand[6]); $F_g(\cdot) \in [0,1]$ captures the proportion of shared parking spaces over all potential unoccupied parking spaces. We assume that $F_g(\cdot)$ is a strictly increasing and continuously differentiable function so that a higher parking revenue activates a higher parking supply.

4.4. Matching vehicles with garages

Between rides, some idle vehicles cruise on the street, while other idle vehicles are matched to vacant parking spaces (if available). At the stationary state, the matching rate of ride-sourcing vehicles with shared parking garages (m^{p-t}) depends on the number of cruising vehicles (N^c) and the number of vacant parking spaces (K^v). This matching process can be captured by a Cobb–Douglas matching function, which is written as:

$$m^{p-t} = M^{p-t}(N^c, K^v) = B(N^c)^{\beta_1}(K^v)^{\beta_2}, \tag{8}$$

where B is the scaling parameter, and β_1 and β_2 are the elasticity parameters within the range of $[0,1]$, reflecting the efficiency of vehicle-garage matching.[7] The matching function (8) that is homogeneous of degree $\beta_1+\beta_2$ is said to exhibit increasing returns to scale if $\beta_1+\beta_2 > 1$, constant returns to scale if $\beta_1+\beta_2 = 1$, and decreasing returns to scale if $\beta_1+\beta_2 < 1$ (Yang and Yang, 2011). Since there is no data to validate whether vehicle-parking matching exhibits increasing, constant, or decreasing returns to scale, we will perturb the parameters of (8) to investigate how they affect the proposed business model. Note that the variables N^c, K^v, and m^{p-t} depend on other endogenous variables. Below we derive these relations separately.

To derive N^c, we first note that in ride-sourcing services, vehicles/drivers experience the following three statuses sequentially and recurrently: (1) status 1: being idle and waiting for passengers; (2) status 2: being assigned with a passenger and picking up the passenger; (3) status 3: being occupied with passengers and delivering passengers. Let μ denote the average duration of ride-sourcing trips. At the stationary state, the conservation of the total vehicle operating hour yields:

$$N = N^I + \lambda w^c + \lambda\mu, \tag{9}$$

where the first term corresponds to the average number of idle drivers (status 1), and the second and third terms are derived based on Little's law, which represent the average number of vehicles on the way to pick up the passenger (status 2), and the average number of vehicles occupied with some passenger (status 3), respectively. Let w^d denote the average driver waiting time between rides, based on Little's law, the number of idle drivers N^I relates to w^d through the following equation:

$$N^I = \lambda w^d. \tag{10}$$

[5] Since each TNC vehicle only parks for several minutes, we assume that the parking rate is imposed on a per-minute basis and the TNC platform pays a prorated rate if the parking spot is occupied for less than one hour.

[6] Ideally, K_0 should come from the optimal allocation of parking spaces between TNC parking and regular parking. For simplicity, this paper does not explicitly model this decision process and regards K_0 as exogenous.

[7] The model parameters of B, β_1, β_2 depend on the matching algorithms of the platform and whether drivers follow the platform's parking recommendation.

We further note that idle drivers in status 1 are either being parked at garages or cruising on the street. If we let N^p and N^c denote the average number of parked and cruising idle vehicles, respectively, then the conservation of the total number of idle vehicles yields:

$$N^I = N^p + N^c. \tag{11}$$

Plugging (10) into (11), the number of cruising vehicles N^c can be derived as:

$$N^c = N^I - N^p = \lambda w^d - N^p. \tag{12}$$

To derive K^v, we note that the total number of parking spaces can be divided into occupied spaces and idle spaces. This leads to the following relation:

$$K^v = K - N^p. \tag{13}$$

To derive m^{p-t}, we denote T_p as the average duration of stay for each parked TNC vehicle (i.e., the average time elapsed after the vehicle finds a parking space and before it is matched to the next passenger). At the stationary state, the arrival rate of parked vehicles is m^{p-t}, and based on Little's law, the number of parked vehicles N^p should equal $m^{p-t}T_p$, thus we have:

$$m^{p-t} = \frac{N^p}{T_p}. \tag{14}$$

We further introduce the following assumption to characterize T_p:

Assumption 2. The status of idle TNC vehicles (either cruising or parked) will not affect their probability of getting matched to the next passenger.

This assumption implies that the vehicle-passenger matching process does not depend on whether the vehicle is parked or cruising. The status of idle TNC vehicles, no matter being cruising or parked, will not affect their probability of being matched with passengers. This is a mild assumption because ride-sourcing platforms typically matches the passenger with the nearest idle vehicle. In this case, the matching probability for TNC vehicles is independent of its status if parking in the garage or not does not change their spatial distribution.[8] Given the above assumption, the waiting time for each vehicle is independent of its status or its history: for a vehicle that just dropped off the last passenger, and a vehicle that just successfully found a parking space, their probability of find the next ride is exactly the same, therefore both of them will wait for an average time of w^d before getting matched to the next passenger, thus we have:

$$T_p = w^d. \tag{15}$$

Substituting (15) into (14) leads to:

$$m^{p-t} = \frac{N^p}{w^d}. \tag{16}$$

Finally, plugging (12), (13) and (16) into the matching function (8) gives the following relation:

$$\frac{N^p}{w^d} = B(\lambda w^d - N^p)^{\beta_1}(K - N^p)^{\beta_2}. \tag{17}$$

4.5. Profit maximization for the ride-sourcing platform

Consider a platform that provides ride-sourcing services to passengers and offers shared parking services to TNC drivers at the same time. In each hour, the platform receives λp_f ride fares from passengers, pays λp_d to drivers, and expend $N^p p_g$ to garage operators. The difference between the revenue and the cost is the average hourly platform profit. The platform jointly determines the ride fare p_f, the per-trip driver payment p_d, and the hourly parking rate p_g to maximize its profit subject to the market equilibrium conditions. This can be formulated as:

$$\max_{p_f, p_d, p_g} \quad \lambda p_f - \lambda p_d - N^p p_g \tag{18}$$

$$\begin{cases} \lambda = \lambda_0 F_p(p_f + \alpha w^c) & \text{(a)} \\ N = N_0 F_d\left(\dfrac{\lambda p_d - (N - N^p)l}{N}\right) & \text{(b)} \\ K = K_0 F_g\left(\dfrac{N^p p_g}{K}\right) & \text{(c)} \\ w^c = \dfrac{A}{\sqrt{\lambda w^d}} & \text{(d)} \\ N = \lambda(w^d + w^c + \mu) & \text{(e)} \\ \dfrac{N^p}{w^d} = B(\lambda w^d - N^p)^{\beta_1}(K - N^p)^{\beta_2} & \text{(f)} \end{cases} \tag{19}$$

[8] This is likely to be the case if the spatial distribution of garages and the spatial distribution of idle vehicles are consistent. However, when this condition does not hold, it requires a network model to characterize how their spatial distributions differ. This is outside the scope of this work.

where p_f, p_d and p_g are decision variables representing the pricing strategy of the platform; λ, N, K, N^p, w^c, and w^d are endogenous variables; α, μ, A, B, β_1, β_2, λ_0, N_0, K_0 and l are model parameters; (19)(d) and (19)(e) are derived by plugging (10) into (3) and (9), respectively. To justify the existence of the market equilibrium, we first introduce the following two lemmas:

Lemma 1. *Given $\lambda, w_d, K > 0$, there exists a unique $N^p \in \left(0, \min\{\lambda w^d, K\}\right)$ that satisfies (19)(f).*

The proof of Lemma 1 can be found in Appendix A. It states that within certain range, there exists a unique N^p that satisfies (19)(f) under given $\lambda > 0, w_d > 0$, and $K > 0$. Note that there may exist other N^p that satisfies (19)(f) but lies outside of this range, i.e., $N_p > \min\{\lambda w^d, K\}$. However, such solution is clearly infeasible as it indicates that either the number of cruising-for-parking vehicles (i.e., $\lambda w_d - N^p$) or the number of idle parking spaces (i.e., $K - N^p$) is negative.

Lemma 2. *Given $\lambda, N, K > 0$, (1) if $N = \left(\sqrt[3]{2} + \sqrt[3]{\frac{1}{4}}\right)(A\lambda)^{\frac{2}{3}} + \lambda\mu$, there exists an unique w^d satisfying (19)(d)–(19)(e); (2) if $N > \left(\sqrt[3]{2} + \sqrt[3]{\frac{1}{4}}\right)(A\lambda)^{\frac{2}{3}} + \lambda\mu$, there exists two strictly positive w^d satisfying (19)(d)–(19)(e). Furthermore, let w_1^d, w_2^d be the two solutions to (19)(d)–(19)(e) and denote P_1, P_2 as the corresponding platform profit, then $P_2 > P_1$ if $w_2^d > w_1^d$.*

The proof of Lemma 2 can be found in Appendix B. It suggests that given λ, N and K, the condition $N \geq \left(\sqrt[3]{2} + \sqrt[3]{\frac{1}{4}}\right)(A\lambda)^{\frac{2}{3}} + \lambda\mu$ guarantees the existence of strictly positive w^d and w^c satisfying (19)(d)–(19)(e). Furthermore, if there admits two solutions of w^d to (19)(d)–(19)(e), the case with the larger w^d and thus the smaller w^c yields higher platform profit. This means that the profit-maximizing platform will always choose to the larger driver waiting time w^d and the smaller passenger waiting time w^c if possible. In this case, w^d and w^c can be uniquely derived as functions of λ and N from (19)(d)–(19)(e), i.e., $w^d = w^d(\lambda, N)$ and $w^c = w^c(\lambda, N)$, if $N \geq \left(\sqrt[3]{2} + \sqrt[3]{\frac{1}{4}}\right)(A\lambda)^{\frac{2}{3}} + \lambda\mu$.

Based on the results of Lemma 2, we have the following results on the existence of the solution to (18):

Proposition 1. *If $F_p\left(\frac{\alpha A}{\sqrt{N_0}}\right) > 0$, there exists strictly positive p_f, p_d, p_g, λ, N, K, N^p, N^c, K^v, w^c and w^d that constitute a market equilibrium satisfying (19). Furthermore, N^p can be uniquely derived as a function of λ, N and K from the market equilibrium (19), i.e., $N^p = N^p(\lambda, N, K)$.*

The proof of Proposition 1 is deferred to Appendix C. The assumption $F_p\left(\frac{\alpha A}{\sqrt{N_0}}\right) > 0$ indicates that when the platform sets the ride fare p_f to 0 and recruits all potential drivers N_0, there will be a positive number of passengers, which guarantees the intersection of the driver supply curve and passenger demand curve. In addition, given λ, N and K, the average number of parked vehicles N^p can be uniquely determined as $N^p(\lambda, N, K)$ at the market equilibrium.

Note that (18) is a non-convex problem due to the nonlinear constraints. However, (18) is a small-scale problem with only three decision variables and several equality constraints. Equivalently, we can consider λ, N and K as decision variables, absorb all the constraints into the objective function and transform the profit maximization problem (18) into an unconstrained optimization:

Lemma 3. *If $F_p(\frac{\alpha}{A\sqrt{N_0}}) > 0$, (18) is equivalent to the following unconstrained optimization problem:*

$$\max_{\lambda,N,K} \quad \Pi(\lambda, N, K) = \lambda\left[F_p^{-1}\left(\frac{\lambda}{\lambda_0}\right) - \alpha w^c(\lambda, N)\right] - NF_d^{-1}\left(\frac{N}{N_0}\right) - Nl + N^p(\lambda, N, K)l - KF_g^{-1}\left(\frac{K}{K_0}\right) \tag{20}$$

The proof of Lemma 3 is omitted since it can be easily derived by plugging (34)(a), (34)(b) and (34)(c) into the objective function $\lambda p_f - \lambda p_d - N^p p_g$ in (18). It derives the platform profit $\Pi(\lambda, N, K)$ as a function of λ, N and K. After this transformation, we can apply a grid search (Bai et al., 2019) to maximize (20), which provides the globally optimal solution to (18).

Remark 5. Lemma 2 and Proposition 1 suggest that there are actually several implicit constraints on λ, N and K for the optimization (20) to guarantee the non-negativity of the endogenous variables, e.g., $N \geq \left(\sqrt[3]{2} + \sqrt[3]{\frac{1}{4}}\right)(A\lambda)^{\frac{2}{3}} + \lambda\mu$ for strictly positive w^d and w^c. However, we argue that these constraints are very weak conditions and the optimal solution to (20) always lies in the interior of the feasible region. This can be easily verified after we find the globally optimal solution by the grid search method. Therefore, we treat (20) as an unconstrained optimization and make it a standing assumption that (20) has an unique interior optimal solution.

5. Market outcomes under shared parking

This section investigates how the integrated business model affects the stakeholders in the ride-sourcing market. First, we present a case study to demonstrate the benefits of shared parking. Second, we complement the numerical study with a formal analysis on the derived insights. Finally, we validate the results in a sensitivity study with respect to B, β_1 and β_2.

5.1. Numerical example

Consider a case study for San Francisco, where an integrated platform coordinates passengers, drivers, and garage operators to jointly offer e-hailing and TNC parking services. In this case, passengers decide whether to take ride-sourcing services based on the total travel cost c. This can be captured by a logit model and consequently the passenger demand function is given by:

$$\lambda = \lambda_0 \frac{e^{-\epsilon c}}{e^{-\epsilon c} + e^{-\epsilon c_0}}, \tag{21}$$

where c is the average generalized travel cost of a ride-sourcing trip given by (1); $\epsilon > 0$ and c_0 are the model parameters. Similarly, drivers choose whether to join the ride-sourcing platform based on the net hourly wage w_n. Under a logit model, the driver supply function can be represented as:

$$N = N_0 \frac{e^{\eta w_n}}{e^{\eta w_n} + e^{\eta w_0}}, \tag{22}$$

where w_n is the average net hourly wage as defined in (5); $\eta > 0$ and w_0 are parameters. For the parking supply function, the cumulative distribution function of a log-normal distribution is introduced to characterize garage operators' willingness to offer unoccupied parking spaces to the ride-sourcing platform. In this case, the parking supply function can be written as:

$$K = K_0 \left[\frac{1}{2} + \frac{1}{2} \mathrm{erf} \left(\frac{\ln(e_g) - u_0}{\sqrt{2}\sigma} \right) \right], \tag{23}$$

where $e_g = \frac{N^p p_g}{K}$ is the average hourly earning of shared parking spaces as defined in (7); $\mathrm{erf}(\cdot)$ is the error function; u_0 and σ are the model parameters. At the equilibrium state, (19)(f) relates the number of parked vehicles N^p to the average driver waiting time w^d, passenger arrival rate λ and the total number of parking spaces K. According to Lemma 3, given λ, N and K, the number of parked vehicles $N^p(\lambda, N, K)$ at market equilibrium can be uniquely determined by solving the polynomial Eq. (19)(f).

Overall, the involved model parameters are:

$$\Theta = \{\lambda_0, N_0, K_0, A, B, \beta_1, \beta_2, \mu, \alpha, l, \epsilon, c_0, \eta, w_0, u_0, \sigma\}. \tag{24}$$

In this numerical example, we set $B = 1\mathrm{e}{-3}$ and $\beta_1 = \beta_2 = 1$ as the nominal value. This corresponds to a matching function that exhibits increasing returns to scale.[9] In this case, the polynomial Eq. (19)(f) is specified to the following quadratic equation with respect to N^p:

$$\frac{N^p}{w^d} = B(\lambda w^d - N^p)(K - N^p). \tag{25}$$

Due to lack of real data, we calibrate μ_0 and σ so that the outcomes produced by the parking supply model lies in a reasonable range. Note that the parking rates of off-street garages in San Francisco range between 2\$/hour–7\$/hour (San Francisco Municipal Transportation Agency, 2021). We set $\mu_0 = 1.1$ and $\sigma = 0.6$ such that the hourly parking revenue under the calibrated parking supply model is comparable but smaller than the hourly rate of regular parking. For the remaining model parameters, we set their values so that the optimal solution to the following profit-maximizing problem without parking services fits the reported San Francisco TNC data:

$$\max_{p_f, p_d} \quad \lambda p_f - \lambda p_d \tag{26}$$

$$\begin{cases} \lambda = \lambda_0 F_p \left(p_f + \alpha w^c \right) & \text{(a)} \\ N = N_0 F_d \left(\dfrac{\lambda p_d - N l}{N} \right) & \text{(b)} \\ w^c = \dfrac{A}{\sqrt{N - \lambda\mu - \lambda w^c}} & \text{(c)} \end{cases} \tag{27}$$

The values of model parameters for the case study of San Francisco are summarized below:

$$\lambda_0 = 944/\mathrm{min}, \quad N_0 = 10{,}000, \quad K_0 = 10{,}000, \quad A = 150, \quad B = 1\mathrm{e}{-3}, \quad \beta_{1,2} = 1, \quad \mu = 11.14 \text{ min},$$
$$\alpha = 2.92, \quad l = \$8/\mathrm{h}, \quad \epsilon = 0.155, \quad c_0 = 16.86, \quad \eta = 0.144, \quad w_0 = 27.36, \quad u_0 = 1.1, \quad \sigma = 0.6.$$

A detailed calibration of the model parameters (aside from μ_0 and σ) can be found in Appendix A of Li et al. (2021).

To evaluate how shared parking affects the ride-sourcing market, we calculate the maximum profit of the integrated platform under different parking supply K. In particular, we solve (18) in the following two steps: first we fix K and apply brute-force search to obtain the platform's optimal pricing and corresponding market outcomes under the specific parking supply K; then we vary K and enumerate all possible K to find the optimal parking supply that maximizes the platform profit. It should be noted that K is essentially an endogenous variable and the optimal parking supply is derived from the platform's profit-maximizing pricing strategy.

[9] Distinct values of B, β_1 and β_2 indicate different levels of matching efficiency between vehicles and garages. A sensitivity analysis on B, β_1 and β_2 is conducted in Section 5.3 to investigate the impacts of matching efficiency.

Figs. 2 and 3 show the platform pricing and the market outcomes under different parking supply. Fig. 2a shows the hourly parking rate and average parking earning of shared parking spots. Fig. 2b presents the changes in the number of parked vehicles and cruising vehicles. The parking ratio (i.e., ratio of parked vehicles out of idle vehicles, defined as N^p/N^I,) and garage occupancy (i.e., N^p/K) over different parking supply are shown in Figs. 2c and 2d. The total number of ride-sourcing vehicles and on-road vehicles are shown in Fig. 2e. The driver net hourly wage is shown in Fig. 2f. Fig. 2g shows the driver waiting time over different parking supply. The ride fare and the per-trip driver payment are shown in Fig. 2h. Fig. 2i presents the passenger waiting time under different parking supply. The passenger travel cost and passenger arrival rate under different parking supply are shown in Figs. 2j and 2k. Finally, the platform profit (left axis) and social welfare[10] (right axis) are shown in Fig. 2l, and the passengers' surplus, drivers' surplus and garage operators' surplus as a function of the parking supply are shown in Fig. 3(a-c), respectively.

Numerical results suggest that to maintain a higher level of parking supply, the platform needs to raise the parking rate to attract more garage operators and consequently garage operators receive a higher average parking earning (Fig. 2a). With the increase of parking supply, more vehicles park at garages and the cruising traffic drops significantly (Fig. 2b). The parking ratio is an increasing function of K (Fig. 2c), and the garage occupancy is a decreasing function of K (Fig. 2d). As more cruising vehicles are parked at shared parking garages, the number of on-road vehicles also decreases (Fig. 2e). In contrast, the total number of ride-sourcing vehicles always increases with the parking supply (Fig. 2e) and drivers have a higher net hourly wage under a higher parking supply (Fig. 2f), which indicates that a higher level of parking supply always yields a higher driver surplus. As for passengers, we find that although the platform will raise the ride fare as the parking supply increases (Fig. 2h), the passenger waiting time drops with the parking supply (Fig. 2i), resulting in a reduced total passenger travel cost (Fig. 2j) and therefore attracting more passengers to choose ride-sourcing services (Fig. 2k). As for the platform, Fig. 2l shows the following two distinct regimes of platform profit under different parking supply: when $K \leq 1680$, the platform earns an increasing profit as the parking supply increases and the maximum profit is achieved at $K = 1680$; when $K > 1680$, the profit decreases with the parking supply, and finally it can be smaller than that in $K = 0$ (without parking service).

5.2. Analysis

Below we summarize the profit-maximizing market outcomes under different parking supply K regarding the platform, the drivers, and the passengers, respectively.

For the platform, on the one hand, shared parking reduces the cruising cost of ride-sourcing vehicles and has the potential of promoting the TNC economy; on the other hand, it also imposes an additional parking cost which suppresses the increase of platform profit. Overall, the platform profit exhibits the following two distinct regimes under different parking supply:

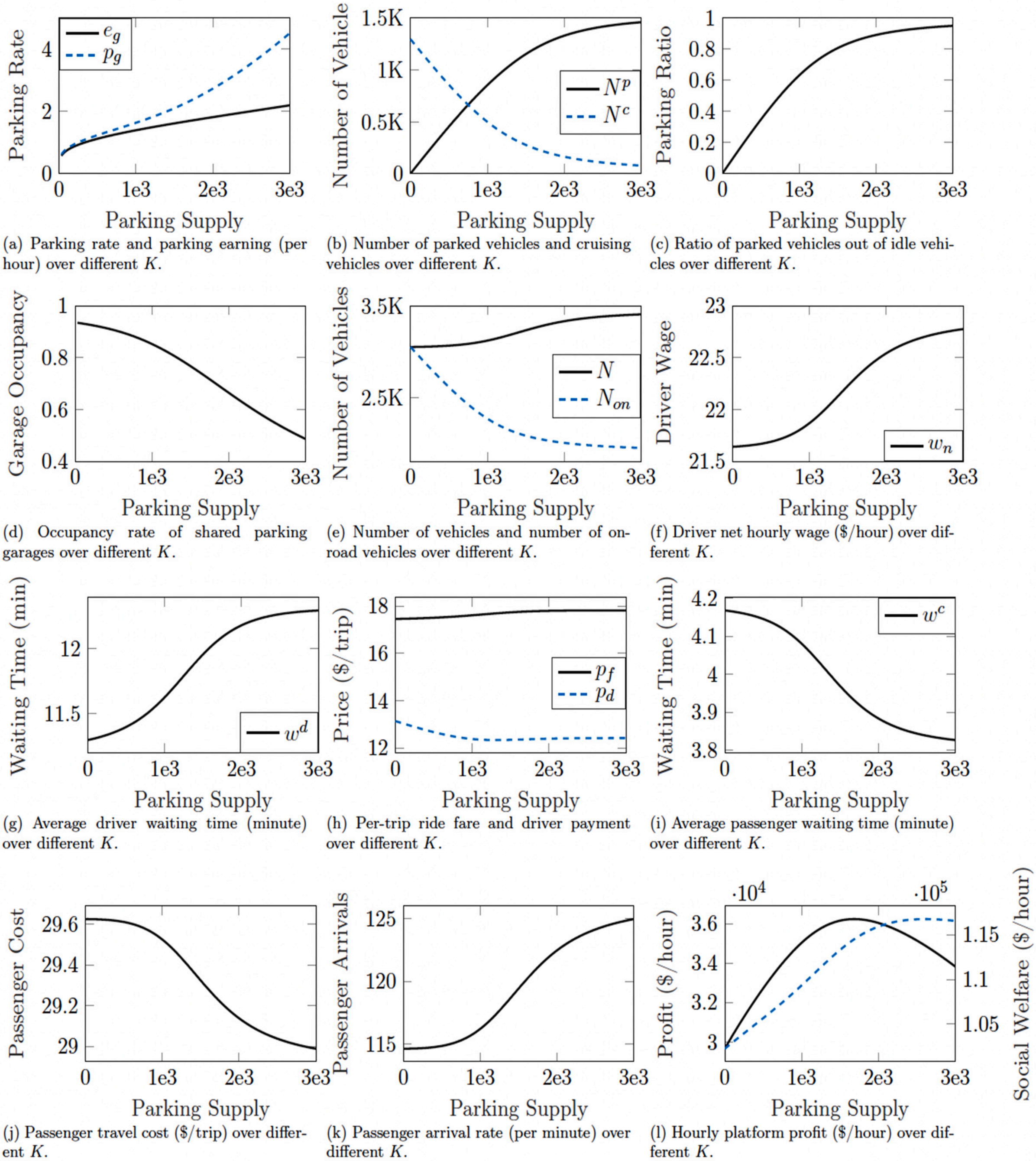
- When $K \leq 1680$, the platform profit increases with the parking supply. When increasing K, the number of parked TNC vehicles sharply increases and the number of cruising vehicles on the street drops significantly (Fig. 2b). This directly reduces the cruising cost of TNC vehicles and such benefits are shared among all stakeholders in the market, which leads to a Pareto improvement that benefits both passengers, drivers, garage operators and the platform. The platform increases the ride fare and decreases the driver payment simultaneously to make higher profit from ride-sourcing services (Fig. 2h). In the meanwhile, it also needs to raise the parking rate to hire more vacant parking spaces (Fig. 2a), and therefore expends more on shared parking services. The marginal benefit of increasing parking supply exceeds the marginal cost, so the platform profit keeps increasing with the parking supply. Finally, the platform profit reaches its maximum at $K = 1680$, at which the marginal benefit curve of increasing parking supply intersects with the marginal cost curve.
- When $K > 1680$, the platform profit decreases with the parking supply. In this regime, shared parking is over-supplied as most cruising vehicles are already parked at the shared parking garages (Fig. 2b). Further increasing the parking supply will incur significant cost for the platform, leading to reduced platform profit. However, passengers and drivers still benefit from the increased parking supply (Figs. 2e and 2k). This indicates that the cost due to over-provision of parking spaces are primarily covered by the integrated platform.

The two regimes identified above involves the over-supply and under-supply of parking spaces. In practice, the integrated platform will avoid these regimes and choose the parking supply that maximizes its profit. In this case, several important conclusions should be emphasized:

- In the presented numerical example, the maximum platform profit is achieved at $K = 1680$. Under the optimal parking supply, we note that the parking spots offered to the platform only account for approximately 1% of the 166,500 parking spaces in the garages and commercial parking lots of San Francisco.[11] This suggests that the demand for shared parking of ride-sourcing vehicles will not disrupt the regular parking demand.
- In comparison with the case without parking services ($K = 0$), passenger arrival rate increases by 5.27% (from 114.65/min to 120.69/min), the total number of ride-sourcing drivers increases by 7.34% (from 3050 to 3274), the passenger travel cost decreases by 1.28% (from \$29.63/trip to \$29.25/trip), the driver net hourly wage increases by 3.33% (from \$21.64\$ to \$22.36\$), and platform profit increases by 22.17% (from \$29,664/h to \$36,240/h). More importantly, the amount of cruising

[10] Social welfare is defined as the sum of passengers' surplus, drivers' surplus, garage operators' surplus and platform profit in this paper. The mathematical formulation of different stakeholders' surpluses are omitted and can be referred to Ke et al. (2019a).

[11] $K_0 = 10{,}000$ is the number of potential unoccupied parking spots out of all spots.



(a) Parking rate and parking earning (per hour) over different K.

(b) Number of parked vehicles and cruising vehicles over different K.

(c) Ratio of parked vehicles out of idle vehicles over different K.

(d) Occupancy rate of shared parking garages over different K.

(e) Number of vehicles and number of on-road vehicles over different K.

(f) Driver net hourly wage ($/hour) over different K.

(g) Average driver waiting time (minute) over different K.

(h) Per-trip ride fare and driver payment over different K.

(i) Average passenger waiting time (minute) over different K.

(j) Passenger travel cost ($/trip) over different K.

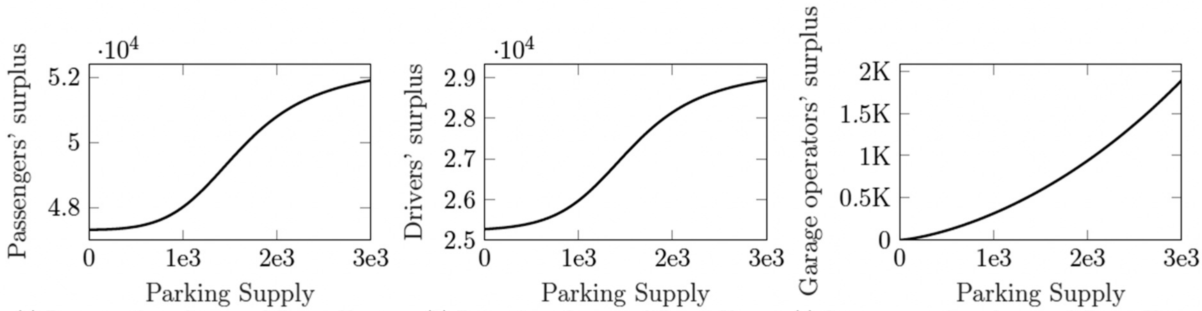
(k) Passenger arrival rate (per minute) over different K.

(l) Hourly platform profit ($/hour) over different K.

Fig. 2. Market outcomes over different parking supply.

traffic decreases by 82.55% (from 1295 to 226) and the number of on-road vehicles drops by 32.95% (from 3050 to 2045), demonstrating the substantial utility of the proposed business model on alleviating the cruising traffic caused by TNC vehicles.

- In Fig. 2l, the parking supply that maximizes the social welfare (i.e., $K = 2610$) is larger than the parking supply that maximizes the TNC profit (i.e., $K = 1680$). This is because when shared parking spaces are over-supplied ($K \geq 1680$), the platform profit begins to decrease but passengers' surplus, drivers' surplus and garage operators' surplus still increase with parking supply (Fig. 3(a-c)). In this case, the welfare-maximizing parking supply is higher than the profit-maximizing parking supply, which indicates that relevant authorities could further enhance the integration of shared parking and ride-sourcing business by regulatory policies or subsidies.



(a) Passengers' surplus over different K. (b) Drivers' surplus over different K. (c) Garage owners' surplus over different K.

Fig. 3. Passengers' surplus, drivers' surplus, and garage operators' surplus over different parking supply.

Numerical results indicate that the cruising traffic on the road network significantly reduces with the parking supply. However, as parking supply increases, the platform profit first increases and then decreases and finally maybe even worse off than the case without parking service ($K = 0$). This means that the integrated platform needs to find a balanced size of the shared parking market to avoid over-supply or under-supply of parking services. We emphasize that this observation regarding the platform remains consistent under a large range of model parameters. To make this precise, let $P(K)$ be the maximized platform profit to (18) as a function of K and let K^* be the profit-maximizing parking supply. Let $\bar{\lambda}$ be the profit-maximizing passenger demand when $K = 0$ and let $\bar{w}^d$ be the corresponding driver waiting time. For the platform profit under the proposed business model, we have the following result:

Proposition 2. *Assume that (18) has a unique solution.*[12] *Given $\beta_1 = \beta_2 = 1$, for any model parameters λ_0, N_0, K_0, α, l, A, and B, any strictly decreasing function $F_p(\cdot)$ that satisfies $F_p\left(\frac{\alpha A}{\sqrt{N_0}}\right) > 0$, any strictly increasing function $F_d(\cdot)$, and any strictly increasing function $F_g(\cdot)$ that satisfies $F_g\left(\frac{lB\bar{\lambda}(\bar{w}^d)^2}{B\bar{\lambda}(\bar{w}^d)^2+1}\right) > 0$ and $F_g(l) < 1$, there exists $0 < K' < \bar{\lambda}\bar{w}^d$ and $0 < K'' < K_0$, so that $P(K) > P(0)\ \forall K \in (0, K')$, and $P(K) < P(0)\ \forall K \in (K'', +\infty)$.*

The proof of Proposition 2 can be found in Appendix D. It indicates that when the matching function between vehicles and garages exhibits increasing returns to scale,[13] integrating more parking spaces brings a higher profit to the platform. Consequently, the platform profit under the optimal parking supply must exceeds the platform profit when no parking service is provided, i.e., $P(K^*) > P(0)$. However, as the parking supply increases, the marginal cost of parking supply rises sharply and finally surpasses the marginal revenue. In this case, increasing parking supply will reduce the platform profit, which may be eventually lower than that of the ordinary ride-sourcing market.

Remark 6. Proposition 2 indicates that shared parking always brings an increased profit to the platform under some extra conditions (for instance, $F_g\left(\frac{lB\bar{\lambda}(\bar{w}^d)^2}{B\bar{\lambda}(\bar{w}^d)^2+1}\right) > 0$) in the case where $\beta_1 = \beta_2 = 1$. This condition implies that when the average hourly parking revenue is $\frac{lB\bar{\lambda}(\bar{w}^d)^2}{B\bar{\lambda}(\bar{w}^d)^2+1}$, there will be a positive number of garage operators that are willing to offer their unused parking spaces to the TNC platform. We argue that this a reasonable assumption since shared parking unlocks the underutilized parking spaces, which would have been otherwise unused. Therefore, the activation parking revenue $F_g^{-1}(0)$ will be relatively low.

As for the driver side, we found that drivers always benefit from the shared parking: higher parking supply stimulates higher driver net hourly wage (Fig. 2f) and thus attracts more drivers to join the platform (Fig. 2e). Specifically, drivers' surplus monotonically increases with the parking supply (Fig. 3b) and in the meanwhile, the driver waiting time increases as more drivers are hired by the platform (Fig. 2g). This indicates that integrating ridesourcing with shared parking will directly benefit drivers as they can save fuel cost and reduce unnecessary cruising. We argue that the above result holds under fairly general conditions, and the monotonicity of the driver supply N as a function of the parking supply K at the profit-maximizing equilibrium holds under a large range of model parameters. To justify this claim, we first define $N(\lambda, K) = \arg\max_N \Pi(\lambda, N, K)$ as the optimal solution of N to (20) when λ and K are specified. The above insights can be formally summarized as the following proposition:

Proposition 3. *Assume that (18) has an unique interior solution.*[14] *Given $\beta_1 = \beta_2 = 1$, for any model parameters λ_0, N_0, K_0, α, l, A, and B, any strictly decreasing function $F_p(c)$ that satisfies $F_p\left(\frac{\alpha A}{\sqrt{N_0}}\right) > 0$, any strictly increasing function $F_d(\cdot)$ and $F_g(\cdot)$, $\partial N/\partial K \geq 0, \forall K \in (0, K_0)$.*

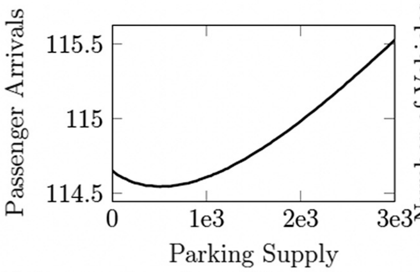
[12] Our experimental results numerically verified this assumption.

[13] The case of constant and decreasing returns to scale will be investigated in Section 5.3.

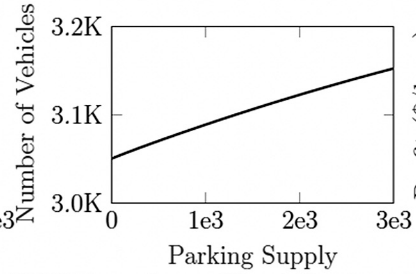
[14] Our experimental results numerically verified this assumption.

The proof of Proposition 3 is deferred to Appendix E. It indicates that for a large range of model parameters, shared parking benefits drivers and improves driver earnings at all supply levels. By deriving the partial derivative $\frac{\partial N}{\partial K}$, we neglect the correlation between passenger demand and driver supply when analyzing the impact of shared parking on the driver supply. This is a reasonable approximation since the second-order indirect effect that we neglect is typically much smaller than the first-order direct effect that we capture. In Section 5.3, a sensitivity analysis is presented to further validate the insights.

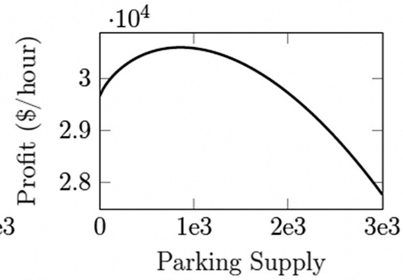
We pay special attention to the market outcomes regarding passengers since they are sensitive to model parameters. In the numerical example with $B = 1e{-3}$ and $\beta_1 = \beta_2 = 1$, numerical results show that as the parking supply increases, although a higher ride fare is charged (Fig. 2h), the passenger waiting time decreases (Fig. 2i), which implies a higher quality of service. The improved service quality lowers the generalized travel cost (Fig. 2j) and thus attracts more passengers to choose ride-sourcing services (Fig. 2k). The passengers' surplus also keeps increasing with the parking supply (Fig. 3a). In this specific numerical example, the passenger demand monotonically increases with the parking supply, which means that shared parking is beneficial to passengers at any parking supply level. However, we find that such results depend on model parameters and passengers may not necessarily always benefit from shared parking. Fig. 4(a-c) show the passenger demand, driver supply, and platform profit under different parking supply with $B = 1e{-3}$ and $\beta_1 = \beta_2 = 0.75$.[15] Other market results under $B = 1e{-3}$ and $\beta_1 = \beta_2 = 0.75$ are shown in Appendix F. Fig. 4a shows that as parking supply increases, the arrival rate of passengers (and therefore passenger surplus) first decreases and then increases. This indicates that at low parking supply and low matching efficiency, the benefit of shared parking is primarily enjoyed by the platform and drivers instead of passengers.



(a) Passenger demand over different K under $B = 1e{-3}$ and $\beta_{1,2} = 0.75$.



(b) Vehicle supply over different K under $B = 1e{-3}$ and $\beta_{1,2} = 0.75$.



(c) Platform profit over different K under $B = 1e{-3}$ and $\beta_{1,2} = 0.75$.

Fig. 4. Passenger demand, vehicle supply, and platform profit over different K under $B = 1e{-3}$ and $\beta_{1,2} = 0.75$.

Remark 7. Given $B = 1e{-3}$ and $\beta_1 = \beta_2 = 0.75$, the numerical results for the platform and drivers are consistent with our previous discussions. In particular, we find that shared parking largely reduces the cruising traffic, drivers always benefit from the TNC-parking integration and the platform profit first increases and eventually decreases with the parking supply. These results coincide with the market outcomes regarding drivers and the platform profit under higher vehicle-parking matching efficiency. This numerically validates the results in Propositions 2 and 3.

An above result is surprising as it indicates that TNC-parking integration may result in a decrease of the passenger surplus when both matching efficiency and parking supply are low. This leads to a quite counter-intuitive market regime: when parking supply increases, the platform recruits more drivers to serve fewer passengers, but at the same time earns a higher profit. To understand this, we comment that increasing the passenger demand may incur a trade-off: on the one hand, it leads to a higher volume of customers and potentially higher revenue; on the other hand, it also leads to higher driver utilization, fewer parked vehicles and consequently a smaller cost saving (due to parking). Note that such trade-off can be explicitly captured by the platform's objective function:

$$\Pi(\lambda, N, K) = \underbrace{\lambda\left[F_p^{-1}\left(\frac{\lambda}{\lambda_0}\right) - \alpha w^c(\lambda, N)\right] - NF_d^{-1}\left(\frac{N}{N_0}\right) - Nl}_{\text{revenue}} + \underbrace{N^p(\lambda, N, K)l}_{\text{cost-saving}} - \underbrace{KF_g^{-1}\left(\frac{K}{K_0}\right)}_{\text{parking cost}} \tag{28}$$

As highlighted above, the platform's profit can be decomposed into three parts: revenue, cost-saving, and parking cost. Given the parking supply K, the parking cost $KF_g^{-1}\left(\frac{K}{K_0}\right)$ is a constant and it does not affect the platform's optimal decision on λ and N. Therefore, λ and N are affected by the parking supply K through the cost-saving $N^p(\lambda, N, K)l$. The platform should carefully adjust its control knobs to lead to a passenger demand level that achieves a trade-off between revenue and cost-saving. As parking supply increases, the platform generally intends to induce more shared parking among idle vehicles in order to better utilize these parking spaces (Figs. 2e and 4b). However, when matching efficiency and parking supply are relatively low, the platform cannot induce enough shared parking by merely increasing the total number of vehicles. It must also reduce the passenger demand to increase the number of *idle* vehicles and incentivize a larger willingness to park among drivers. In this case, the platform prefers to reduce the passenger demand and motivate a higher degree of shared parking in order to induce a larger cost saving (Fig. 4a). On the other

[15] The values of other model parameters remains unchanged as in Section 5.1.

hand, when the matching efficiency or parking supply is relatively high, the number of parked vehicles and the corresponding cost savings are less sensitive to the increase of passenger demand. In the meanwhile, the driver supply keeps increasing and gradually becomes the dominating factor as the parking supply increases. This raises the marginal revenue of passenger demand and promotes the platform to stimulate a higher passenger demand to pursue a higher revenue (Figs. 4a and 2k).

5.3. Perturbing model parameters

Propositions 2 and 3 assume that the matching function between vehicles and garages exhibits increasing returns to scale. To investigate how this assumption affect our conclusions, we perform a sensitivity analysis with respect to the model parameters B, β_1, and β_2. For the parameter B, we log-linearly increase its value from 1e−4 to 1e+2. For parameters β_1 and β_2, we keep $\beta_1 = \beta_2$ and vary the value of $\beta_1 + \beta_2$ to cover different cases of decreasing ($\beta_1 + \beta_2 < 1$), constant ($\beta_1 + \beta_2 = 1$), and increasing ($\beta_1 + \beta_2 > 1$) returns to scale matching functions. We solve the profit maximization problem (18) under distinct combinations of B, β_1 and β_2. Fig. 5(a-d) present the platform profit and profit-maximizing passenger demand, driver supply and parking supply under different B, β_1 and β_2. Several key findings are summarized below:

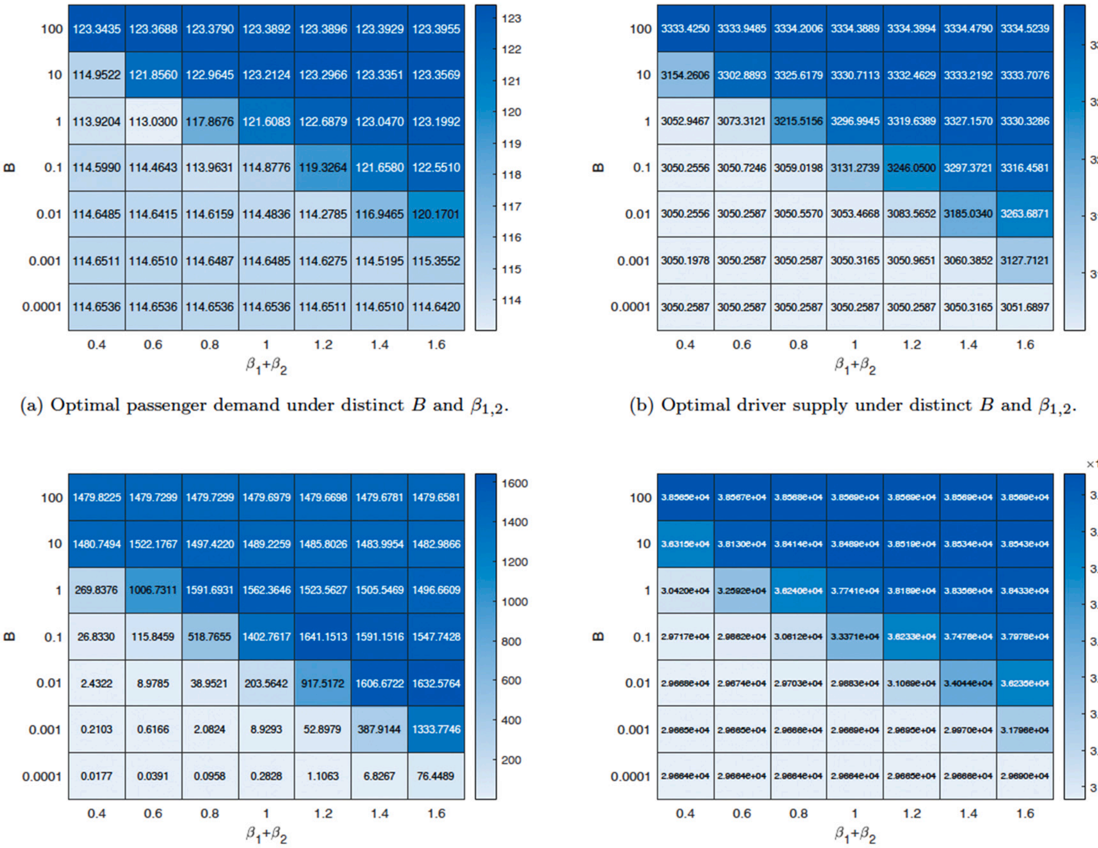
- The platform profit increases with the matching efficiency (Fig. 5d). Under different cases of decreasing, constant and increasing returns to scale, the matching function with a larger B or $\beta_1 + \beta_2$ always brings a higher profit to the platform. This motivates the platform to improve online dispatching techniques for more efficient matching between cruising vehicles and vacant parking spaces.
- The profit-maximizing driver supply monotonically increases with the matching efficiency (Fig. 5b). A higher matching efficiency always yields a higher driver surplus. This result is intuitive as the drivers are the direct beneficiaries of shared parking services. A higher matching efficiency enables more idle vehicles to park at the garages to reduce cruising cost.
- As the vehicle-garage matching efficiency improves, the profit-maximizing passenger demand first decreases and then increases (e.g., column 3 or row 3 of Fig. 5a). This leads to a valley of passenger demand at $B = 1, \beta_1 + \beta_2 = 0.6$ in Fig. 5a. Similar to our previous analysis on the passenger side, this can be explained as a consequence of the trade-off between revenue and cost-saving: increasing the passenger demand may lead to a higher revenue, but it also increases the vehicle occupancy, reduces the need for parking, and suppresses the cost-saving due to shared parking. When the matching efficiency is relatively low, the optimal parking supply is also low (Fig. 5c). In this case, the platform prefers to reduce the passenger demand, enhance shared parking for ride-sourcing vehicles (see Fig. 5c), and pursue a higher cost-saving. However, when the matching efficiency is high, cruising vehicles have a high probability of being matched. In this case, increasing passenger demand has a smaller impact on shared parking and increasing revenue becomes the dominant objective of the platform.
- As the vehicle-garage matching efficiency improves, the optimal parking supply first increases and then decreases (e.g., column 3 or row 3 of Fig. 5c). The reason behind this result is straightforward: when the matching efficiency is low, the parking garages are not efficiently utilized. In this case, the platform is unwilling to hire a large number of garages as the cost exceeds the benefit. On the other hand, when the matching efficiency is high, the platform can hire fewer parking spaces to accommodate the same number of TNC vehicles. In this case, the platform does not need to hire a large number of parking garages. Overall, the platform is only willing to hire a large number of parking garages when the matching efficiency is in the intermediate level. This corresponds to the peak of parking supply at $B = 0.1, \beta_1 + \beta_2 = 1.2$ in Fig. 5c.

The sensitivity analysis indicates that our conclusions regarding the platform and the driver are not affected by the values of B, β_1 and β_2. For the platform, its profit always exceeds that of the *ordinary* ride-sourcing platform profit ($2.9664e+04/h) without shared parking (Fig. 5d) and the optimal parking supply is always positive (Fig. 5c) for distinct combinations of B, β_1 and β_2. For drivers, the profit-maximizing driver supply is always higher than the optimal driver supply (3050.2586) without shared parking (Fig. 5b). It indicates that regardless of the vehicle-garage matching efficiency, TNC-parking integration always benefits drivers and brings an increased profit. This further validates some of the results in Propositions 2 and 3. However, we find that under some matching efficiencies, the passenger demand at the profit-maximizing parking supply may be lower than the passenger demand (114.6536) in the *ordinary* ride-sourcing market (Fig. 5a). For instance, when $B = 1$ and $\beta_1 + \beta_2 = 0.6$, the passenger demand is 113.0300, which is smaller than 114.6536. This indicates that when the matching efficiency is low, the passengers may be hurt from the proposed business model.

6. Conclusion

This paper proposes a novel business model that integrates shared parking into the ride-sourcing platform to reduce cruising traffic. The integrated platform offers app-based interfaces to passengers, drivers, and garage operators and implements online dispatching algorithms to coordinate real-time vehicle-passenger matching and vehicle-garage matching. An economic equilibrium model is developed to understand and investigate the underlying implications of the novel business model. The incentives of both passengers, drivers, and garage operators are captured and the matching process between vehicles and garages is characterized by the Cobb–Douglas matching function. The platform's optimal pricing strategies and the corresponding market outcomes are obtained by solving the profit maximization problem.

We use the developed model to quantify the impact of shared parking on the ride-sourcing market. We find that TNC-parking integration can offer a Pareto improvement: in certain regime, it leads to higher passenger surplus, higher driver surplus, higher



(a) Optimal passenger demand under distinct B and $\beta_{1,2}$.

(b) Optimal driver supply under distinct B and $\beta_{1,2}$.

(c) Optimal parking supply under distinct B and $\beta_{1,2}$.

(d) Platform profit (\$/hour) under distinct B and $\beta_{1,2}$.

Fig. 5. Optimal passenger demand, driver supply, parking supply, and the platform profit under distinct B and $\beta_{1,2}$.

garage surplus, higher platform profit, and reduced cruising traffic. The numerical study for San Francisco shows that the integrated platform hires 1680 parking spots for shared parking of ride-sourcing vehicles to maximize its profit. Under the profit-maximizing parking supply, compared to the *ordinary* ride-sourcing business without parking service, passenger demand increases by 5.27% (from 114.65/min to 120.69/min), the total number of drivers increases by 7.34% (from 3050 to 3274), the platform profit increases by 22.17% (from \$29,664/h to \$36,240/h). In the meanwhile, the amount of cruising traffic decreases by 82.55% (from 1295 to 226) and the number of on-road vehicles drops by 32.95% (from 3050 to 2045). This demonstrates that by unlocking the potential of underutilized parking spaces, the business model can promote the ride-sourcing economy while largely curbing cruising traffic at the same time. We further investigate the impacts of the vehicle-garage matching efficiency on the integrated business model. Model parameters regarding the vehicle-garage Cobb–Douglas matching function are perturbed and evaluated. We find that higher matching efficiency always yields higher driver surplus and increased platform profit. However, as we improve the matching efficiency, the passenger demand first decreases and then increases, while the parking supply initially increases and then decreases.

This paper gives a comprehensive investigation of the business model integrating ride-sourcing services with shared parking services. Under the proposed business model, we mainly focus on the platform's profit-maximizing pricing and the corresponding market outcomes at the aggregate level. One future direction is extending the current mathematical model to a network equilibrium model to characterize the spatial heterogeneity of the ride-sourcing business. Another extension would be investigating the operation strategies and market response under social welfare maximization. We hope this work could stimulate more discussion in the synergistic operation of on-demand mobility services for traffic improvement.

CRediT authorship contribution statement

Jing Gao: Methodology, Investigation, Formal analysis, Writing – original draft. **Sen Li:** Conceptualization, Methodology, Investigation, Supervision, Funding acquisition, Writing – review & editing. **Hai Yang:** Methodology, Investigation, Formal analysis, Writing – review & editing.

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Appendix A. Proof of Lemma 1

Note that (19)(f) is a polynomial equation with respect to N^p given $\lambda, K, w^d > 0$. The LHS of Eq. (19)(f) is a continuous and increasing function of N^p while the RHS of (19)(f) is a polynomial function of N^p (denoted as RHS(N^p)) which is monotonically decreasing when $N^p \in \left(0, \min\{\lambda w^d, K\}\right)$. Given $\lambda, K, w^d > 0$, RHS(0) $= Bw^d(\lambda w^d)^{\beta_1}(K)^{\beta_2} > 0$ and RHS($\min\{\lambda w^d, K\}$) $= 0$. By continuity of RHS(N^p), the 45-degree line $y = N^p$ and the polynomial function curve $y = \text{RHS}(N^p)$ have and only have one intersection within $\left(0, \min\{\lambda w^d, K\}\right)$. Thus, there is a unique N^p satisfying (19)(f) within the range of $\left(0, \min\{\lambda w^d, K\}\right)$.

Appendix B. Proof of Lemma 2

Plugging (19)(d) into (19)(e) yields

$$N = \lambda w^d + \frac{\lambda A}{\sqrt{\lambda w^d}} + \lambda \mu. \tag{29}$$

Given $\lambda, N, \mu > 0$, the LHS of Eq. (29) is a constant and the RHS of (29) is a function of w^d, denoted as RHS(w^d). Note that RHS(w^d) first decreases and then increases with w^d when $w^d > 0$. The minimum of RHS(w^d) is achieved at $w^d = \sqrt[3]{\frac{1}{4}}\lambda^{-\frac{1}{3}}A^{\frac{2}{3}}$ by solving the first order condition RHS$'(w^d) = 0$. The corresponding minimum value is $\text{RHS}(w^d)_{\min} = \left(\sqrt[3]{2} + \sqrt[3]{\frac{1}{4}}\right)(A\lambda)^{\frac{2}{3}} + \lambda\mu$. In this case, when $N = \left(\sqrt[3]{2} + \sqrt[3]{\frac{1}{4}}\right)(A\lambda)^{\frac{2}{3}} + \lambda\mu$, the horizontal line $y = N$ intersects with the curve of $y = \text{RHS}(w^d)$ at the minimum point and (29) has an unique solution $w^d = \sqrt[3]{\frac{1}{4}}\lambda^{-\frac{1}{3}}A^{\frac{2}{3}}$; When $N > \left(\sqrt[3]{2} + \sqrt[3]{\frac{1}{4}}\right)(A\lambda)^{\frac{2}{3}} + \lambda\mu$, there are two intersections of $y = N$ and $y = \text{RHS}(w^d)$ and thus (29) has two distinct solutions.

Given λ, N, K and w^d, based on Lemma 1, there exists a unique feasible N^p, thus the platform profit is well defined. Let w_1^d, w_2^d be the two solutions to (29) and denote P_1, P_2 as the corresponding platform profit when $N > \left(\sqrt[3]{2} + \sqrt[3]{\frac{1}{4}}\right)(A\lambda)^{\frac{2}{3}} + \lambda\mu$. We consider $w_2^d > w_1^d$ without loss of generality and show that $P_2 > P_1$. Let $(w_1^c, p_{f1}, p_{d1}, p_{g1}, N_1^p)$ and $(w_2^c, p_{f2}, p_{d2}, p_{g2}, N_2^p)$ be the corresponding endogenous variables under w_1^d and w_2^d, respectively. We have $w_2^c < w_1^c$ since $w_c = \frac{A}{\sqrt{\lambda w^d}}$. The ride fare p_f can be derived from (19)(a) as:

$$p_f = F_p^{-1}\left(\frac{\lambda}{\lambda_0}\right) - \alpha w^c \tag{30}$$

Since $w_2^c < w_1^c$, $p_{f2} > p_{f1}$. The driver payment p_d can be derived from (19)(b) as:

$$p_d = \frac{NF_d^{-1}\left(\frac{N}{N_0}\right) + (N - N^p)l}{\lambda}, \tag{31}$$

where N^p is determined by the polynomial Eq. (19)(f) given λ, N and w^d. By implicit function theorem, we can derive the partial derivative $\frac{\partial N^p}{\partial w^d}$ from (19)(f):

$$\frac{\partial N^p}{\partial w^d} = \frac{B(\lambda w^d - N^p)^{\beta_1}(K - N^p)^{\beta_2} + B\beta_1\lambda w^d(\lambda w^d - N^p)^{\beta_1 - 1}(K - N^p)^{\beta_2}}{B\beta_1 w^d(\lambda w^d - N^p)^{\beta_1 - 1}(K - N^p)^{\beta_2} + B\beta_2 w^d(\lambda w^d - N^p)^{\beta_1}(K - N^p)^{\beta_2 - 1} + 1}. \tag{32}$$

Based on Lemma 1, there exists a unique N_p that satisfies (19)(f), and at this solution, we have $\lambda w^d - N^p > 0$ and $K - N^p > 0$. Thus, we have $\frac{\partial N^p}{\partial w^d} > 0$ and $w_2^d > w_1^d$ implies that $N_2^p > N_1^p$. Since $N_2^p > N_1^p$, $p_{d2} < p_{d1}$ according to (31). Further, from (19)(c), we have:

$$N^p p_g = KF_g^{-1}\left(\frac{K}{K_0}\right), \tag{33}$$

which indicates that $N_1^p p_{g1} = N_2^p p_{g2}$ given λ, N and K. Finally, the platform profit under w_1^d is $P_1 = \lambda p_{f1} - \lambda p_{d1} + N_1^p p_{g1}$ and the profit under w_2^d is $P_2 = \lambda p_{f2} - \lambda p_{d2} + N_2^p p_{g2}$. Since $p_{f2} > p_{f1}$, $p_{d2} < p_{d1}$ and $N_1^p p_{g1} = N_2^p p_{g2}$, we have $P_2 > P_1$. This completes the proof.

Appendix C. Proof of Proposition 1

To guarantee the existence of the market equilibrium, we need to show that there exists strictly positive p_f, p_d, p_g, λ, N, K, N^p, N^c, K^v, w^c and w^d satisfying (19). We have shown in Lemma 2 that there exists strictly positive w^d and w^c satisfying (19)(d)–(19)(e) if $N \geq \left(\sqrt[3]{2}+\sqrt[3]{\frac{1}{4}}\right)(A\lambda)^{\frac{2}{3}}+\lambda\mu$ and w^d, w^c can be uniquely derived from (19)(d)–(19)(e) as a function of λ and N respectively, denoted as $w^d(\lambda, N)$ and $w^c(\lambda, N)$. In this case, the market equilibrium condition (19) is equivalent to the following:

$$\begin{cases} p_f = F_p^{-1}\left(\dfrac{\lambda}{\lambda_0}\right) - \alpha w^c(\lambda, N) & \text{(a)} \\ p_d = \dfrac{N F_d^{-1}\left(\frac{N}{N_0}\right) + (N - N^p) l}{\lambda} & \text{(b)} \\ p_g = \dfrac{K F_g^{-1}\left(\frac{K}{K_0}\right)}{N^p} & \text{(c)} \\ w^d = w^d(\lambda, N) & \text{(d)} \\ N^p = B \cdot w^d(\lambda, N)\left(\lambda \cdot w^d(\lambda, N) - N^p\right)^{\beta_1} (K - N^p)^{\beta_2} & \text{(e)} \\ N \geq \left(\sqrt[3]{2}+\sqrt[3]{\dfrac{1}{4}}\right)(A\lambda)^{\dfrac{2}{3}}+\lambda\mu & \text{(f)} \end{cases} \tag{34}$$

where $F_p^{-1}(\cdot)$, $F_d^{-1}(\cdot)$ and $F_g^{-1}(\cdot)$ are inverse function of $F_p(\cdot)$, $F_d(\cdot)$ and $F_g(\cdot)$, respectively. For the profit maximization problem (18), we can equivalently treat λ, N and K as decision variables/free variables. To prove Proposition 1, it suffices to show that there exists $\lambda \in (0, \lambda_0)$, $N \in (0, N_0)$ and $K \in (0, K_0)$ such that

$$\begin{cases} p_f = F_p^{-1}\left(\dfrac{\lambda}{\lambda_0}\right) - \alpha w^c(\lambda, N) > 0 & \text{(a)} \\ p_d = \dfrac{N F_d^{-1}\left(\frac{N}{N_0}\right) + (N - N^p) l}{\lambda} > 0 & \text{(b)} \\ p_g = \dfrac{K F_g^{-1}\left(\dfrac{K}{K_0}\right)}{N^p} > 0 & \text{(c)} \\ N^p = B \cdot w^d(\lambda, N)\left(\lambda \cdot w^d(\lambda, N) - N^p\right)^{\beta_1} (K - N^p)^{\beta_2} & \text{(d)} \\ N^p > 0 & \text{(e)} \\ N^c = \lambda w^d - N^p > 0 & \text{(f)} \\ K^v = K - N^p > 0 & \text{(g)} \\ N \geq \left(\sqrt[3]{2}+\sqrt[3]{\dfrac{1}{4}}\right)(A\lambda)^{\dfrac{2}{3}}+\lambda\mu & \text{(h)} \end{cases} \tag{35}$$

We first show that there exists $0 < \lambda < \lambda_0$ and $0 < N < N_0$, such that $N \geq \left(\sqrt[3]{2}+\sqrt[3]{\frac{1}{4}}\right)(A\lambda)^{\frac{2}{3}}+\lambda\mu$, i.e., condition (35)(h). This is obvious since the LHS of (35)(h) is a continuous and strictly increasing function of N and the RHS of (35)(h) is also a continuous and strictly increasing function of λ. By continuity, there must exist $0 < N_h < N_0$ and $0 < \lambda_h < \lambda_0$ such that $N \geq \left(\sqrt[3]{2}+\sqrt[3]{\frac{1}{4}}\right)(A\lambda)^{\frac{2}{3}}+\lambda\mu$ when $N \in (N_h, N_0)$ and $\lambda \in (0, \lambda_h)$. In this case, we have uniquely determined $w^d = w^d(\lambda, N) > 0$ and $w^c = w^c(\lambda, N) > 0$ (according to Lemma 2, the platform would choose the larger w^d for higher profit). Based on Lemma 1, $N^p = N^p(\lambda, N, K)$ can be also derived as a function of λ, N and K by solving (35)(d).

So far, we have shown that when $\lambda \in (0, \lambda_h)$ and $N \in (N_h, N_0)$, conditions (35)(d)–(35)(h) hold and $N^p = N^p(\lambda, N, K)$ can be derived as a function of λ, N and K. Next, we show that there exists $\lambda \in (0, \lambda_0)$, $N \in (0, N_0)$ and $K \in (0, K_0)$ such that $p_f, p_d, p_g > 0$, i.e., conditions (35)(a)–(35)(c).

For (35)(a), note that $w^c(\lambda, N)$ is derived from (19)(d)–(19)(e). Substituting (19)(e) into (19)(d), we have

$$w^c = \frac{A}{\sqrt{N - \lambda\mu - \lambda w^c}}. \tag{36}$$

Plugging (36) into (35)(a) leads to

$$p_f = F_p^{-1}\left(\frac{\lambda}{\lambda_0}\right) - \frac{\alpha A}{\sqrt{N - \lambda\mu - \lambda w^c}}. \tag{37}$$

By assumption $F_p(\cdot)$ is a strictly decreasing function, therefore $F_p^{-1}\left(\frac{\lambda}{\lambda_0}\right)$ monotonically decreases with λ. Thus, p_f is a decreasing function with respect to λ and an increasing function with respect to N. The maximum of p_f is

$$p_{f\,\max} = \lim_{\lambda \to 0} F_p^{-1}\left(\frac{\lambda}{\lambda_0}\right) - \frac{\alpha A}{\sqrt{N_0 - \lambda\mu - \lambda w^c}} = F_p^{-1}(0) - \frac{\alpha A}{\sqrt{N_0}}. \tag{38}$$

To guarantee the existence of positive p_f, we need that $p_{f\max} = F_p^{-1}(0) - \frac{\alpha A}{\sqrt{N_0}} > 0$. This is equivalent to $F_p\left(\frac{\alpha A}{\sqrt{N_0}}\right) > 0$. Under this assumption, there must exist $0 < \lambda_a < \lambda_0$ and $0 < N_a < N_0$ such that $p_f > 0$ when $\lambda \in (0, \lambda_a)$ and $N \in (N_a, N_0)$.

For (35)(b), since $F_d(\cdot)$ is defined on positive real numbers, we have $F_d^{-1}\left(\frac{N}{N_0}\right) > 0$ for any $N \in (0, N_0)$. Further, we have shown that when $\lambda \in (0, \lambda_h)$ and $N \in (N_h, N_0)$, $w^d > 0$, $w^c > 0$ and $N^p < \lambda w^d$. This implies that $N^p < \lambda w^d < \lambda(w^d + w^c + \mu) = N$ when $\lambda \in (0, \lambda_h)$ and $N \in (N_h, N_0)$. Therefore, $p_d > 0$ when $\lambda \in (0, \lambda_h)$ and $N \in (N_h, N_0)$.

For (35)(c), similarly, $F_g^{-1}(\frac{K}{K_0}) > 0$ for any $K \in (0, K_0)$ as $F_g(\cdot)$ is defined on positive real numbers. Since $N^p > 0$ when $\lambda \in (0, \lambda_h)$ and $N \in (N_h, N_0)$, we have $p_g > 0$ when $\lambda \in (0, \lambda_h)$ and $N \in (N_h, N_0)$.

Finally, we take the intersection of all the derived conditions to show the feasibility of (35). Let $\lambda_{\min} = \min\{\lambda_a, \lambda_h\}$ and $N_{\max} = \max\{N_a, N_h\}$. Given the assumption that $F_p\left(\frac{\alpha A}{\sqrt{N_0}}\right) > 0$, we conclude that (35) hold when $\lambda \in (0, \lambda_{\min})$, $N \in (N_{\max}, N_0)$ and $K \in (0, K_0)$. This completes the proof.

Appendix D. Proof of Proposition 2

Without loss of generality, we denote $\lambda(K)$, $N(K)$, $p_f(K)$, $w_n(K)$ and $w^d(K)$ as the optimal to (18) as a function of K. For simplicity of expression, let $\bar{\lambda} = \lambda(0)$, $\bar{N} = N(0)$, $\bar{p}_f = p_f(0)$, $\bar{w}_n = w_n(0)$ and $\bar{w}^d = w^d(0)$ be the optimal to (18) when $K = 0$. According to Lemma 3, the platform profit as a function of λ, N and K is given by

$$\Pi(\lambda, N, K) = \lambda\left[F_p^{-1}\left(\frac{\lambda}{\lambda_0}\right) - w^c(\lambda, N)\right] - NF_d^{-1}\left(\frac{N}{N_0}\right) - Nl + lN^p(\lambda, N, K) - KF_g^{-1}\left(\frac{K}{K_0}\right). \tag{39}$$

First, we show that there exists $K' > 0$ such that $P(K) > P(0)$ when $K \in (0, K')$. When $K = 0$, $N^p(\lambda, N, 0) = 0$ and $P(0) = \Pi(\bar{\lambda}, \bar{N}, 0) = \bar{\lambda}\bar{p}_f - \bar{w}_n\bar{N} - \bar{N}l$. Since $P(K)$ is the maximized platform profit under parking supply K, $\forall K \in [0, +\infty)$, we have:

$$\begin{aligned} P(K) = \Pi(\lambda(K), N(K), K) &\geq \Pi(\lambda(0), N(0), K) = \bar{\lambda}\bar{p}_f - \bar{w}_n\bar{N} - \bar{N}l + lN^p(\bar{\lambda}, \bar{N}, K) - KF_g^{-1}\left(\frac{K}{K_0}\right) \\ &= P(0) + lN^p(\bar{\lambda}, \bar{N}, K) - KF_g^{-1}\left(\frac{K}{K_0}\right). \end{aligned} \tag{40}$$

Now it suffices to show that there exists $K' > 0$ such that $lN^p(\bar{\lambda}, \bar{N}, K) - KF_g^{-1}\left(\frac{K}{K_0}\right) > 0$ when $K \in (0, K')$. Given $\beta_1 = \beta_2 = 1$, $N^p(\bar{\lambda}, \bar{N}, K)$ is a function of K which is derived from the following quadratic equation:

$$N^p = B\bar{w}^d(\bar{\lambda}\bar{w}^d - N^p)(K - N^p). \tag{41}$$

Note that the LHS of (41) is a linear function of N^p and the RHS is a quadratic function of N^p with two roots $\bar{\lambda}\bar{w}^d$ and K. Based on Lemma 1, there is an unique solution to (41) within $(0, \min\{\bar{\lambda}\bar{w}^d, K\})$. When $0 < K < \bar{\lambda}\bar{w}^d$, we have $N^p < K < \bar{\lambda}\bar{w}^d$. This indicates that the RHS of (41) satisfies

$$\text{RHS} = B\bar{w}^d(\bar{\lambda}\bar{w}^d - N^p)(K - N^p) \geq B\bar{w}^d(\bar{\lambda}\bar{w}^d - K)(K - N^p), \tag{42}$$

In this case, solving the linear equation $N^p = B\bar{w}^d(\bar{\lambda}\bar{w}^d - K)(K - N^p)$ gives a lower bound of $N^p(\bar{\lambda}, \bar{N}, K)$ for $K \in (0, \bar{\lambda}\bar{w}^d)$:

$$N^P(\bar{\lambda}, \bar{N}, K) > \frac{B\bar{w}^d(K - \bar{\lambda}\bar{w}^d)}{B\bar{w}^d(K - \bar{\lambda}\bar{w}^d) - 1}K. \tag{43}$$

Therefore, for $K \in (0, \bar{\lambda}\bar{w}^d)$, we have:

$$lN^P(\bar{\lambda}, \bar{N}, K) - KF_g^{-1}\left(\frac{K}{K_0}\right) > K\left[\frac{lB\bar{w}^d(K - \bar{\lambda}\bar{w}^d)}{B\bar{w}^d(K - \bar{\lambda}\bar{w}^d) - 1} - F_g^{-1}\left(\frac{K}{K_0}\right)\right]. \tag{44}$$

Note that the term in the square brackets in the RHS of (44) is a function of K (denoted as RHS(K)). RHS(K) is a decreasing function with

$$\begin{cases} \text{RHS}(0) = \dfrac{-lB\bar{\lambda}(\bar{w}^d)^2}{-B\bar{\lambda}(\bar{w}^d)^2 - 1} - F_g^{-1}(0) \\ \text{RHS}(\bar{\lambda}\bar{w}^d) = -F_g^{-1}(\dfrac{\bar{\lambda}\bar{w}^d}{K_0}) < 0. \end{cases} \tag{45}$$

By continuity, if $\text{RHS}(0) = \frac{-lB\bar{\lambda}(\bar{w}^d)^2}{-B\bar{\lambda}(\bar{w}^d)^2-1} - F_g^{-1}(0) > 0$, there must exist $0 < K' < \bar{\lambda}\bar{w}^d$ such that RHS(K) > 0 when $K \in (0, K')$. This is equivalent to $F_g\left(\frac{lB\bar{\lambda}(\bar{w}^d)^2}{B\bar{\lambda}(\bar{w}^d)^2+1}\right) > 0$. Therefore, under the assumption that $F_g\left(\frac{lB\bar{\lambda}(\bar{w}^d)^2}{B\bar{\lambda}(\bar{w}^d)^2+1}\right) > 0$, there exists $0 < K' < \bar{\lambda}\bar{w}^d$ such that $P(K) > P(0)$ when $K \in (0, K')$.

Second, we show that there exists $0 < K'' < K_0$ such that $P(K) < P(0)$ when $K \in (K'', K_0)$. Similarly, since $P(0)$ is the maximized platform profit when $K = 0$, we have:

$$\begin{aligned} P(0) = \Pi(\lambda(0), N(0), 0) &\geq \Pi(\lambda(K), N(K), 0) = \lambda(K)p_f(K) - w_n(K)N(K) - lN(K) \\ &= P(K) - \left[lN^p(\lambda(K), N(K), K) - KF_g^{-1}\left(\frac{K}{K_0}\right)\right]. \end{aligned} \tag{46}$$

Since $N^p(\lambda(K), N(K), K) < K$, we have:

$$\begin{aligned} P(K) &\le P(0) + \left[l N^p(\lambda(K), N(K), K) - K F_g^{-1}\left(\frac{K}{K_0}\right)\right] \\ &< P(0) + K\left[l - F_g^{-1}\left(\frac{K}{K_0}\right)\right]. \end{aligned} \tag{47}$$

It suffices to show that there exists $K'' > 0$ such that $l - F_g^{-1}\left(\frac{K}{K_0}\right) < 0$ when $K > K''$. Since $F_g^{-1}\left(\frac{K}{K_0}\right)$ is a strictly increasing function of K, if $F_g^{-1}\left(\frac{K}{K_0}\right)_{\max} = F_g^{-1}(1) > l$, there must exist $0 < K'' < K_0$ such that $F_g^{-1}\left(\frac{K}{K_0}\right) > l$ when $K \in (K'', K_0)$. This is equivalent to $F_g(l) < 1$. Therefore, under the assumption that $F_g(l) < 1$, there exists $0 < K'' < K_0$ such that $P(K) < P(0)$ when $K \in (K'', K_0)$. This completes the proof.

Appendix E. Proof of Proposition 3

Based on Lemma 3, (18) can be transformed into the unconstrained optimization problem (20):

$$\max_{\lambda,N,K} \quad \Pi(\lambda, N, K) = \lambda\left[F_p^{-1}\left(\frac{\lambda}{\lambda_0}\right) - w^c(\lambda, N)\right] - N F_d^{-1}\left(\frac{N}{N_0}\right) - Nl + l N^p(\lambda, N, K) - K F_g^{-1}\left(\frac{K}{K_0}\right)$$

By definition, $N(\lambda, K)$ is the optimal interior solution to (20) given (λ, K). The first-order condition leads to

$$\left.\frac{\partial \Pi}{\partial N}\right|_{N(\lambda,K)} = 0. \tag{48}$$

Note that (48) is an implicit function of λ and K. By implicit function theorem, we have:

$$\frac{\partial N}{\partial K} = -\frac{\partial^2 \Pi}{\partial N \partial K} \Big/ \frac{\partial^2 \Pi}{\partial N^2} = -l \frac{\partial^2 N^p}{\partial N \partial K} \Big/ \frac{\partial^2 \Pi}{\partial N^2}. \tag{49}$$

Since $\Pi(\lambda, N, K)$ is maximized at $N(\lambda, K)$, we have $\frac{\partial^2 \Pi}{\partial N^2} < 0$. To prove that $\frac{\partial N}{\partial K} \ge 0\ \forall K \in (0, K_0)$, it is equivalent to show that $\frac{\partial^2 N^p}{\partial N \partial K} \ge 0\ \forall K \in (0, K_0)$.

To derive $\frac{\partial^2 N^p}{\partial N \partial K}$, we start with the equilibrium condition

$$N^p = w^d m^{p-t} = w^d M^{p-t}(K^v, N^c) = w^d M^{p-t}(K - N^p, \lambda w^d - N^p), \tag{50}$$

where $w^d = w^d(\lambda, N)$ is a function of λ and N, m^{p-t} is the vehicle-garage matching rate, and $M^{p-t}(\cdot)$ is the Cobb–Douglas matching function. Taking derivative of both sides with respect to N yields:

$$\frac{\partial N^p}{\partial N} = \frac{\frac{\partial w^d}{\partial N} m^{p-t} + \lambda w^d \frac{\partial w^d}{\partial N}\frac{\partial M^{p-t}}{\partial N^c}}{1 + w^d\left(\frac{\partial M^{p-t}}{\partial K^v} + \frac{\partial M^{p-t}}{\partial N^c}\right)}. \tag{51}$$

Further taking derivative of $\frac{\partial N^p}{\partial N}$ with respect to K, we have:

$$\frac{\partial^2 N^p}{\partial N \partial K} = \frac{\partial w^d}{\partial N} \frac{\left(\frac{\partial m^{p-t}}{\partial K} + \lambda w^d \frac{\partial^2 M^{p-t}}{\partial N^c \partial K}\right)\left[1 + w^d\left(\frac{\partial M^{p-t}}{\partial K^v} + \frac{\partial M^{p-t}}{\partial N^c}\right)\right] - \left(m^{p-t} + \lambda w^d \frac{\partial M^{p-t}}{\partial N^c}\right)\left[w^d\left(\frac{\partial^2 M^{p-t}}{\partial K^v \partial K} + \frac{\partial^2 M^{p-t}}{\partial N^c \partial K}\right)\right]}{\left[1 + w^d\left(\frac{\partial M^{p-t}}{\partial K^v} + \frac{\partial M^{p-t}}{\partial N^c}\right)\right]^2}. \tag{52}$$

Similarly, we can derive $\frac{\partial N^p}{\partial K}$ and $\frac{\partial M^{p-t}}{\partial K}$ from (50) as

$$\frac{\partial N^p}{\partial K} = \frac{w^d \frac{\partial M^{p-t}}{\partial K^v}}{1 + w^d\left(\frac{\partial M^{p-t}}{\partial K^v} + \frac{\partial M^{p-t}}{\partial N^c}\right)}, \tag{53}$$

and

$$\frac{\partial m^{p-t}}{\partial K} = \frac{\partial N^p}{\partial K} \Big/ w^d = \frac{\frac{\partial M^{p-t}}{\partial K^v}}{1 + w^d\left(\frac{\partial M^{p-t}}{\partial K^v} + \frac{\partial M^{p-t}}{\partial N^c}\right)}. \tag{54}$$

Given $\beta_1 = \beta_2 = 1$, the Cobb Douglass function is specified as $m^{p-t} = M^{p-t}(K^v, N^c) = BK^v N^c$. The involved partial derivatives are summarized below:

$$\begin{cases} \dfrac{\partial M^{p-t}}{\partial K^v} = BN^c & \text{(a)} \\ \dfrac{\partial M^{p-t}}{\partial N^c} = BK^v & \text{(b)} \\ \dfrac{\partial N^p}{\partial K} = \dfrac{w^d BN^c}{1 + w^d(BN^c + BK^v)} & \text{(c)} \\ \dfrac{\partial m^{p-t}}{\partial K} = \dfrac{BN^c}{1 + w^d(BN^c + BK^v)} & \text{(d)} \\ \dfrac{\partial^2 M^{p-t}}{\partial K^v \partial K} = \dfrac{\partial (BN^c)}{\partial K} = -B\dfrac{\partial N^p}{\partial K} = \dfrac{-B^2 w^d N^c}{1 + w^d(BN^c + BK^v)} & \text{(e)} \\ \dfrac{\partial^2 M^{p-t}}{\partial N^c \partial K} = \dfrac{\partial (BK^v)}{\partial K} = B\dfrac{\partial K^v}{\partial K} = B\left(1 - \dfrac{\partial N^p}{\partial K}\right) = \dfrac{B(1 + Bw^d K^v)}{1 + w^d(BN^c + BK^v)} & \text{(f)} \end{cases} \tag{55}$$

Plugging (55) into (52) leads to

$$\frac{\partial^2 N^p}{\partial N \partial K} = \frac{\partial w^d}{\partial N} \frac{BN^c + w^d B^2 (N^c)^2 + B^3 K^v (N^c)^2 (w^d)^2 + B^2 (w^d)^2 K^v (\lambda - m^{p-t}) + \lambda w^d B + \lambda B^2 (w^d)^2 N^c + 2\lambda B^3 (w^d)^3 N^c K^v}{[1 + w^d(BN^c + BK^v)]^3} \tag{56}$$

Since $\lambda - m^{p-t} = \lambda - \frac{N^p}{w^d} = \lambda - \frac{\lambda w^d - N^c}{w^d} = \frac{N^c}{w^d} > 0$, the second term in the RHS of (56) is a positive number. Now to prove that $\frac{\partial^2 N^p}{\partial N \partial K} \geq 0$ $\forall K \in (0, K_0)$, it suffices to show that $\frac{\partial w^d}{\partial N} \geq 0$.

Based on Lemma 2, $w^d = w^d(\lambda, N)$ is uniquely derived from Eq. (29) and $w^d \geq \sqrt[3]{\frac{1}{4}} \lambda^{-\frac{1}{3}} A^{\frac{2}{3}}$. Taking derivative of both sides of (29) with respect to N yields:

$$\frac{\partial w^d}{\partial N} = \frac{1}{\lambda - \frac{A}{2}\lambda^{\frac{1}{2}} (w^d)^{-\frac{3}{2}}}. \tag{57}$$

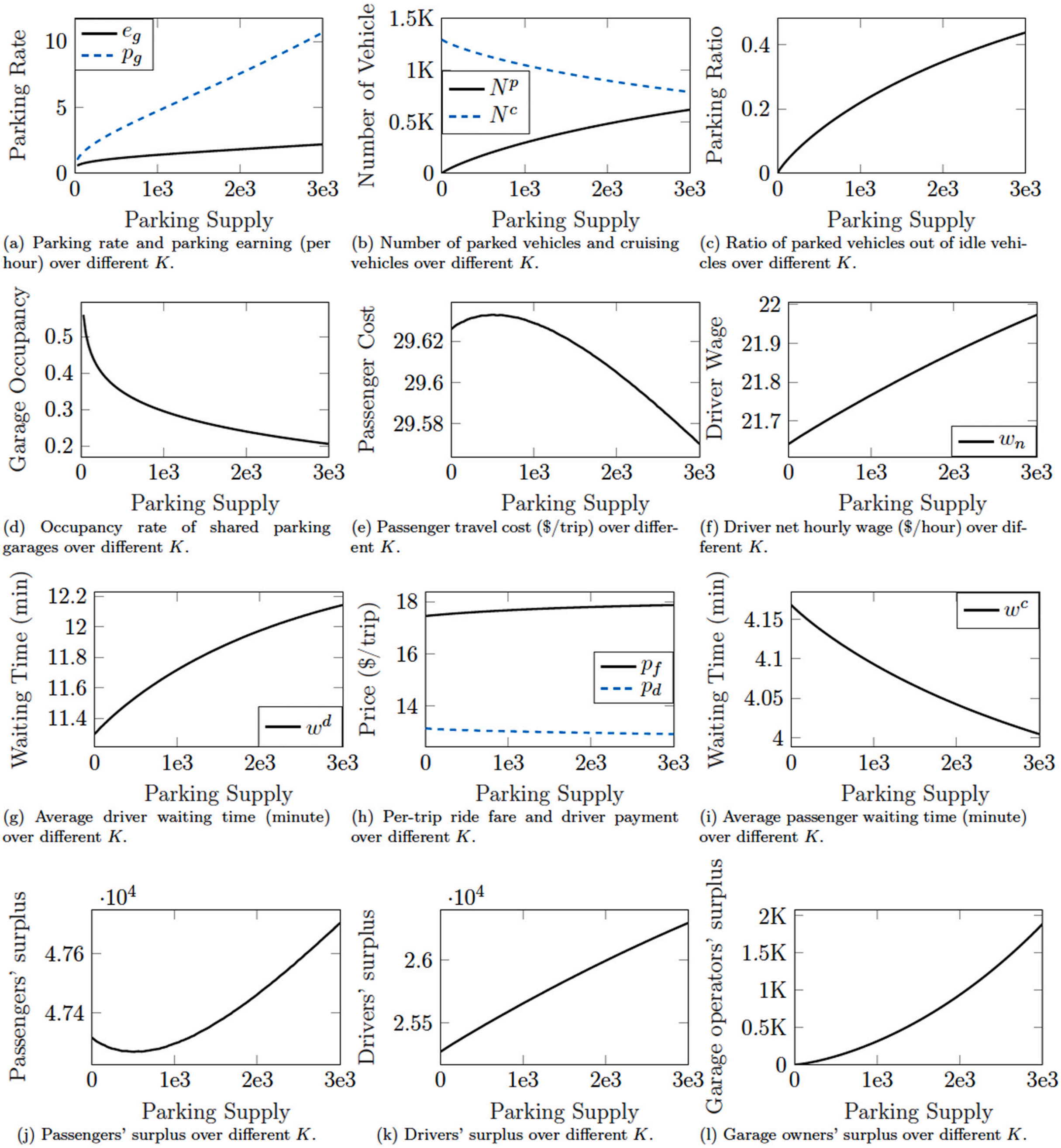
Since $w^d \geq \sqrt[3]{\frac{1}{4}} \lambda^{-\frac{1}{3}} A^{\frac{2}{3}}$, we have:

$$\lambda - \frac{A}{2}\lambda^{\frac{1}{2}} (w^d)^{-\frac{3}{2}} \geq \lambda - \frac{A}{2}\lambda^{\frac{1}{2}} \left(\sqrt[3]{\frac{1}{4}} \lambda^{-\frac{1}{3}} A^{\frac{2}{3}} \right)^{-\frac{3}{2}} = 0, \tag{58}$$

which indicates that $\frac{\partial w^d}{\partial N} \geq 0$. This completes the proof.

Appendix F. Market outcomes under B = 0.001 and $\beta_{1,2}$ = 0.75

See Fig. F.6.



(a) Parking rate and parking earning (per hour) over different K.
(b) Number of parked vehicles and cruising vehicles over different K.
(c) Ratio of parked vehicles out of idle vehicles over different K.
(d) Occupancy rate of shared parking garages over different K.
(e) Passenger travel cost ($/trip) over different K.
(f) Driver net hourly wage ($/hour) over different K.
(g) Average driver waiting time (minute) over different K.
(h) Per-trip ride fare and driver payment over different K.
(i) Average passenger waiting time (minute) over different K.
(j) Passengers' surplus over different K.
(k) Drivers' surplus over different K.
(l) Garage owners' surplus over different K.

Fig. F.6. Market outcomes over different parking supply under $B = 0.001$ and $\beta_{1,2} = 0.75$.

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